On the Diffusion of Electronic Commerce*

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February 2006

Abstract

This paper analyzes retailers' adoption of e-commerce in a technology adoption race framework. An Internet-based firm with no traditional market presence competes with an established traditional firm to adopt the e-commerce technology and sell to a growing number of consumers with on-line shopping capability. The focus of the analysis is on identifying how consumer loyalty, differences in firms' technology and consumers' preferences across the traditional versus the virtual market, and expansion in market size made possible by the Internet can affect the timing and sequence of adoption by firms, as well as the post-adoption evolution of prices. The model's implications are used to discuss empirical evidence on adoption patterns across different product categories and firm types.

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*We thank the editor Stephen Martin and an anonymous referee for helpful comments.
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1 Introduction

Since early 1990’s, the ‘electronic commerce’ technology has been adopted by many traditional firms and also by new, entirely Internet-based firms.\(^1\) Yet, traditional and Internet-based firms have exhibited different tendencies to embrace the new technology and the rates of adoption by these two types of firms have also varied considerably by product category. In some industries, such as book and CD retailing, pure Internet-based firms like Amazon.com were successful early adopters. In other industries, such as clothing and apparel, established traditional firms like Gap adopted the technology early. Internet-based firms tended to be the first movers. On the other hand, established firms sometimes moved first, but usually they followed, either quickly or with some delay. This delay in the adoption of e-commerce by established firms has drawn attention in the literature.\(^2\) Fear of cannibalizing an existing sales channel or technological incompatibility of the two sales channels may prevent or delay going on-line. Internet-based firms also face obstacles to adoption, such as the lack of an established brand, trust, a loyal customer base, or a network of warehouses and distribution facilities. These adoption patterns pose several questions: How do traditional and new firms differ in their adoption patterns? In what type of environments are we likely to observe early adoption by new or by established firms? Are adoption patterns systematically related to the main differences between traditional and virtual markets? What can be said about inter-industry differences in the diffusion of e-commerce?

We develop a model of the adoption of e-commerce technology by retailers to assess the patterns of adoption in the early days of e-commerce. Since the decision of whether and when to adopt is dynamic in nature, the analysis of adoption patterns require a dynamic framework rich enough to incorporate basic differences between traditional and Internet-based firms and between traditional and virtual market environments. We use a continuous-time technology adoption model, where at each point in time firms decide whether to adopt the e-commerce technology and then choose prices, given the adoption decisions up to that point. We derive the implications of the model for the timing and sequence of adoption by firms to understand which firm is likely to adopt first, whether the gap between adoption times is large, and how the adoption times depend on the parameters characterizing the traditional and virtual market environments. The predictions of the model explain observed adoption patterns across firm types and product categories. The result is a simple characterization of market environments that encourages early adoption by established firms and market environments that facilitate early entry by new firms. The tractable model can be applied to other market settings.

\(^1\)The e-commerce technology can be defined as a technology that allows business transactions based on processing and transmission of digital data on the Internet, in contrast to the traditional business technology, whose logistics are based on the physical environment.

to address questions of adoption.

The structure of the model reflects important differences between traditional and virtual markets, as well as between traditional and Internet-based firms. While a long list of such differences can be made, the main elements we consider reflect a desire to maintain analytical tractability and empirical relevance. First, firms’ costs and consumers’ utility across traditional and virtual markets can differ. Potential cost savings can arise in the virtual market from low-cost electronic transactions and diminishing need for inventory, retail space, and labor, as well as elimination of intermediaries. For some goods, convenience of on-line transactions and savings in shopping time and transportation costs may enhance utility, but for other goods, delayed consumption or the inability to inspect the good physically may result in a utility loss. Second, some consumers have a preference for the good sold by the established firm, resulting from the established firm’s reputation or from consumers’ trust in an established brand name built during the firm’s long presence in the traditional market. The importance of such reputation and brand name effects in on-line markets has been emphasized in recent empirical literature. We refer to such brand preference by consumers as “loyalty”. Initially, we assume that loyalty is specific to the established firm, so a prior presence of the established firm in the traditional market creates an important asymmetry between the two firms. An extension allows the new firm also to acquire loyal customers over time. Third, the Internet can increase an established firm’s market size by extending its geographic reach or by expanding hours of shopping that the Internet makes possible. The market expansion effect is represented by a set of consumers who have no access to the traditional firm’s physical shop and can only buy on-line, provided that they have Internet access.

