Merger stability in a three firm game

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Abstract

We compare different notions of stability in three firm merger games. We discuss some of their shortcomings and introduce an alternative notion of stability which overcomes them. The paper concludes with an illustrative example.

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1 Introduction

In the merger games literature, the identity of the merging firms and the stability of the post-merger market structure are dealt with in two different ways. On the one hand, some of the research is based on non-cooperative game theory. For instance, Kamien & Zang (1990) and (1993) consider a simultaneous game where each firm submits a bid for any rival and asks a selling price. Acquisitions will take place at the highest bid, provided that it exceeds the price the acquired firm asked for. A similar kind of game is presented in Fridolfsson & Stennek (2002), but instead of bidding for all firms, each firm chooses a rival with some probability and makes its bid. Other authors have modelled mergers as resulting from a sequential game, as in Gowrisankaran (1999), in which a size-related ranking for the different firms is exogenously defined. The first firm then decides if it wants to acquire any rival. If there is an acquisition,
the resulting firm will decide whether it wants to keep on acquiring other rivals. Otherwise, it is the second firm’s turn to decide and the process continues until the last firm makes a decision. Sequential merger games are also studied, for instance, in Bloch (1996) and Rodrigues (2001).

Along different lines, Barros (1998), Horn and Persson (2001) (henceforth H&P) and Socorro (2004) examine which mergers tend to be stable once they take place. These papers are focused not in the process that leads to one merger instead of another, but in defining which market structures and, hence, which mergers are stable. Although related, the stability concepts presented in these papers are somewhat different, implying different predictions over which mergers will be stable.

Our paper departs from theirs in that we propose a stability concept which overcomes the different shortcomings that can be identified in the stability concepts discussed in each of those papers. We then show how these different stability concepts can result in different predictions over which mergers will occur in a standard three-firm merger game.

2 Merger stability

To study stability in an $n$-firm merger game means to characterize the set of coalition structures (defined as partitions of the set of all players, i.e., firms), or ownership structures, immune to deviations leading either to the break up of any coalition or to the formation of alternative coalitions. We assume payoffs are shared between the merged firms through some ‘binding’ agreement between them, which is taken as costless for simplicity reasons. Our goal is to characterize the set of stable coalition structures, without analyzing possible bargaining processes leading to merger agreements between firms. Before proceeding we need to introduce some terminology.

Let us take the simple three firm case.$^1$ As most mergers involve only two firms, and monopolization is typically frowned upon, we will rule out from our analysis the grand coalition, i.e., a merger between all three firms. Let $\Pi_{m}^{i+j}$ denote the profit accruing to firm $m$ when firms $i$ and $j$ merge, for all $i,j$ in $\{1,2,3\}$ with $i \neq j$, and $m$ in $\{1,2,3\}$ or $m = i + j$, in which case $\Pi_{i+j}^{i+j}$ represents merger $i + j$ aggregate profits. Let $s_{i}^{i+j}$ in $[0,1]$ be firm $i$’s share of the aggregate profits from merger $i + j$. Naturally, in a two-firm merger case, $s_{i}^{i+j} = 1 - s_{j}^{i+j}$ (i.e., there is no free-disposal). Finally, let $\Pi_{i}$, for $i$ in $\{1,2,3\}$, denote the profits accruing to firm $i$ in the absence of any merger, i.e., in the no-merger coalition structure.

$^1$The three firm case has been used extensively in this literature, e.g. Barros [1998], Socorro [2004], Possajennikov [2001], Lommerud et al [2005] and Straume [2003].
Since we have ruled out the grand coalition, we only need to study the stability of coalition structures \{1, 2, 3\} and \{k, i + j\}, for \(k, i, j = 1, 2, 3\) and \(k, i, j\) all different. Let us first consider coalition structure \{k, i + j\} resulting from a merger between firms \(i\) and \(j\). Once firms \(i\) and \(j\) merge, firm \(k\) might have an incentive to break such merger by making an attractive enough offer to either of the insiders. This offer is defined as a share \(s_{i+k}^j\) (or \(s_{j+k}^i\)) of the joint profits \(\Pi_{i+k}^j\) (or \(\Pi_{j+k}^i\)), offered by \(k\) to \(i\) (or \(j\)). Because we rule out side payments between independent firms, it follows that firm \(k\) cannot pay \(i\) to leave \(j\) and stay on its own.

