Investment and Welfare Implications of the Ownership Structure of Overlapping Networks*

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Abstract

We analyze the impact of the ownership structure of cable television firms on the incentives to upgrade the cable networks to offer telecommunication services. First, we show that dual ownership of a local telephone network and a cable network, compared with separate ownership, may increase or decrease incentives to invest in upgrading the cable television network. Coordination economies benefit dual ownership, and business-stealing benefits separate ownership. Second, we perform a welfare analysis of the investment decision and third, a welfare analysis of the ownership structure.

Key Words: Cable Networks, Local Access, Competition

JEL Classification: L43, L96

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1 Introduction

The Telecommunications Act of 1996 in the US, and the liberalization of the telecommunications industry in the EU, e.g., Directives 90/388/EEC and 96/19/EC, promoted entry of new firms into the local access market through: (i) resale of the incumbent’s services, and (ii) unbundling of the incumbent’s local loop. Removing the high entry barriers in the local access market, associated with scale, density, and scope economies, would give entrants time to develop their customer base and to build their own infrastructures.

However, these two alternatives rely on the Open Network Provision, e.g., the Access Directive 2002/19/EC, which is hard to enforce. Besides, the two alternatives, particularly resale, considerably constrain the entrants’ marketing options. Either due to the incumbent’s obstruction or due to its intrinsic limitations, resale and the unbundling of the local loop has so far produced very modest results.

Facilities-based entry in local access markets for residential clients is probably unprofitable, unless the network is used to provide other services. In this respect, networks for the distribution of cable television services, Cable Networks for short, can play an important role.\textsuperscript{1} Cable networks have three characteristics that allow them to compete in the short run with the public switched telephone network. First, cable networks can be upgraded to offer interactive services, like fixed telephony or broadband access to the Internet. Second, upgrading an existing network to carry a return signal is faster and cheaper than deploying a new telecommunications network.\textsuperscript{2} Third, in many countries the cable networks deployed offer almost complete coverage.\textsuperscript{3}

The Telecommunications Act of 1996 recognized the importance of cable networks for providing an alternative infrastructure to the incumbent’s local access network. And it also recognized a potential conflict of interests. Section 302 imposes structural separation between firms

\textsuperscript{1}See Brito and Pereira (2007) for a discussion of the policy issues related to the importance of cable networks in promoting facilities based entry.

\textsuperscript{2}There are many ways of endowing a cable network with bidirectionality, ranging from extensively overhauling the network, to deploying copper wire pair intertwined with the coaxial cable.

\textsuperscript{3}The example of the UK is revealing. After seven years of a legal duopoly, 1984-1991, Mercury, the rival of British Telecom, gained a 1% market share. In 1991, cable television firms were allowed to offer fixed telephone services. In 1992, British Telecom had a revenue market share of 99.2% for UK geographic calls. In the second quarter of 2003, cable companies NTL and Telewest had, jointly, a revenue market share of 13.4% for UK geographic calls, whereas British Telecom had 65.7. These numbers are remarkable since cable networks only pass around half of the households in the UK. The data on NTL is from the firm’s "Quarterly Report", the data on Telewest is from ECCA’s "Web Site", and the data on market shares is from OFCOM’s "Market Information". 

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that own local telecommunications networks and firms that own cable networks. The situation in Europe is more ambiguous. The European Commission indicated in the Cable Directive 95/51/EC, Article 2, and in the Communication 98/C 71/04, a preference for structural separation. However, the legislation of the European Community, namely Directive 2002/77/CE, Article 8, only imposed legal separation.

The Commission’s preference for structural separation, stated in Directive 99/64/EC, preamble, paragraph 10, is based on the argument that a firm that owns both networks has no incentive to upgrade the cable network. The investment in the cable network is unlikely to generate additional net revenues; it will only redirect revenues. For an independently owned cable television firm there is no conflict of interests. Telecommunication services will attract new clients.

This perspective of the Commission seems to be based on two beliefs. The first belief is that the self-cannibalization effect will reduce the incentives of a firm that owns both networks to upgrade the cable network. However, there are other factors, such as coordination economies in the joint operation of the two networks, that can mitigate, or overcome, the impact of this effect and give a firm that owns both networks more incentive to upgrade the cable network.

The second belief of the Commission is that upgrading of the cable network is socially desirable under both ownership structures. This is unclear because separate ownership leads to more intense competition on the traditional services.\(^4\) However, lower prices for these products inhibit the adoption of the new services enabled by the upgrade of the cable network, which are more valued by consumers and possibly cheaper to produce than are the traditional services. Furthermore, it is also unclear whether it is cheaper to produce the new services using only the upgraded cable network, or using the upgraded network and the telephone network jointly.

To analyze the impact of the ownership structure of cable television firms on the incentives to upgrade cable networks to offer telecommunication services, we have developed a model in which there are two networks: (i) a telephone network, and (ii) a cable television network. The telephone network belongs to the telecommunications firm and provides fixed telephony services. The cable network belongs to the cable firm and provides subscription television services. However, the cable network can be upgraded to provide additional services. Of these additional services we will focus on telephone services because they have been at the center of policy debates. But much of our article applies to other services, such as broadband access to

\(^4\)Pereira and Ribeiro (2007b) used a rich consumer level database to evaluate the impact on the market of broadband access to the Internet of the structural separation of the telephone and cable television networks of the Portuguese telecommunications incumbent. They found that the structural separation may lead to substantial prices reduction in the order of 14%.
The two firms may be owned separately, or dually, through a holding company. We assume that the holding company behaves like a multiproduct monopolist, and that firms do not engage in price discrimination. There are scope economies in the joint provision of subscription television services and telephony services, i.e., a bundle, over the cable network. In addition, there are coordination economies in the dual ownership of the two networks. Scope economies refer to the cost benefits of producing the two services jointly, rather than separately, and coordination economies refer to the cost benefits of operating the two networks jointly, rather than separately.

We find that a firm which owns only a cable network may or may not have more incentives to invest in upgrading the network than a firm that owns both networks. The investment generates four effects. Both firms benefit from scope economies, and from the consumers having a higher valuation for the bundle than for the two services separately. The trade-off is between the two remaining effects; the holding company benefits from coordination economies with respect to the bundle, and the independently owned cable firm benefits from stealing business from the telecommunications firm. The balance between these effects determines which firm has more incentive to upgrade the cable network. But the net effect is potentially ambiguous. The absence of coordination economies is a sufficient condition for the independent firm to have more incentive to upgrade the network, while the absence of the business stealing effect is a sufficient condition for the opposite to occur.

The welfare analysis shows that either ownership structure may generate the largest incremental welfare benefit from upgrading the cable network. In addition, both ownership structures may lead to socially inefficient investment, where, in particular, the cable network is not upgraded.