The analysis of the model reveals that either firm can lead in adoption depending on the parameters. We identify conditions under which the only subgame perfect adoption equilibria are in open-loop strategies which depend only on calendar time. In these equilibria, one of the firms is always a leader in adoption. We also identify conditions under which a firm always wants to pre-empt its rival. The new firm is a leader in adoption when it faces an established firm with low levels of loyalty in an environment where the physical shop of the established firm has low marginal cost, the virtual market provides low incremental profit over the traditional one, and there is little opportunity for market expansion through e-commerce. On the other hand, the established firm leads if it enjoys relatively high loyalty, it has relatively high marginal cost in the traditional market, the incremental profit from e-commerce is relatively high, and the market expansion effect is relatively large. We assess the relevance of these predictions in explaining the entry patterns in various industries.

We also consider several extensions to the model and the analysis. One of them is a discussion of the closed-loop strategies that allow firms to condition their adoption decisions on their rival’s actions,

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3We discuss these elements in more detail in Section 2.1.
4See, e.g., Smith and Brynjolfsson (2001).
in addition to calendar time. Equilibria in closed-loop strategies emerge under certain parameter restrictions and involve either pre-emptive or waiting strategies by firms. We compare the nature of these equilibria with the equilibria in open-loop strategies and discuss their empirical relevance. As another extension, we consider growing loyalty for the new firm. Many Internet-based firms have invested heavily in loyalty programs and some early adopters, such as Amazon.com, seem to enjoy significant loyalty. Allowing the new firm to develop loyalty over time results in earlier adoption by the new firm, regardless of whether it is a leader or a follower in adoption, but does not alter the adoption behavior of the established firm. We also discuss how our results may change when more than two firms can adopt the technology.

The technology adoption framework we develop is related to earlier models of technology adoption, such as Fudenberg and Tirole (1985), Quirmbach (1986), Jensen (1982) and Reinganum (1981). Our analysis extends these models to analyze entry decisions in richer market environments. The approach here differs from the earlier literature in three main ways. First, in earlier models, the consequences of competition between the firms at any point in time was summarized by an exogenously given reduced form profit function. Here, the market game at any point in time, and the resulting profit functions, are endogenously determined by the adoption decisions, as well as by the fundamentals of the model. This allows us to investigate the sensitivity of our results to the parameters of the market game. Second, the loyalty for the established firm introduces an important asymmetry between firms that leads to pure strategy adoption equilibria, in contrast to only mixed strategy equilibria in previous models which assumed symmetry. The existence of pure strategy equilibria helps to explain observed adoption patterns. Third, while previous models assume market stationarity, we incorporate non-stationarity by allowing the market to grow over time. In this sense, the model can be generalized to analyze firms’ entry decisions into growing markets when strategic interaction is important.

As in Baye, Kovenock and deVries (1992), Narasimhan (1988) and Varian (1981), mixed pricing strategies emerge naturally in our model. Recent empirical evidence suggests the relevance of mixed strategy pricing in on-line markets. Mixed strategy equilibria also lead to simpler period payoff structures for firms and makes the dynamic model tractable, compared to pure strategy equilibria that emerge in common vertical and horizontal product differentiation models. The evolution of online prices in the model is also broadly consistent with the emerging empirical evidence.

The rest of the paper is organized as follows. Section 2 discusses major factors influencing adoption and empirical evidence motivating our model. Section 3 presents the model, followed by the characterization of equilibrium in Section 4. In Section 5, we analyze the model and its empirical implications.

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Extensions to the model are considered in Section 6. Section 7 reconciles the model with the empirical evidence. Section 8 concludes. All proofs are in the Appendix.

2 Patterns of adoption

2.1 Important factors influencing adoption

The first factor we focus on is the “incremental per consumer profitability” of the virtual market with respect to the physical market, a term we use to refer to the gap, negative or positive, in per consumer profit that results from the differences in firms’ costs and consumers’ utility across the two markets. If the incremental profitability is lower, an established firm may choose not to adopt because of the possibility of “cannibalization”: if the firm sells only to some of its existing consumers through the virtual shop at a lower profit per consumer than the physical shop, the net effect is a loss. An assumption behind the cannibalization effect is that the virtual market does not create any new sales. In general, however, the profit per consumer may be higher in the virtual market and a firm’s market size may increase beyond its local physical market. For a new firm, cannibalization is not an issue.