For any given coalition structure under consideration, an offer made by one firm to another is called admissible if the firm making such offer becomes weakly better off when the offer is accepted than under the status quo. Let \(O_{k\rightarrow i}\) denote the set of admissible offers made by \(k\) to \(i\). Set \(O_{k\rightarrow i}\) is then defined as follows:

\[
O_{k\rightarrow i} \equiv \{s_{i+k}^j \in [0, 1] : (1 - s_{i+k}^j) \Pi_{i+k}^j \geq \Pi_{k}^{i+j}\}\]

The highest admissible offer \(k\) will make to \(i\) is defined as \(s_{i+k}^j = 1 - \Pi_{k}^{i+j}/\Pi_{i+k}^j\). That is, \(s_{i+k}^j\) is the profit share offered by firm \(k\) to firm \(i\) such that \(k\)’s remaining profit share is the same as under the original coalition structure \{\(k, i + j\)\}.

A counter offer \(s_{i+j}^j\) made by firm \(i\)’s partner, namely firm \(j\), to firm \(i\), is said to be admissible if it makes firm \(j\) no worse off than under the alternative coalition structure \{\(j, i + k\)\}, i.e., the coalition structure formed after the break up of merger \(i + j\) and the merging of firms \(i\) and \(k\), leaving \(j\) as the outside firm. The set of all admissible counter offers is then defined as follows:

\[
C_{j\rightarrow i} \equiv \{s_{i+j}^j \in [0, 1] : (1 - s_{i+j}^j) \Pi_{i+j}^j \geq \Pi_{j}^{i+j}\}\]

The highest admissible counter offer made by firm \(j\) to firm \(i\) is defined by \(s_{i+j}^j = 1 - \Pi_{j}^{i+j}/\Pi_{i+j}^j\).

Finally, an offer \(s_{i+k}^{i}\) in \(O_{k\rightarrow i}\) made by firm \(k\) to firm \(i\) will be successfully countered by firm \(j\) if and only if:

\[
(\exists s_{i+j}^j \in C_{j\rightarrow i}) : s_{i+k}^j \Pi_{i+k}^j \leq s_{i+j}^j \Pi_{i+j}^j
\]

Otherwise, firm \(i\) will accept firm \(k\)’s merger proposal with an offered joint profit share equal to \(s_{i+k}^j\). In case of a draw, we assume the original coalition structure (i.e., the status quo) prevails.

2.1 Weak stability

A merger is said to be externally stable if and only if every admissible offer can be successfully countered. A merger is said to be internally stable if and only if it is profitable, i.e., if and
only if there is an internal gain. Finally, a merger is said to be stable if and only if it is both internally and externally stable. A coalition structure is said to be (weakly) stable if and only if it either contains a stable merger or it is the no-merger coalition and it is not profitable for any two firms to merge.

We show in a Proposition included in the appendix, that a coalition structure is (weakly) stable in the sense defined above, if and only if it is in the core as defined in H&P. More precisely, coalition structure \( \{ k, i + j \} \) is in the core if and only if the following conditions are simultaneously satisfied:

\[
\Pi_{i+j}^{i+j} + \Pi_{k}^{i+j} \geq \Pi_{i+k}^{i+k} + \Pi_{j}^{i+k} \quad (4)
\]
\[
\Pi_{i+j}^{i+j} + \Pi_{k}^{i+j} \geq \Pi_{j+k}^{i+k} + \Pi_{i}^{i+k} \quad (5)
\]
\[
\Pi_{i+j}^{i+j} \geq \Pi_i + \Pi_j \quad (6)
\]

On the other hand, the no-merger coalition structure \( \{ 1, 2, 3 \} \) is in the core if and only if the following inequalities hold: \( \Pi_{i+j}^{i+j} \leq \Pi_i + \Pi_j \), for all \( i, j = 1, 2, 3 \) and \( i \neq j \).