Either ownership structure may be socially optimal, even if one restricts the welfare criterion to consumer surplus. Despite the assumption that the holding company benefits from coordination economies in the production of subscription television services and the bundle, separate ownership may be the optimal ownership structure. This may occur if the cable network is upgraded only under this type of ownership structure, and if coordination economies and the upgrading costs are sufficiently low. Upgrading the cable network is desirable because it allows the introduction of the bundle, and allows scope economies.

There is a lack of clarity regarding which ownership structure: (i) provides more incentives to upgrade the cable network, (ii) generates the larger incremental welfare benefit from upgrading the cable network, (iii) and is socially optimal. This indeterminacy can only be resolved through
empirical analysis aimed at determining the relative magnitudes of the opposing effects involved.

This article relates to the literature on the relative advantages of the various forms of entry into the telecommunications industry and the literature on intermodal competition. Regarding the first literature strand, Faulhaber (2003) analyzes regulatory initiatives to open segments of the telecommunications market to competition and mentions cable networks as a viable alternative to the incumbents’ local access telecommunications network. Bourreau & Dogan (2004, 2005), using a dynamic model of technology adoption, study the incentives of an entrant to lease the incumbent’s local loops and compete “service-based”, or to build a more efficient infrastructure and compete “facilities-based”. They show that the incumbent can delay the entrant’s adoption of new superior technology by setting low rental prices for the local loops. Dessein (2004) considers competition between two established horizontally differentiated networks and shows how customer heterogeneity affects nonlinear pricing strategies. Regarding intermodal competition, Loomis & Swann (2005) develop and estimate a model of local competition. They find that there is substantial competition between incumbents and entrants using wireless and high-speed services.\footnote{For a discussion of optimal pricing with intermodal competition, see Braeutigam (1979).} Finally, although we use a model of a multiproduct monopolist, note that our setting has several nonstandard features, such as scope and coordination economies.

The paper is organized as follows: Section 2 presents the model, section 3 characterizes the equilibria, Section 4 conducts the analysis and section 5 concludes our study. All proofs are in the appendix.

## 2 The Model

### 2.1 Environment

Consider a telecommunications industry where there are two firms: the Telecommunications Company and the Cable Company. The telecommunications company owns a public switched telephone network, PSTN, which provides fixed telephony services, denoted by \( f \). The cable company owns a cable network, \( CN \), which provides subscription television services, denoted by \( t \). If upgraded and endowed with bidirectionality, the \( CN \) also provides fixed telephony services, denoted by \( \phi \), and a bundle of fixed telephony services and subscription television services, denoted by \( b \). We index the products with subscript \( j = f, t, \phi, b \).

There are two possible ownership structures for the two companies: (i) Separate Ownership,
and (ii) Dual Ownership, through a Holding Company. We denote $i$, the independently owned cable company, and $h$, the cable company owned by the holding company. We index the two types of cable companies with superscript $k = i, h$.

The game has two stages, which unfold as follows under both ownership structures. In stage 1, the cable company decides whether or not to upgrade the CN. In stage 2, both companies choose prices simultaneously.

2.2 Consumers

There is a large number of consumers, formally, a continuum, normalized to measure 1. All consumers have access to both networks.

Consumers view telephony services provided through the PSTN and the CN as perfect substitutes. We denote the consumers’ valuation for fixed telephony and for subscription television services by $v_f$ and $v_t$, respectively, and denote the consumers’ valuation for the bundle by $v_b := v_f + v_t + \theta$, where $\theta$ is a parameter uniformly distributed on $[0, 1]$. The higher valuation for the bundle, which differs among consumers, is due to the convenience of interacting with a single firm, i.e., signing a single contract, paying a monthly bill, calling one maintenance service, etc.\(^6\) When indifferent between buying and not buying the bundle, consumers choose the former.

We abstract from network effects since, typically, the PSTN has interconnection obligations.

2.3 Firms

Upgrading the CN involves a fixed cost, $\varphi$ on $(0, +\infty)$, independently of the ownership structure of the cable company.\(^7\) All players observe whether the CN is upgraded. We denote by $u$ the case where the CN was upgraded, and by $n$ the case where it was not. We index these two cases with superscript $z = u, n$.

The marginal production costs are constant for the four products. We denote by $c_f$ and $c_\varphi$, the marginal cost of telephony services in the PSTN and the upgraded CN, respectively. Let $c_f = c_\varphi$. For subscription television and for the bundle, we distinguish between the marginal

\(^6\)Cooper (2003) provides evidence that, following the introduction of new advanced services, cable television firms were able to raise their fees by more than the increase in capital expenditures required to make these services available.

\(^7\)If the holding company has easier access to funding than the independently owned cable company, because it is a larger firm, or because it has been in the market for a longer time, it could have a lower cost of upgrading the CN.
cost of an independently owned cable company, and a cable company owned by the holding company. We denote by \( c^k_j \) on \([0, +\infty)\), the marginal cost of product \( j \) produced by cable company \( k \), with \( j = t, b \), and \( k = i, h \).

There are Coordination Economies in the dual ownership of the two networks. Coordination economies stem from the cable company integrating the CN with the PSTN, e.g., sharing resources, if it is owned by the holding company.\(^8\) This means that a cable company will have lower marginal costs if it is owned by the holding company, than if it is owned independently. More specifically: (i) \( c^h_t := c^i_t - \delta_t \), and (ii) \( c^h_b := c^i_b - \delta_b \). Parameter \( \delta_t \) on \([0, c^i_t)\) captures coordination economies with respect to cable television services, and parameter \( \delta_b \) on \([0, c^i_b)\) captures coordination economies with respect to the bundle.

There are Scope Economies in the joint provision of subscription television services and fixed telephony services over the CN. Scope economies stem from joint marginal costs of offering multiple services over the same network.\(^9\) This means that the production cost of the bundle on the CN is no larger than the sum of the production costs of subscription television services and telephony services. More specifically: \( c^k_b := c_f + c^k_t - \gamma \), where parameter \( \gamma \) on \([0, c_f + c^i_b - \delta_b)\) captures scope economies.

The owner of the upgraded CN can offer telephony services separately, i.e., can practice mixed bundling. Value \( c^i_b + \gamma_m \) is the unit cost of producing subscription television and telephony services separately on the CN, where parameter \( \gamma_m \) on \((0, \gamma)\) captures the diseconomies of producing these two services separately, instead of in a bundle, over the CN.

The cost of producing the bundle is lower than the cost of producing the two services separately through the CN, and the latter is lower than the cost of producing the two services through different networks: \( c^h_b < c^i_b + \gamma_m < c^i_b + \gamma = c_f + c^i_t \). Producing subscription television and telephony services separately on the same network involves scope economies, but of smaller magnitude than those of producing both services in a bundle. This is a reasonable assumption given that, if both services are produced separately, there will be a duplication of some of the

\(^8\) There is a duplication of resources between the two networks, e.g., the local loops, switches, or backbone networks. If such resources are managed jointly, they can be used more efficiently. And if both networks are digitalized then, bandwidth considerations aside, either network can carry the two types of traffic. This allows the optimization of the traffic flow over the two networks.