Relative profitability depends on both the technology and the preference structures. The virtual market can be more cost-effective compared to the traditional market in at least two ways. First, the e-commerce technology can reduce transaction costs in making payments, record keeping, ordering, invoicing, and exchanging information with customers, employees and suppliers. Second, the technology can reduce the dependence on traditional inputs of retailing, such as physical space and sales force, which constitute a large fraction of retailers’ costs. Cost reductions made possible by the Internet are believed to be wide-spread. Litan and Rivlin (2001) discuss several case studies supporting this view. Garicano and Kaplan (2001) find important cost advantages in Internet automobile auctions. Lucking-Riley and Spulber (2001) discuss further evidence on declining transaction costs. Many Internet-based firms have wholesalers and manufacturers handle the inventory and shipping services, which cuts down costs significantly (see Randall, Netessine, and Rudi (2002)). Overall, for tangible goods, the Internet is estimated to reduce distribution costs by more than 25% (see Geyskens et al. (2002)). While we continue under the assumption that the marginal cost is lower in the virtual market, this assumption is not crucial for our analysis of adoption decisions.

On the preference side, on-line shopping can affect a consumer’s utility, depending on the nature of the product and the services tied to it. For products whose features cannot be inferred without a physical inspection or for which shipping delay can be substantial, consumers may have to give

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7See several references therein.
8In fact, from the modeling perspective, differences in technology and preferences are not separately identified, as will be clear in Section 3.
up some utility. For other products, the convenience of on-line shopping can result in higher utility compared to the physical market.\footnote{See Litan and Rivlin (2001) for more on the convenience of Internet shopping.} For instance, a category of products that are especially suitable for e-commerce is digital goods, as they can be easily delivered and returned in electronic form, and virtual product demos enable consumers to verify the features and quality of the product. Utility gain can also arise from increased variety within the same product category, as emphasized by Brynjolfsson, Smith, and Hu (2003).

The second factor we consider is consumers’ loyalty to the established firm in the form trust and brand recognition, which may result from the firm’s long presence in the traditional market. Loyalty creates an important asymmetry between an established firm and a new firm, and may have been especially important in the early phases of e-commerce when consumers were reluctant to release personal information to relatively unknown web-sites. The emergence of third-party certification of trust by on-line intermediaries (e.g., Trust-E.com) highlights the importance of such intangible and non-contractible assets. Our emphasis on loyalty is based on the empirical evidence suggesting its importance. Smith and Brynjolfsson (2001) document the importance of brand recognition for homogeneous products among users of price-comparison search engines. Using data for book prices, the authors report that, while price is the strongest predictor of customer choice, only 49% of customers choose the cheapest vendor. Consumers were willing to pay 5% more to purchase from Amazon, rather than from the lowest priced vendor, and 3% more to purchase from Barnes & Noble or Borders. Johnson et al. (2001) report that 70% of CD shoppers, 70% of the book shoppers, and 42% of the travel shoppers, were observed as being loyal to just one site in their data. Shankar, Rangaswamy, and Pusateri (1998) find that consumers with prior positive experience with an established brand in the physical market had lower price sensitivity in on-line markets, where it may be difficult to evaluate a retailer’s reliability. These findings suggest that loyalty can be an important factor in the adoption decision of both established and new firms.

The third factor is the expansion in market size made possible by the Internet. Such expansion can take place in the geographic space, the time window consumers can shop, or the demographic dimension.\footnote{While 90 percent of Victoria’s Secret’s store customers in 1999 were women, 60 percent of its Internet buyers in the holiday season were men (see Kaufman (1999)).} Market expansion effect is relevant for the adoption decision of established firms, which must consider the magnitude of market expansion as well as the per consumer profitability of the virtual market. Established firms, such as Nike, initially emphasized that market expansion was one of their primary motives in going on-line. The evidence is scant, however, on how important the market expansion effect might have been quantitatively. An early investigation by Jupiter Communications estimated that only 6% of on-line sales in 1999 were incremental and therefore non-cannibalizing.
We acknowledge other factors that matter for adoption, but do not focus on them in this paper. It is well-known that financial constraints matter for young firms, but in the case of e-commerce capital markets initially favored Internet-based start-ups. ‘Organizational inertia’ of established firms can also delay adoption (see, e.g. Henderson (1993)). ‘Channel conflict’ that arises from the resistance of intermediaries to a direct channel of sale can also prevent adoption. This conflict especially applies to manufacturers that by-pass retailers by selling directly through the Internet and is less of a concern for retailers.