The above notion of weak stability states that for any given merger to be externally stable, it is enough that any admissible offer is successfully countered by an admissible counter offer. Employing terminology from conflict theory, this notion of external stability requires that any possible ‘attack’ be countered by a successful ‘defense’. Hence, it falls short of requiring that ‘there is a successful defense for all possible attacks’. This highlights what we regard as a main shortcoming of the (weak) stability concept and, hence, of the concept of core as introduced by H&P in the context of merger games, and used more recently in Straume (2003), Huck & Konrad (2003), and Lommerud et al. (2005). Namely, that there might not exist any way of both merged firms \( i \) and \( j \) splitting their joint profit so as to successfully counter both an offer to \( i \) and an offer to \( j \) made by the outside firm \( k \). This lack of robustness would imply the need for a constant revision of the way both inside firms share their joint profit as \( k \) made successive offers to either of them. However, if we were to adopt the requirement that ‘there is a successful defense for all possible attacks’, that would mean defining a set of admissible and successful counter offers against *all* admissible offers by the outside firm, avoiding the need for a constant revision of counter offers.
2.2 Strong stability

A coalition structure \( \{k, i+j\} \) will be called strongly stable, if and only if the following condition holds:

\[
(\exists s_i^{i+j} \in C_{j-i}) \ (\exists s_j^{i+j} \in C_{i-j}) \ (\forall s_i^{i+k} \in O_{k-i}) \ (\forall s_j^{i+k} \in O_{k-j}) : s_i^{i+j} = (1 - s_j^{i+j}) \ ; \ s_i^{i+k} \Pi^{i+k} \leq s_i^{i+j} \Pi^{i+j} \ ; \ s_j^{i+j} \Pi^{i+j} \leq s_j^{i+k} \Pi^{i+k} ; \ s_i^{i+j} \Pi^{i+j} + s_j^{i+k} \Pi^{i+k} \geq \Pi_i ; \ s_j^{i+j} \Pi^{i+j} + s_i^{i+k} \Pi^{i+k} \geq \Pi_j.
\]

By using the notions of ‘highest admissible’ offer and counter offer, the above condition is equivalent to the following one: \((\exists s_i^{i+j} \in (0, 1))\) such that the following set of double inequalities is satisfied:

\[
\Pi_i^{i+j} \leq s_i^{i+j} \Pi^{i+j} \leq s_i^{i+k} \Pi^{i+k} \leq \Pi_j^{i+k}
\]

(7)

\[
\Pi^{i+k} - \Pi_k^{i+j} \leq s_i^{i+j} \Pi^{i+j} - s_i^{i+k} \Pi^{i+k} + \Pi^{i+j} \leq \Pi_i^{i+j}
\]

(8)

\[
\Pi_i \leq s_i^{i+j} \Pi^{i+j} \leq \Pi_j^{i+j} - \Pi_j
\]

(9)

On the other hand, and similarly to the case of weak stability, the no-merger coalition \( \{1, 2, 3\} \) will be called strongly stable if and only if it is not profitable for any two firms to merge. Whether or not the set of strongly stable coalition structures is non-empty in any given three firm merger game, depends on the values attained by the different profit levels composing those double inequalities and the different firms’ profit levels as singletons. Finally, weak stability is a necessary but not sufficient condition for strong stability.

Requiring the existence of a ‘successful defense against all possible attacks’ is not new in the literature. In Barros, merger stability requires that “the firm external to the merger cannot offer to either of the firms participating in the merger a more profitable alternative.” (see op. cit., p. 115). In the subsequent analysis of merger stability, he requires the existence of a profit sharing profile such that the merger is mutually advantageous, i.e., profitable for the insiders, and that both insiders, when comparing their individual gain within the merger to what the outsider can offer, have an incentive to simultaneously reject any outside offer. This amounts to requiring the existence of a ‘defense against all admissible attacks,’ as described above, and corresponds to the fulfilment of the double inequalities (8) and (9) by the profit sharing profile \( s_i^{i+j} \). The fulfilment of the double inequality (7) is conditional on the creation, by the merger, of a negative payoff externality on the outsider.\(^2\) If this is the case, then double inequality (9) implies double inequality (7) and our notion of strong stability will be equivalent to the notion of merger stability introduced in Barros (and discussed in Possajennikov). However, in the