\(^9\) Pereira and Ribeiro (2007b) found scope economies between cable television and broadband access to the Internet through cable modem. According to Cluny (1995) for a multiple services operator, about 10% of its operating costs are incremental to subscription television, 20% to telephone and 70% or more are non-attributable common costs. See also Woroch (1997) for a description of the technological advances that allow scope economies between video and voice services.
activities required to produce the services.

We assume that for the holding company, producing fixed telephony services and subscription television services separately using both networks is cheaper than producing both services separately using the upgraded CN: \( c_f + c_h^t < c_f + c_t^i - \gamma + \gamma_m \), i.e., \( \delta_t > \gamma - \gamma_m \).

Table 1 summarizes the cost structure.

\[ \text{Table 1} \]

To simplify the notation, we define the monopoly profit margins for telephony and cable television as: \( \mu_f := v_f - c_f \), and \( \mu_t^k := v_t - c_t^k \), with \( k = i, h \). The following assumptions on the parameters values close the model.

C: Let: (i) \( c_f < v_f \), (ii) \( c_t^i < v_t \), (iii) \( \delta_b + \gamma < 1 \).

Assumptions C: (i)-(iii), and the assumptions on the cost structure imply that: \( c_h^b < c_i^b < v_b \).

Assumption C: (iii) ensures that under dual ownership, some consumers purchase the separate components instead of the bundle.

The holding company behaves like a multiproduct monopolist.\(^{10}\) Firms do not engage in price discrimination, i.e., firms charge all consumers the same price for identical services. Denote the price of product \( j = f, t, \phi, b \) by \( p_j \).

The stage 1 strategy of the cable company is an investment decision of whether to invest, given the ownership structure. A firm’s stage 2 strategy is a pricing rule, \( p_j^* \), that states, for each of its products, which price a firm should charge, given the investment decision of stage 1, and given the ownership structure.

### 2.4 Equilibrium

A subgame perfect Nash equilibrium in pure strategies is: (i) an investment decision for the cable company, and (ii) a set of pricing rules for each product available in the market, such that for each ownership structure:

(E1) each firm chooses its pricing rule to maximize profit, given the rival’s pricing rule, and given the stage 1 investment decision;

\(^{10}\)Perfect coordination among legally separate, jointly owned firms, may be hard to achieve as it is difficult to align the incentives of members of legally separate firms. However, casual empiricism suggests that some level of coordination is possible. And typically, this is enough for firms to promote their common interests.
(E2) the cable company chooses an investment decision to maximize profit, given the stage 2 pricing rules.

3 Characterization of the Equilibrium

In this section, we characterize the game’s equilibria, which we solve by working backward for the two alternative ownership structures. For \( z = u, n \), we denote by \( \pi_{j}^{z,k} \), the profit of service \( j = f, t, \phi, b \) for cable company \( k = i, h \), and by \( CS^{z,k} \), the consumer surplus for the products of firm \( k \).

3.1 Stage 2: The Pricing Game

3.1.1 Non–Upgraded CN

The next Lemma presents the price equilibrium.

**Lemma 1:** In equilibrium, if the CN was not upgraded, then the telecommunications and the cable companies charge, respectively: (i) \( p_f^* = v_f \), and (ii) \( p_t^* = v_t \).

The price equilibrium is simple, and is the same under both ownership structures. The telecommunications and the cable companies are monopolists and charge their monopoly prices.\(^{11}\)

Using Lemma 1, the equilibrium profits of the telecommunications and cable companies when the CN was not upgraded are, respectively: \( \pi_f^n = \mu_f \), and \( \pi_t^{n,k} = \mu_t^k \) for \( k = i, h \). Due to coordination economies, the holding company has larger profits from selling subscription television services than the independently owned cable company, \( \mu_t^h > \mu_t^i \). The consumers’ surplus, \( CS^{n,k} \), is trivially zero.

3.1.2 Upgraded CN with dual Ownership

We denote by \( \Phi(\cdot) \), the uniform cumulative distribution function with support on \([0, 1]\). The following demand functions apply both to the cases of dual and separate ownership. To simplify the notation, assume initially that \( p_f < p_\phi \). The demand for telephony services over the PSTN,

\(^{11}\)The holding company with a non-upgraded CN does not offer the bundle for two reasons: First, it seems unlikely that two legally separated firms could offer consumers the benefits described above. Second, there are no scope economies if each service is provided over a different network. Hence, bundling the services or selling them separately is the same as far as consumer valuation and marginal costs are concerned.
and for the subscription of television services over the CN is: $D^n_j(p_f, p_t, p_b) = \Phi(p_b - p_t - p_f)$, with $j = f, t$. The demand for the bundle is: $D^n_b(p_f, p_t, p_b) = 1 - \Phi(p_b - p_t - p_f)$. The case $p_f \geq p_\phi$ is straightforward.\(^{12}\)

The assumption that the holding company behaves like a multiproduct monopolist implies that it maximizes joint profits, given by: $\pi_{f}^{u,h} + \pi_{t}^{u,h} + \pi_{b}^{u,h} = \sum_{j=f,t,b}(p_j - c_j)D^n_j$.\(^{13}\)

The next Lemma presents the price equilibrium.

**Lemma 2:** In equilibrium, under dual ownership, if the CN was upgraded, then the holding company charges: (i) $p^*_f = v_f$, (ii) $p^*_t = v_t$, and (iii) $p^*_b = c_b^h + \mu_t^h + \mu_f + \frac{1+\delta_b+\gamma}{2}$.

The price equilibrium is again, simple. The holding company sets $p_f$ and $p_t$ at their monopoly levels, extracting all the surplus of consumers who do not purchase the bundle, while at the same time increasing the demand for the bundle. Assume initially that the holding company sets $p_b$ below $v_t + v_f$. Since the bundle is less expensive than the separate components, all consumers purchase the bundle. An increase in $p_b$ until $v_t + v_f$ is hence profitable, since no consumer will switch to the separate components. As $p_b$ increases above $v_t + v_f$, the bundle becomes increasingly expensive, and consumers with lower bundle valuation will switch to the separate components. Hence, there is the usual trade-off between profit margin and volume of sales. Given assumption C: (iii), it is more profitable for the holding company to sell the bundle at a price above $v_t + v_f$, and to have only a fraction of consumers purchase the bundle.\(^{14}\)

The next Corollary gives the equilibrium profits.

**Corollary 1:** In equilibrium, under dual ownership, if the CN is upgraded, then the holding company has profits: $\pi_{f}^{u,h} + \pi_{t}^{u,h} + \pi_{b}^{u,h} = \mu_f + \mu_t^h + \frac{(1+\delta_b+\gamma)^2}{4} - \varphi$. \(\blacksquare\)

The increase in the profits of the holding company from offering the bundle after the upgrade consists of the increase in the profit margin, $p_b^* - c_b^h - \mu_t^h - \mu_f = \frac{1+\delta_b+\gamma}{2}$, times the proportion

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\(^{12}\)One merely has to replace $p_f$ by $p_\phi$. Given that $f$ and $\phi$ are perfect substitutes, consumers only demand the cheapest product and only its price is relevant when determining the demand for the bundle.