2.2 Empirical evidence on adoption patterns

The share of e-commerce in total sales for any individual retail industry is still very small and differences across industries are not highly perceptible. Table 1 presents the percentage of sales accounted by e-commerce by category, considering only the firms classified as “electronic and mail-order houses”. According to the statistics provided by the Census Bureau, the diffusion of e-commerce sales was relatively rapid and wide-spread among electronic and mail-order houses compared to other retail sectors. Therefore, differences across product categories in the share of e-commerce should be more visible in this industry. In 2001, the highest shares were observed in books and magazines, electronics, and music and videos. Relatively low shares were observed in food, beer and wine, clothing and apparel, and drugs. The nature of the product appears to matter for the extent of the diffusion. The timing of adoption also depends on product category and a firm’s presence in the traditional market. Table 2 provides a list of major adopters. In most sectors, a new firm was a leader, such as Amazon.com in books or Netflix.com in movies. In others, established firms were quick in adoption, such as Gap.com in clothing and Charles Schwab in brokerage. In some sectors, established firms took longer to adopt, such as Blockbuster in movies and Borders in books. The main message from Table 2 is that new firms usually came first, and established firms followed either quickly or with some delay.

To give a glimpse of how the elements discussed in the previous section can explain the adoption patterns, we now consider some examples. Consider first book retailing. Amazon.com was the first major adopter, whereas Barnes and Noble, an established firm, came much later. Books are

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11 Recent work analyzes this inertia in the context of e-commerce. For instance, Lasry (2001) finds that larger and younger firms have lower levels of inertia and are more willing to accommodate e-commerce. Lieberman (2002) investigates the role of first mover advantages in the adoption of e-commerce across several business sectors.

12 According to the US Census Bureau, the electronic and mail order houses industry (NAICS 454110) includes all catalog and mail order houses and other direct retailers, many of which sell in multiple channels, as well as pure Internet-based firms and “brick and click” retailers, if the e-commerce group operates as a separate unit and is not engaged in the online selling of motor vehicles.

13 A more detailed discussion of empirical evidence on adoption patterns can be found in Dinlersoz and Pereira (2005).

14 See Mendelson and Meza (2001) for the history of Amazon.com.
homogenous products. There is little need to physically inspect the product and if needed, consumers can often view part of the contents of a book on-line. The consumer can enjoy the conveniences of on-line shopping, such as ordering a locally unavailable book or savings in transportation costs. These benefits are result in higher utility. Marginal cost of selling books on-line may be lower, as firms can economize on costs of labor, inventory and real estate. Loyalty also appears to be important for established firms even for homogenous goods such as books (see, again, Smith and Brynjolfsson (2001)). But its effect may not have been as pronounced as in the case of a good that requires physical inspection or service, both of which tend to increase the importance of trust. Early emergence of Amazon.com can be attributed to the convenience of on-line shopping for books and/or lower marginal costs, which resulted in high incremental profitability for the virtual market, and to the relative homogeneity of books as a product category, which may have rendered loyalty less influential. This reasoning may also apply to CD and movie retailing.\footnote{Netflix.com, an early Internet-only adopter, was followed by Walmart (see Tedeschi (2003)). Blockbuster has also recently adopted e-commerce, several years after Netflix.com.}

Consider now the case of clothing and apparel for which the Internet may be a relatively less effective channel. Established firms such as Gap were early adopters. In clothing, Internet retail sales are much more concentrated in traditional retailers than pure Internet-based ones (see Ramanathan (2000)). Clothing products are relatively heterogenous and customized. Since a product’s fit to personal taste, and good service – such as convenient return policies and alteration possibilities – are more serious concerns for these goods, a physical market presence, reputation, and trust are likely to give advantage to established retailers in adoption. The case of Gap highlights how product type and customer loyalty can play a role in adoption. According to McIntyre and Perlman (2000), the following factors favored Gap in early adoption: return policy, i.e. the product could be returned to the physical stores, services, e.g., alterations could be done at the nearest physical store, loyalty to a trusted brand name, and pre-shopping and in-store promotions. Of these factors, loyalty and promotions are quite general and apply to other product categories.

Finally, consider digital products such as software, MP3 music, stocks, and downloadable movies. For such goods, on-line demos, easy delivery and return enhance consumers’ utility. Cost reductions can also be large, as there is no need for physical storage and physical transportation. In brokerage industry, Internet-based firms such as Datek.com, Ameritrade.com, and E-trade.com adopted early, as well as Charles Schwab, an established, traditional discount broker. Other established firms, such as Merrill Lynch and Morgan Stanley Dean Witter were late. Mendelson, Techopitayakul, and Meza (2000) provide reasons why Internet-based firms adopted early. First, the Internet appears to have reduced both the fixed and variable costs of brokerage. Second, consumers in this industry are very heterogenous in terms of price sensitivity and about 40% of them are highly sensitive, whereas the
rest exhibit low sensitivity and loyalty. Early adoption by Internet-based firms and Charles Schwab
can be explained by their emphasis on low cost trading, the convenience of on-line shopping and
the availability of a large pool of price-sensitive consumers. The Internet initially was not an ideal
medium for loyal consumers who value full-service and personalized advice, causing delay in adoption
by full-service brokers such as Merrill Lynch.