\(^2\)We are in the presence of a negative (positive) payoff externality for the outsider \( k \) if \( \Pi_k^{i+j} < (>) \Pi_k \).
presence of a positive payoff externality on the outsider, such implication does not hold. This means that, by not including this latter double inequality in the definition of merger stability, one cannot guarantee that any of the two insiders will have an incentive, even though it will have the means, to successfully counter an offer made by the outsider to his partner in the merger. The possible lack of incentive would follow from the higher profit level an insider could guarantee for itself by letting its partner accept the outsider’s offer. Hence, by requiring the verification of the double inequality (7), we guarantee that such an incentive is in place, implying that the set of strongly stable coalition structures in a three firm merger game is smaller, possibly strictly smaller, than the set of stable coalition structures in Barros.

Finally, note that stability a la Barros does not imply weak stability. In fact, when there is a positive externality, Barros does not require successful counter offers to be admissible, whereas under weak stability only admissible counter offers are considered. In this sense, weak stability is more demanding than stability a la Barros. On the other hand, because the latter notion of stability requires the existence of a profit sharing rule that can successfully counter any outside offer, it is more demanding than weak stability. Summing up, strong stability implies both stability a la Barros as well as weak stability, but these two notions of stability are not comparable with each other, except when there is a negative externality.

3 A comparative example

Consider a Cournot three firm oligopoly, where firms differ in their productive efficiency levels, as in Barros and in Possajennikov, in that \(0 \leq c_1 < c_2 < c_3\), where \(c_i\) denotes firm \(i\)'s constant marginal constant. In particular, assume, as in Barros, that \(c_3 = c_2 + \delta = c_1 + 2\delta\), for some \(\delta > 0\). Assume also that the market inverse demand function is given by \(P(Q) = 1 - Q\), where \(Q = q_1 + q_2 + q_3\), with \(q_i\) denoting firm \(i\)'s output level. Finally, assume that \(c_1 = 0\) and that \(\delta \leq 0.2\) so that all firms have positive equilibrium market shares. It is straightforward to characterize the set of ‘stable’ coalition structures as a function of \(\delta\), using (i) the concept of weak stability presented above; (ii) the alternative concept of strong stability; and, finally, (iii) the concept of stability as in Barros and in Possajennikov. For simplicity we will focus our analysis on the stability of the merger between firms 1 and 2. Figure 1 indicates the values of \(\delta\) for which this merger is stable under the alternative notions of stability.

The particular case of \(\delta = 0.15 \in [0.1333, 0.1716]\) helps highlight the differences between them. Pre-merger payoffs for the three firms are \(\Pi_1 = 473.06\), \(\Pi_2 = 162.56\) and \(\Pi_3 = 14.063\)

\footnote{Such is the case in e.g. Salant et al (1983) and Perry and Porter (1985).}
while the post-merger profits are those in the following table:\(^4\)

<table>
<thead>
<tr>
<th></th>
<th>Merger 1+2</th>
<th>Merger 1+3</th>
<th>Merger 2+3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insiders</td>
<td>(\Pi_{1+2}^{1+2} = 676)</td>
<td>(\Pi_{1+3}^{1+3} = 529)</td>
<td>(\Pi_{2+3}^{2+3} = 196)</td>
</tr>
<tr>
<td>Outsider</td>
<td>(\Pi_3^{1+2} = 64)</td>
<td>(\Pi_2^{1+3} = 196)</td>
<td>(\Pi_1^{2+3} = 529)</td>
</tr>
</tbody>
</table>