\(^{13}\)Given that consumers perceive the two forms of fixed telephony as perfect substitutes, and given that fixed telephony and television services are less expensive to produce using both networks, in equilibrium the holding company does not offer telephony services over the upgraded CN.

\(^{14}\)If the bundle is significantly less expensive to produce than the separate components, i.e., if $\delta_b + \gamma$ is large, it is costly for the holding company to have consumers switching from the bundle to the separate components. Therefore, a large $\delta_b + \gamma$ discourages increases in $p_b$. If $\delta_b + \gamma \geq 1$, the holding company is unwilling to increase the bundle’s price above $v_t + v_f$ and all consumers would purchase the bundle.
of consumers that buy the bundle, \(1 - \Phi = \frac{1 + \delta_b + \gamma}{2}\).

As expected, the profits of the holding company after the upgrade are increasing in scope economies, \(\gamma\), and in coordination economies with respect to the bundle, \(\delta_b\).

For the consumers that purchase fixed telephony or subscription television services separately, the surplus is 0. For those who purchase the bundle, the surplus is \(\frac{(1 + \gamma + \delta_b)^2}{8}\). Hence, \(CS_{u,h} = \frac{(1 + \gamma + \delta_b)^2}{8}\).

### 3.1.3 Upgraded CN with Separate Ownership

The next Lemma presents the price equilibrium.

**Lemma 3:** In equilibrium, under separate ownership, if the CN is upgraded, then the telecommunications and the cable companies charge: (i) \(p_f^* = c_f\), (ii) \(p_t^* = v_t\), (iii) \(p_b^* = c_b^t + \mu_t^i + \gamma - \gamma_m + \frac{1 + \gamma_m}{2}\), and (iv) \(p_b^* = c_f - \varepsilon\), with \(\varepsilon \rightarrow 0^+\).

The independently owned cable company faces no competition in subscription television services, and therefore charges the monopoly price. This also increases the demand for the bundle. Price competition between telephony services over the CN and telephony services over the PSTN lowers prices to the marginal cost of the least efficient firm which, in this case, is the telecommunications company, since it does not benefit from scope or coordination economies.

The price of the bundle can be decomposed into: the marginal cost, \(c_b^i\), the opportunity cost of selling the bundle, \(p_t^* + p_f^* - c_t^i - c_f + \gamma - \gamma_m = \mu_t^i + \gamma - \gamma_m\),\(^{15}\) and the profit margin, \(\frac{1 + \gamma_m}{2}\).

The next Corollary gives the equilibrium profits.

**Corollary 2:** In equilibrium, under separate ownership, if the CN is upgraded, then the cable company has profits: \(\pi^{u,i}_b + \pi^{u,i}_t + \pi^{u,i}_b = \mu_t^i + \gamma - \gamma_m + \frac{(1 + \gamma_m)^2}{4} - \varphi\).

The increase in the profits of the cable company from selling the bundle after the upgrade consists of two effects: the extraction of part of the consumers’ valuation for fixed telephony, \(\gamma - \gamma_m\), and the increase in the profit margin when selling the bundle instead of its components, \(p_b^* - c_b^i - \mu_t^i - \gamma + \gamma_m = \frac{1 + \gamma_m}{2}\), times the proportion of consumers that buy the bundle, \(1 - \Phi = \frac{1 + \gamma_m}{2}\). The increase in the profit margin when selling the bundle is due to its lower production costs and to higher consumer valuation. The first effect increases with scope economies, \(\gamma\), and

\(^{15}\)If the independently owned cable company sells the bundle, it does not sell subscription television or fixed telephony services over the CN.
decreases with the diseconomies of producing the two services separately, instead of in a bundle, over the $CN, \gamma_m$. The second effect increases with $\gamma_m$. The overall effect of $\gamma_m$ is negative.

For the consumers that purchase subscription television services separately, the surplus is 0. For those that purchase fixed telephony services separately, the surplus is $\frac{1-\gamma_m}{2} \mu_f$. Finally, for those who purchase the bundle, the surplus is $\frac{1+\gamma_m}{2} \mu_f + \frac{(1+\gamma_m)^2}{8}$. Thus, $CS_{u,i} = \mu_f + \frac{(1+\gamma_m)^2}{8}$.

### 3.2 The Impact of the Ownership Structure on Prices

The next Corollary compares the price of the bundle under the two ownership structures, when the $CN$ was upgraded. As stated in Lemmas 2 and 3, the price of television subscription services is $v_t$, independent of the ownership structure. The price of fixed telephony is $v_f$ under dual ownership, and $c_f$ under separate ownership.

**Corollary 3:** (i) The equilibrium price of the bundle is lower under separate ownership than under dual ownership, if and only if, $\mu_f > \frac{\gamma_m + \delta_b}{2}$. (ii) The equilibrium mark-up of the bundle is lower under separate ownership than under dual ownership.

Based only on competition considerations, one would expect prices to be lower under separate ownership than under dual ownership. However, coordination economies lead to lower production costs under dual ownership. Consequently, depending on the parameter values, the price of the bundle can be lowest under either ownership structure. The price of the bundle is lower under separate ownership than under dual ownership if: the opportunity cost of the holding company selling the bundle, $\mu_f$, is large, the diseconomies of producing the two services separately, instead of in a bundle, over the $CN, \gamma_m$, are large, scope economies, $\gamma$, are small, and coordination economies with respect to the bundle, $\delta_b$, are small.

### 3.3 Stage 1: The Investment Decision

The incremental profit of upgrading the $CN$ for the holding company is:

$$\Delta \pi^h := \pi^u_f + \pi^u_t + \pi^u_b - \pi^u_f - \pi^t_f - \varphi = \frac{(1 + \delta_b + \gamma)^2}{4} - \varphi.$$ 

The incremental benefit has three effects: (i) the Scope Economies effect, associated with $\gamma$, (ii) the Coordination Economies effect, associated with $\delta_b$ and (iii) the Bundle Value effect.

\footnote{If the holding company sells the bundle it does not sell fixed telephony.}
associated with $\theta$.$^{17}$

The incremental profit of upgrading the $CN$ for the independently owned cable company is:

$$\Delta \pi^i := \pi^{u,i}_{\phi} + \pi^{u,i}_{b} + \pi^{u,i}_t - \pi^{n,i}_t - \varphi = (\gamma - \gamma_m) + \frac{(1 + \gamma_m)^2}{4} - \varphi.$$  

The incremental benefit also has three parts, which consist of: (i) the Scope Economies effect, (ii) the Bundle Value effect, and (iii) the Business Stealing effect, associated with $\gamma - \gamma_m$.$^{18}$ The intuition of this last effect is as follows. By upgrading the $CN$, the cable company is able to extract some surplus from the consumers of fixed telephony, who, before the upgrade, chose the telecommunications company. Price competition drives down the prices of fixed telephony to $c_f$. Due to the economies of scope, the independently owned cable company sells this service with a markup of $\gamma - \gamma_m$.$^{19}$

Under dual ownership, the holding company should upgrade the $CN$ if $\Delta \pi^h \geq 0$. Under separate ownership, the cable company should upgrade the $CN$ if $\Delta \pi^i \geq 0$.

4 Analysis

In this section, we analyze, (i) the impact of the ownership structure on the firms’ incentives to upgrade the $CN$, (ii) the social and private incentives to upgrade the $CN$, and (iii) the optimal ownership structure.

4.1 Ownership Structure and Incentives to Upgrade the $CN$

The difference between the incremental profit of upgrading the $CN$, for the holding company, and for the independently owned cable company, is given by $\Delta \pi^h - \Delta \pi^i$.

$^{17}$If $\theta$ is supported on $[0, \Theta]$, with $\Theta$ on $(0, +\infty)$, the Bundle Value effect is made more explicit. We normalized $\Theta = 1$, to avoid an unnecessary proliferation of parameters.

$^{18}$A cost reduction typically has two effects on a firm’s profit: (i) the units that the firm produced initially are now produced at a lower cost, and (ii) the firm may capture some surplus from consumers that initially selected other suppliers. Although it is motivated by lower costs, we call the latter effect the Business Stealing Effect.

$^{19}$To be exact, it is not possible to disentangle the Scope Economies Effect and the Business Stealing Effect because we chose not to separate the economies of scope obtained with fixed telephony services and with television services. For expository purposes we associate $\gamma - \gamma_m$ with the Business Stealing effect. We assume implicitly that when both services are sold separately, the equilibrium markup, $v_t + c_f - (c_t + c_f - \gamma + \gamma_m)$, can be divided into $v_t - c_t$ for television services, and $\gamma - \gamma_m$ for fixed telephony services. As for the bundle, we assume analogously that the markup, $v_t - c_t + \gamma - \gamma_m + \frac{1 + \gamma_m}{2}$, can be divided into $v_t - c_t$ for television services, $\gamma - \gamma_m$ for fixed telephony services, and $\frac{1 + \gamma_m}{2}$ for the additional convenience of purchasing the services bundled.
The next Proposition compares the incentives to upgrade the CN under the two ownership structures.

**Proposition 1:** The independently owned cable company has more incentives to upgrade the CN than does the cable company owned by the holding company, i.e., \( \Delta \pi^i > \Delta \pi^h \), if and only if,

\[
\delta_b < \sqrt{4 \gamma + (1 - \gamma_m)^2} - \gamma - 1.
\]

As seen in section 3.3, the incremental profit of upgrading the CN involves four effects that may have opposing signs, and whose magnitude depends on the ownership structure. Depending on the parameter values, either ownership structure can generate more incentive to upgrade the CN. The independently owned cable company has more incentive to upgrade the CN than does the holding company, if the coordination economies with respect to the bundle, \( \delta_b \), are small, and if the diseconomies of producing the two services separately, instead of in a bundle, over the CN, \( \gamma_m \), are small.\(^{20}\) Economies of scope, \( \gamma \), benefit both types of ownership structure. On the one hand, scope economies are relatively more profitable for the holding company, because, in the absence of competition, the extent to which the cost reductions translate into lower price is smaller. On the other hand, the holding company only benefits from economies of scope with the consumers that purchase the bundle, whereas the independent ownership benefits from scope economies with all consumers, although of a smaller magnitude when the bundle components are sold separately. Hence, the effect of economies of scope is ambiguous.

The next Corollary sharpens Proposition 1 by considering three particular cases.

**Corollary 4:** (i) If there are no coordination economies, \( \delta_b = 0 \), then the independently owned cable company has more incentive to upgrade the CN than the cable company owned by the holding company. (ii) Either if there is no business stealing effect, \( \gamma = \gamma_m \), or, if there are no economies of scope, \( \gamma = 0 \), then the cable company owned by the holding company has more incentive to upgrade the CN than the independently owned cable company.

Coordination economies are specific to dual ownership, whereas the business stealing effect is specific to separate ownership. Corollary 4, therefore, frames the trade-off between the two

---

\(^{20}\)Note that only coordination economies with respect to the bundle are relevant here. The holding company already benefitted from coordination economies with respect to television services, \( \delta_t \), before the upgrade.
ownership structures in terms of providing incentives to upgrade the CN.

4.2 Social and Private Incentives to Upgrade the CN

Next, we show that the private decision to upgrade the CN may be socially inefficient. For \( z = u, n \), denote by \( W^{z,k} \), the social welfare, i.e., the sum of consumer surplus and industry profits.

Social welfare under dual ownership is given by:

\[
W^{u,h} = \mu_f + \mu_i + (1 - \Phi^h) \left( \frac{1 + \Phi^h}{2} + \gamma + \delta_b \right) - \varphi
\]  

(1)

with \( \Phi^h := \frac{1 - \gamma - \delta_b}{2} \). Social welfare under separate ownership is given by:

\[
W^{u,i} = \mu_f + \mu_i + (1 - \Phi^i) \left( \frac{1 + \Phi^i}{2} + \gamma \right) + \Phi^i (\gamma - \gamma_m) - \varphi
\]  

(2)

with \( \Phi^i := \frac{1 - \gamma_m}{2} \).

Social welfare is the difference between the consumers’ valuations and the production cost, for all the units of all services sold by both companies. Under both ownership structures, all consumers purchase fixed telephony and purchase subscription television services. This accounts for the first two terms in (1) and (2). A fraction \( (1 - \Phi^k) \) of consumers purchases these two products in a bundle. As explained above, the bundle is more valued than the two products sold separately, and it is less costly to produce. Given our assumptions about the distribution of \( \theta \), the average valuation for the bundle, conditional on purchasing it, is \( \frac{1 + \Phi^k}{2} \), while the cost savings obtained from scope economies and coordination economies are of \( \gamma \) and \( \delta_b \), respectively. This explains the third term in (1) and (2), which represents the welfare generated by the production and consumption of the bundle. The fourth term in (2) represents the smaller scope economies that arise when the CN sells both products unbundled. The final term in both expressions is the cost of upgrading the CN.

The net incremental welfare benefit of upgrading the CN, i.e., the difference between the welfare level when the CN is upgraded and when it is not, denoted by \( \Delta W^r \), \( k = i, h \), is obtained by deleting the first two terms in (1) and (2). As expected, both \( \Delta W^h \) and \( \Delta W^i \) are more likely to be positive: the lower the upgrading costs, \( \varphi \), and the higher the extent of scope economies.\(^{22}\) The net incremental welfare benefit depends on the trade-off between: the cost of the upgrading the CN, the consumers’ marginal valuation for the bundle, scope economies, and

\(^{21}\)For any parameter values, the holding company sells the bundle to a higher fraction of consumers: \( \Phi^h \leq \Phi^i \).