As Figure 1 illustrates, merger 1+2 is in the core and it is also stable according to the Barros’ stricter notion of stability. What exactly does this mean? Firstly, that profits may be shared in such a way that both firms 1 and 2 are better off after the merger than before: this happens if and only if their profit sharing agreement is such that \(69.98\% \leq s_{1+2}^{1+2} \leq 75.95\%\). Also, even if firm 3 made the highest possible offers to both firms 1 and 2, these firms could still divide their aggregate profit in such a way that no firm would choose to accept firm 3’s offer. To make things clear, firm 3 is making a profit of only \(\Pi_3^{1+2} = 64\) if firms 1 and 2 do not merge. In order to entice firm 1 to break up this merger and merge with firm 3 instead, this firm is willing to offer firm 1 up to \(\Pi_{1+3}^{1+3} - \Pi_3^{1+2} = 465\) of the aggregate profit of \(\Pi_{1+3}^{1+3} = 529\). If, on the other hand, the offer is made to firm 2, then firm 3 is willing to make an offer of up to \(\Pi_{2+3}^{2+3} - \Pi_3^{1+2} = 132\). It is possible that firms 1 and 2 share their profits in such a way that neither of these proposals is successful: As long as firm 1 receives more than 465 and firm 2 receives more than 132, that is, as long as \(68,79\% \leq s_{1}^{1+2} \leq 80.47\%\), both firms will divide their merger profits in such a way that they are both better off with the current merger than with the highest possible profit share in the alternative merger with firm 3.\(^5\) The question is: will they make such division?

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\(^4\)For numerical convenience, all payoffs have been multiplied by 3600.

\(^5\)As any \(69.98\% \leq s_{1}^{1+2} \leq 75.95\%\) verifies both conditions, firms 1 and 2 may divide profits in such a way that they are both better off with the current merger than with the alternative merger or with the no-merger case.
Consider that an offer is made to firm 2. Firm 1 may beat this offer, as we have seen. But a merger between firms 2 and 3 creates a positive externality on firm 1 (instead of \( \Pi_1 = 473.06 \), it will have a profit of \( \Pi_1^{2+3} = 529 \)). This happens because a merger, in this setting, merely removes the least efficient of the insiders, which benefits the outside firm due to the reduction in the number of competitors. Hence, in order to stay with firm 2, firm 1 will not be willing to forego as large a share of the profit as it would if the no-merger scenario was the relevant alternative. In fact, it will not accept to receive less than \( \Pi_1^{2+3} = 529 \), i.e., \( s_1^{1+2} \geq 78.25\% \).

But this leaves firm 2 with at most \( \Pi_1^{1+2} - \Pi_1^{2+3} = 147 \) which is clearly unacceptable: firm 2 would then prefer to stay alone. Note that if the merger involved a negative externality on the outsider, a possibility in the presence of substantial synergies for the insiders, both notions would be equivalent: the fact that the profit sharing rule lead both insiders to have a higher payoff than in the no-merger case is sufficient for both of them to prefer to stay merged, rather than being outsiders to an alternative merger.

4 Concluding remarks

Different concepts of stability have been used in the recent literature on merger games, leading to possibly different conclusions on which mergers are likely to occur. The present note highlights some of the main differences between these concepts and their shortcomings in a standard three firm case. Within this context, we propose an alternative definition of merger stability, somewhat stronger than two main stability concepts used in the literature, by arguing that it is not enough to require that each possible offer targeting any of the two merger insiders by the outside firm be met by a successful counter offer from the non-targeted insider. Rather, we argue that one ought to require the existence of a joint profit sharing profile that can be successfully employed by any of the insiders as a counter offer to any admissible offer made by the outside firm. Moreover, we require that any of the insiders will have an incentive to make such a counter offer. This requirement is particularly relevant in the presence of a positive payoff externality on the outsider. By way of a comparative example, we show how the concept of strong stability leads to different results from the ones used in the recent literature on merger games.
Appendix

Proposition 1  In a merger game with three players, a coalition structure belongs to the core if and only if it is weakly stable.

Proof. We will start by showing that a coalition structure containing a merger and belonging to the core is weakly stable. W.l.o.g. assume such merger is between firms $i$ and $j$. If the corresponding coalition structure $\{k, i + j\}$ is in the core, then conditions (4), (5) and (6) above must be simultaneously verified.