\(^{22}\)And also the higher the valuation for the bundle, if one allowed \( \theta \) to be supported on \([0, \Theta]\).
coordination economies. This trade-off depends on the ownership structure, and is clarified in
the following Proposition.

**Proposition 2:** The net incremental welfare benefit is larger when the independently owned
cable company upgrades the CN, i.e., \( \Delta W^i > \Delta W^h \), if and only if:

\[
\delta_b < \sqrt{\frac{8\gamma + \gamma_m (3\gamma_m - 2) + 3}{3}} - \gamma - 1. \tag{3}
\]

Upgrading the CN allows production of the bundle, under both ownership structures, and
allows reduction of the costs of producing both components separately, under independent
ownership. The first effect is greater under dual ownership than under independent ownership
for two reasons. The first reason is that more consumers purchase the bundle, which has a
positive effect on welfare by itself. The second reason is that the production costs of the bundle
decrease by a larger amount.\(^{23}\) Regarding the second effect, for the consumers that purchase
the two components separately, if the CN is upgraded there is a cost reduction of \( \gamma - \gamma_m \) in
the case of independent ownership. This effect accounts for the possibility that the upgrade
increases welfare by a larger amount when it is undertaken by an independent cable company.
Figure 1 illustrates these effects.

[Figure 1]

When the costs of upgrading the CN, \( \varphi \), are very low, the upgrading increases welfare under
either ownership structure. When the costs of upgrading the CN are sufficiently high, the up-
grading always has the opposite effect. The interesting cases lie in-between. The next Corollary
describes the impact on welfare of upgrading the CN under the two ownership structures.

**Corollary 5:** (i) If (3) holds, there are values for the upgrading cost, \( \varphi \), such that welfare
increases if the independently owned cable company upgrades the CN, and decreases if the holding
company upgrades the CN. (ii) If (3) is reversed, there are values for the upgrading cost, \( \varphi \), such that welfare decreases if the independently owned cable company upgrades the CN, and
increases if the holding company upgrades the CN.

\(^{23}\)In the case of dual ownership, the bundle is cheaper to produce by \( \gamma + \delta_b \) than the separate components
before the upgrade, whereas in the case of separate ownership, the bundle is cheaper to produce by \( \gamma \) than the
separate components before the upgrade.
This result shows that upgrading the CN may reduce welfare under both ownership structures. The result has, therefore, two important implications. First, upgrading the CN should not be an objective in itself, because it may in fact decrease welfare. Second, changes in the ownership structure should not be regarded merely as a means to promote the upgrade of the CN, but also as a factor affecting the desirability of the upgrade itself. The upgrade is not equally desirable under both ownership structures. As a consequence of these two implications, the choice of the ownership structure should not focus only on the impact it has on the private incentives to upgrade the CN, but also on whether the private incentives are aligned with the social incentives.

The next Corollary sharpens Proposition 2 by considering two particular cases.

**Corollary 6:** (i) If either there is no business stealing effect, $\gamma_m = \gamma$, or there are no economies of scope, $\gamma = 0$, then the net incremental welfare benefit is less when the independently owned cable company upgrades the CN than when the cable company owned by the holding company upgrades the CN. (ii) If either there is no business stealing effect, $\gamma_m = \gamma$, or there are no economies of scope, $\gamma = 0$, then there are values for the upgrading cost, $\varphi$, so that welfare decreases, if the independently owned cable company upgrades the CN, and increases, if the holding company upgrades the CN.

Next we will verify, for the different ownership structures, if the privately optimal decision of the owner of the cable company to upgrade the CN or not is socially efficient.

**Definition 1:** (i) Investment is socially optimal, or Optimal for short, if the owner of the cable company upgrades the CN, and by upgrading the CN, increases welfare. (ii) There is Underinvestment if the owner of the cable company does not upgrade the CN, when upgrading the CN means increasing welfare. (iii) There is Overinvestment if the owner of the cable company upgrades the CN, when this means decreasing welfare.

The next Corollary compares the private and social incentives to upgrade the CN under the two ownership structures.

**Corollary 7:** Under both ownership structures, the cable company may invest optimally or underinvest.
Part of the increase in profits of the independently owned cable company from upgrading the CN is a transfer of profits from the telecommunications company, with no social value. This business stealing effect may lead the independently owned cable company to overinvest from a social perspective. But, with mixed bundling, this effect is small because competition on the separate components drives down the price of fixed telephony, and hence, also the price of the bundle. For the holding company, however, this transfer of profits never occurs, since it is the only firm in the market.

If the CN is upgraded, the cable company captures part of the social surplus that results from production of the bundle. However, since there is no price discrimination, the consumers who purchase the bundle also obtain some surplus. The fact that under both ownership structures the cable company does not capture all the increase in the consumer surplus may lead to underinvestment.

4.3 Optimal Ownership Structure

In the previous section, we established that the positive impact on welfare of upgrading the CN may be larger under independent ownership than under dual ownership. However, without upgrading the CN, welfare is larger under dual ownership. It is, therefore, unclear which ownership structure is optimal. Next, we investigate which ownership structure is optimal from the social perspective and from the consumers’ perspective. We start with the first approach.

Depending on the ranking of \( \Delta \pi^i \) and \( \Delta \pi^h \), and on the value of \( \varphi \), four cases may occur regarding the upgrading decision: (i) the CN is not upgraded under both ownership structures, \( \max \{ \Delta \pi^h, \Delta \pi^i \} < 0 \); (ii) the CN is upgraded only by the holding company, \( \Delta \pi^i < 0 \leq \Delta \pi^h \); (iii) the CN is upgraded only by the independently owned company, \( \Delta \pi^h < 0 \leq \Delta \pi^i \); (iv) the CN is upgraded under both ownership structures, \( 0 \leq \min \{ \Delta \pi^h, \Delta \pi^i \} \).

The following Lemma establishes the conditions under which separate ownership is optimal.

Proposition 3: Separate ownership is the optimal ownership structure, if and only if: (a) the CN is upgraded only by the independently owned company, \( \Delta \pi^h < 0 \leq \Delta \pi^i \), and (b) 
\[
\gamma - \gamma_m + \frac{3(1+\gamma_m)^2}{8} > \delta_t + \varphi.
\]

Four comments are in order. First, when the CN is not upgraded under either ownership structure, dual ownership is optimal due to coordination economies in subscription television

\[^{24}\text{If mixed bundling is ruled out, prices are higher under separate ownership. This increases the business stealing effect and the independently owned CN might overinvest.}\]
services. In the absence of these economies, the two ownership structures are equivalent.

Second, when the $CN$ is upgraded by the holding company but not by the independently owned cable company, dual ownership is optimal, not only due to coordination economies, but also due to the introduction of the bundle, which is valued by some consumers and cheaper to produce than the separate components.

Third, when the $CN$ is upgraded by the independently owned cable company but not by the holding company, the result is potentially ambiguous. On the one hand, dual ownership allows cost savings associated with coordination economies in the production of subscription television, $\delta_t$, and saves on the upgrade costs, $\varphi$. On the other hand, independent ownership leads to the introduction of the bundle, valued by the consumers and produced with economies of scope. Each of the $(1 - \Phi^i)$ units of the bundle produced in equilibrium results in an increase of $\frac{1 + \Phi^i}{t}$ in the average consumer valuation, and a decrease of $\gamma$ in the unit cost. Additionally there is also a reduction of $\gamma - \gamma_m$ in the costs of producing both components separately. Naturally, for sufficiently low values of $\delta_t$ and $\varphi$, separate ownership is optimal. Note, parenthetically, that this seems to be the case that the European Commission has in mind.

And fourth, when the $CN$ is upgraded under both ownership structures, dual ownership is optimal due to the lower production costs for all products.

Next, we focus on the consumers’ perspective to see if, by restricting the welfare criterion, one can reach more definite conclusions regarding the optimality of the ownership structure.

The next Corollary establishes under which conditions separate ownership and dual ownership are optimal from the consumers’ perspective.

**Corollary 8:** Separate ownership is the optimal ownership structure from the consumers perspective: (a) if the $CN$ is upgraded only by the independently owned company, or (b) if the $CN$ is upgraded under both ownership structures, and $\mu_f > \frac{(1 + \gamma + \delta_h)^2 - (1 + \gamma_m)^2}{8}$. ■

If the $CN$ is not upgraded, consumers have no surplus. This is due to the assumption of the rigidity of the demand for fixed telephony and subscription television. When the bundle is introduced, however, consumer heterogeneity and the absence of price discrimination ensure that the cable company is not be able to extract all the consumers’ surplus. This explains why consumers prefer the ownership structure that leads to the introduction of the bundle. When both structures lead to the upgrade of the $CN$, competition between the cable company and the telecommunications company pushes the price of fixed telephony down to marginal cost, which also affects the price of the bundle. Consumers will then have a surplus of $\mu_f$. If $\mu_f$
is sufficiently large, it may compensate for the fact that less consumers purchase the bundle under independent ownership. However, if $\mu_f$ is small then, as seen in Corollary 3, the price of the bundle is lower under dual ownership, and consumers are better off under this ownership structure.

Restricting the welfare criterion to the consumers’ surplus does not solve the indeterminacy regarding which ownership structure is the socially optimal one.

5 Concluding Remarks

In this article, we studied the impact of the ownership structure of cable television firms on the incentives to upgrade the cable networks to offer telecommunication services. We identified four different effects that the upgrade of the cable network has on the owner’s profit: the Scope Economies effect, the Coordination Economies effect, the Bundle Value effect, and the Business Stealing effect. We find that the magnitudes of these effects depend on the ownership structure of the cable network. In particular, they depend on whether the owner of the cable network also owns the telecommunications network. Contrary to the European Commission’s perspective that a firm that owns both networks has no incentive to upgrade the cable network, we showed that there is no simple relation between the ownership structure and the incentives to upgrade the cable network. This occurs because coordination economies are specific to dual ownership, the business stealing effect is specific to separate ownership, and economies of scope play an ambiguous role.

The analysis of these four effects demonstrates that the impact of the upgrade on welfare is not independent of the ownership structure. For the same upgrade cost, either structure may be the one under which welfare increases the most. In addition, whether the upgrade is desirable or not may depend on the ownership structure.

Finally, we analyzed the optimality of the decision to invest in upgrading the network and showed that under both ownership structures, the cable company may invest optimally or underinvest. We further established that structural separation is optimal if, but not only if, the cable network is upgraded solely by the independently owned cable company. This structure allows for the introduction of the bundle, valued by the consumers and produced with economies of scope, which should be traded-off against the cost savings originated by dual ownership. These savings are associated with coordination economies in the production of subscription television and with the absence of upgrade costs.

Brito and Pereira (2007) present reasons unrelated to the model of this article, which might
increase or decrease the firms’ incentives to upgrade the cable network.
A Appendix

In the appendix we prove the Lemmas, Propositions and Corollaries in the main text. We restrict our analysis to the prices $p_t$ on $[c_t, v_t]$, $p_f$ on $[c_f, v_f]$ and $p_b$ on $[c_b, v_t + v_f + 1]$

**Lemma 2**: The Holding Company, controlling both the Telecommunications Company and the Cable Company, and thus acting as a monopolist, will maximize its profits given by

$$
\pi^h = \begin{cases} 
  m_b & p_b - p_t - p_f < 0 \\
  m_t + m_f + (1 - \Phi) (m_b - m_t - m_f) & 0 \leq p_b - p_t - p_f \leq 1 \\
  m_t + m_f & p_b - p_t - p_f > 1 
\end{cases}
$$

where $m_i = p_i - c_i$ for $i = f, t, b$ and $\Phi$ is an abbreviation for $\Phi(p_b - p_t - p_f)$. In the first and third “branches” all derivatives in prices are trivially nonnegative as the demand is infinitely rigid. For $0 \leq p_b - p_t - p_f \leq 1$, the derivatives in $p_t$, $p_f$ and $p_b$ are, respectively,

$$
\frac{\partial \pi^h}{\partial p_t} = \Phi - m_t - m_f + m_b \\
\frac{\partial \pi^h}{\partial p_f} = \Phi - m_t - m_f + m_b \\
\frac{\partial \pi^h}{\partial p_b} = (1 - \Phi) + m_t + m_f - m_b
$$

Assume initially that $p_b = p_t + p_f - \delta_b - \gamma$. It is easy to check that $\frac{\partial \pi^h}{\partial p_b}$ evaluated at $p_b = p_t + p_f - \delta_b - \gamma$ is equal to $(1 - \Phi)$, which is nonnegative for all admissible $p_f$ and $p_t$. As $\frac{\partial^2 \pi^h}{\partial p_t^2} = -2 < 0$, a profit maximizing company should set $p_b \geq p_t + p_f - \delta_b - \gamma$. But this is equivalent to $-m_t - m_f + m_b \geq 0$ which implies that $\frac{\partial \pi^h}{\partial p_t} > 0$ and $\frac{\partial \pi^h}{\partial p_f} > 0$. Hence, $p_t$ and $p_f$ should be placed at their admissible maximum which is, respectively, $v_t$ and $v_f$. Profits, as a function of $p_b$, then become

$$
\pi^h = \begin{cases} 
  \mu^h_t + \mu_f + (m_b - \mu^h_t - \mu_f) & p_b < v_t + v_f \\
  (\mu^h_t + \mu_f) \Phi + m_b (1 - \Phi) & v_t + v_f \leq p_b \leq 1 + v_t + v_f \\
  \mu^h_t + \mu_f & p_b > 1 + v_t + v_f 
\end{cases}
$$

In order to obtain the Holding Company’s optimum $p_b$ we solve

$$
\frac{\partial \pi^h}{\partial p_b} = \mu^h_t + \mu_f + (1 - p_b + v_t + v_f) - (p_b - c^h_b) = 0 \iff \\
 p^*_b = c^h_b + \mu^h_t + \mu_f + \frac{1 + \delta_b + \gamma}{2}.
$$

Note that $p^*_b - v_t - v_f = \frac{1 - \delta_b - \gamma}{2}$. This is clearly smaller than 1 and nonnegative given assumption C: (iii).
Corollary 1: Obvious, with $\Phi = \frac{1-\delta_b-\gamma}{2}$.