Therefore, by condition (6), this merger is internally stable. We will now assume that this merger is not externally stable. This means that there exists an admissible offer made by firm $k$, to either firm $i$ or firm $j$, for which there is no admissible counter offer which is successful. Assuming, w.l.o.g., that firm $k$’s offer is extended to firm $i$, the following must then be true:

$$\exists s_{i+k}^i \in O_{k-i} \ (\forall s_{i+j}^i \in C_{j-i} ) : s_{i+k}^i \Pi_{i+k}^i > s_{i+j}^i \Pi_{i+j}^i$$

In particular it must be true that such an offer made by firm $k$ beats the highest admissible counter offer $s_{i+j}^i$ made by firm $j$; that is:

$$\exists s_{i+k}^i \in O_{k-i} : s_{i+k}^i \Pi_{i+k}^i > (1 - \Pi_{i+j}^j / \Pi_{i+j}^j) \Pi_{i+j}^j$$

Given that firm $k$’s offer is admissible, we know that $s_{i+k}^i \Pi_{i+k}^i \geq \Pi_{i+j}^i$. Adding these last two inequalities yields:

$$\Pi_{i+k}^i > (\Pi_{i+j}^i - \Pi_{i+j}^k) + \Pi_{i+j}^i$$

$$\iff \Pi_{i+k}^i + \Pi_{i+j}^j > \Pi_{i+j}^i + \Pi_{i+j}^k$$

But this contradicts condition (4). Hence, such merger is externally stable. Therefore, coalition structure $\{k, i + j\}$ is weakly stable.

We will now show that a coalition structure containing a merger and which is weakly stable belongs to the core. Again, and w.l.o.g., take coalition structure $\{k, i + j\}$. If merger $i + j$ is internally stable, it must be that:

$$\Pi_{i+j}^i \geq \Pi_i + \Pi_j.$$

External stability means that all of firm $k$’s admissible offers to either $i$ or $j$, have at least one successful admissible counter offer. Once more, assume, w.l.o.g., that an admissible offer
targets firm $i$. Then:

$$(\forall s_i^{i+k} \in O_{k-i}) \ (\exists s_i^{i+j} \in C_{j-i}) : s_i^{i+k} \Pi_{i+k}^{i+k} \leq s_i^{i+j} \Pi_{i+j}^{i+j}$$

In particular, there is an admissible counter offer by firm $j$ that beats the highest possible admissible offer $s_i^{i+k}$ made by firm $k$:

$$\exists s_i^{i+j} \in C_{j-i} : (1 - \Pi_{i+k}^{i+j}/\Pi_{i+i+k}^{i+j}) \Pi_{i+i+k}^{i+k} \leq s_i^{i+j} \Pi_{i+i+k}^{i+j}$$

$$\iff (\exists s_i^{i+j} \in C_{j-i}) : \Pi_{i+i+k}^{i+i+k} - \Pi_{i+i+k}^{i+j} \leq s_i^{i+j} \Pi_{i+i+j}^{i+j}$$

(14)

But, as $s_i^{i+j} \in C_{j-i}$ it must be that $(1 - s_i^{i+j}) \Pi_{i+i+j}^{i+i+j} \geq \Pi_{i+i+k}^{i+i+k}$.

Adding these two inequalities yields:

$$\Pi_{i+i+j}^{i+i+j} + \Pi_{i+i+k}^{i+i+k} \geq \Pi_{i+i+k}^{i+i+k} + \Pi_{i+j+k}^{i+j+k}$$

(15)

If firm $k$ targeted firm $j$ instead of $i$ we would get:

$$\Pi_{j+i+k}^{j+i+k} + \Pi_{j+i+k}^{j+i+j} \geq \Pi_{j+i+k}^{j+i+k} + \Pi_{i+i+k}^{i+i+k}$$

(16)

Conditions (13), (15) and (16) are the same as conditions (4), (5), and (6). Hence, coalition structure $\{k,i+j\}$ belongs to the core.

In order to complete the proof, we must show that the no-merger coalition structure $\{1,2,3\}$ is in the core if and only if it is weakly stable. This equivalence is immediate by recalling that such coalition structure is in the core if and only if the following inequalities hold: $\Pi_{i+i+j}^{i+i+j} \leq \Pi_i + \Pi_j$, for all $i,j = 1,2,3$ and $i \neq j$. □
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