Lemma 3: Start by noting that the demand for the bundle depends on $\min \{p_f, p_\phi\}$. Let the telecommunications company set any price $p_f < v_f$. The cable company will never set a higher price: if it sets a higher price, it will not sell to fixed telephony, separately, and it will not increase the demand for the bundle. Setting $p_\phi = p_f - \varepsilon$ will have a negligible effect on the demand for the bundle but will, at the same time, increase significantly its fixed telephony sales. This is profitable provided that $p_f > c_f - \gamma + \gamma_m$. Hence, the cable company will always undercut any $p_f > c_f$. As for the telecommunications company, setting $p_f > p_\phi$ will lead to no sales. Undercutting is always the best response until $p_f^* = c_f$. Given that $p_f^* = c_f$ and $p_\phi^* = c_f - \varepsilon$, with $\varepsilon \to 0^+$, $p_b$ will be set to maximize profits given by:

$$(p_b - c_b^i) (1 - p_b + v_t + c_f) + (v_t + c_f - c_b^i - \gamma_m)(p_b - v_t - c_f),$$

from where we obtain,

$$p_b^* = c_b^i + \mu_t^i + \gamma - \gamma_m + \frac{1+\gamma_m}{2}.$$ 

Corollary 2: Obvious.

Corollary 3: (i) The prices for the bundle under separate ownership are lower than under dual ownership if and only if $c_b^i + \mu_t^i + \gamma - \gamma_m + \frac{1+\gamma_m}{2} < c_b^h + \mu_t^h + \mu_f + \frac{1+\delta_b+\gamma}{2}$ which is equivalent to

$$\mu_f > \frac{\gamma - \gamma_m + \delta_b}{2}.$$ 

(ii) The mark-up for the bundle under separate ownership is lower than under dual ownership if and only if $\mu_t^i + \gamma - \gamma_m + \frac{1+\gamma_m}{2} < \mu_f + \mu_t^h + \frac{1+\gamma+\delta_b}{2}$ which is equivalent to

$$0 < \mu_f + \delta_t - \frac{\gamma - \gamma_m}{2} + \frac{\delta_b}{2}.$$ 

As $\delta_t > \gamma - \gamma_m$ this is always true.

Proposition 1: The independently owned Cable Company has more incentives if the increase in its profit resulting from upgrading the CN exceeds the increase the holding company would have. This happens when

$$\Delta \pi^h < \Delta \pi^i \iff \left( \frac{1+\gamma + \delta_b}{2} \right)^2 - (\gamma - \gamma_m) - \left( \frac{1+\gamma_m}{2} \right)^2 < 0 \iff$$

$$\delta_b < \sqrt{4\gamma + (1-\gamma_m)^2 - \gamma - 1}.$$
Corollary 4: (i) If \( \delta_b = 0 \), the condition becomes

\[
0 < \sqrt{4\gamma + (1 - \gamma_m)^2 - \gamma - 1} \iff 0 < \sqrt{(\gamma + 1)^2 + (\gamma - \gamma_m) (2 - \gamma - \gamma_m) - (\gamma + 1)},
\]

which is always true given that \( 1 > \gamma > \gamma_m \) (ii) If \( \gamma = \gamma_m \), the condition becomes

\[
\delta_b < \sqrt{4\gamma + (1 - \gamma)^2 - \gamma - 1} \iff \delta_b < 0,
\]

which is impossible. Note that \( \gamma = 0 \) is a particular case of \( \gamma = \gamma_m \).

Proposition 2: The net incremental welfare benefit is larger when the independently owned cable company upgrades the CN, i.e., \( \Delta W^i > \Delta W^h \), if and only if:

\[
(1 - \Phi^i) \left( \frac{1 + \Phi^i}{2} + \gamma \right) + \Phi^i (\gamma - \gamma_m) > (1 - \Phi^h) \left( \frac{1 + \Phi^h}{2} + \gamma + \delta_b \right).
\]

This can be simplified to

\[
(\gamma - \gamma_m) + \frac{3 (1 + \gamma_m)^2}{8} > \frac{3 (1 + \gamma + \delta_b)^2}{8},
\]

which is equivalent to

\[
\delta_b < \sqrt{\frac{8\gamma + 3\gamma_m^2 - 2\gamma_m + 3}{3} - \gamma - 1}.
\]

Corollary 7: Note that

\[
\Delta W^i > 0 \iff (\gamma - \gamma_m) + \frac{3}{2} \left( \frac{1 + \gamma_m}{2} \right)^2 > \varphi,
\]

\[
\Delta \pi^i > 0 \iff (\gamma - \gamma_m) + \left( \frac{1 + \gamma_m}{2} \right)^2 > \varphi,
\]

\[
\Delta W^h > 0 \iff \frac{3}{2} \left( \frac{1 + \gamma + \delta_b}{2} \right)^2 > \varphi,
\]

\[
\Delta \pi^h > 0 \iff \left( \frac{1 + \delta_b + \gamma}{2} \right)^2 > \varphi.
\]

Clearly \( \Delta \pi^k > 0 \Rightarrow \Delta W^k > 0 \), for \( k = i, h \). This proves that whenever the holding company or the separately owned cable company invest, welfare increases: there is no overinvestment.

Lemma 1, Corollary 5, Corollary 6, Proposition 3 and Corollary 8: Obvious.
References


[8] Cooper, M., 2003, Cable Mergers, Monopoly and Price Increases, Consumer Federation of America


Tables

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Table 1: Unit Costs under both Ownership Structures

Figures
Figure 1: Increases in welfare resulting from the upgrade of the CN. The areas filled with horizontal lines refer to the holding and those with vertical lines to the independent ownership case. Rectangular shapes at the bottom refer to cost reductions and trapezoidal shapes at the top to the increase in valuation for the bundle. The thick border outlines welfare after an upgrade by the holding company while the dashed border outlines welfare after upgrade by an independent cable company. The upgrade cost, $\varphi$, is not illustrated.