3 The model

This section presents a model stylized to analyze how the main factors outlined can generate the
observed adoption patterns.

3.1 The environment

Consider a retail market in which initially a single firm can sell to consumers through a physical
shop. The physical shop operates in a traditional, physical environment: consumers can visit the shop,
physically inspect the good, and interact with a sales person. We refer to the firm with the physical
shop as the “old firm”, to emphasize that this firm has been in the market before the e-commerce
technology arrives. There is also a “new firm”, which has no physical market presence, but can enter
the market by adopting the e-commerce technology.

The e-commerce technology enables a firm to operate a virtual shop. A virtual shop consists of
a web-site through which consumers can interact with the firm, learn about the product, place and
track orders. After this new technology arrives, the new firm can open a virtual shop and compete
with the old firm for consumers with Internet access.\footnote{We preclude the possibility that this new firm opens a physical shop in addition to the virtual shop because of little empirical relevance.} The old firm also has the option of opening a
virtual shop, in addition to its physical shop.

Time is continuous and is denoted by $t \in [0, \infty)$. The e-commerce technology arrives at $t = 0$.
Every period $t$ consists potentially of 2 stages. In the first stage, firms decide whether to adopt the
new technology, if they have not done so before. In the second stage, firms choose prices for their open
shops and consumers make their decisions.

3.2 Consumers

There is a continuum of identical risk-neutral consumers with unit demand.\footnote{The analysis generalizes to downward-sloping demand functions with no additional insight.} Each consumer has
a reservation price of 1 for the good sold in the physical shop and a reservation price of $1 - v$ for the
good sold in a virtual shop, where $v$ is in $[-1, 1]$. This specification implies that consumers may either
gain or lose some utility when they purchase on-line. Case \( v > 0 \) is representative of physical goods. For these goods, delays in consumption due to waiting for shipment, or the consumer’s inability to physically inspect the good to infer its features, may cause a utility loss. Examples are books and CD’s, for which \( v \) may be small, or apparel and furniture, for which \( v \) may be large. Case \( v < 0 \) is representative of digital goods, such as software, for which easier delivery and return via the Internet may result in higher utility.

There are three types of consumers. Some consumers, called “loyals”, have access to the physical shop, and always prefer buying from the old firm. These consumers view the product of the new firm as an unacceptable substitute for the product of the old firm.\(^{18}\) Loyalty may result from the old firm’s reputation or from trust in an established brand name built during the old firm’s long presence in the traditional market. We assume that loyalty is an asset specific to the old firm and cannot be easily imitated by the new firm. In the extensions, we also consider loyalty for the new firm. Let \( \lambda \) in \((0, 1)\) denote the measure of loyals in the market.

Another set of consumers are called “local switchers”. These consumers have access to the physical shop, and view the products of the two firms as perfect substitutes. Let \( \sigma \) in \((0, 1)\) be the measure of such consumers in the market. We assume that \( \lambda + \sigma = 1 \), so that the size of the “local market” for the old firm’s physical shop is normalized to one.\(^{19}\)

Finally, another set of consumers are referred to as “distant switchers”. Just as local switchers, these consumers view the products of the two firms as perfect substitutes. However, they are unable to buy from the old firm’s physical shop. The inability to buy from the physical shop may be due to high transportation costs or inconvenient shopping hours. These consumers can only be reached via a virtual shop. Let \( \alpha \in (0, 1) \) denote the measure of distant switchers. The magnitude of \( \alpha \) gives the strength of the market expansion effect made possible by the Internet.

At the beginning of each period, a cohort of consumers enters the market and then leaves at the end of the period to be replaced by a new cohort of consumers in the next period. Each cohort has the same proportion of loyals, local switchers, and distant switchers. We assume that loyalty persists across cohorts through some mechanism like word-of-mouth communication or reputation effects that persist over generations.\(^{20}\) At any time, only a fraction of consumers have access to the Internet.

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\(^{18}\)Such strong loyalty is a stark way of specifying consumer inertia. Our analysis can be generalized to a case where consumers differ in their degree of loyalty: some may be less loyal than others and can switch if the price is sufficiently low.

\(^{19}\)A higher value of \( \lambda \) represents a more established old firm which has converted a larger fraction of its local market into loyals.

\(^{20}\)The fact there is full consumer turnover each period simplifies our analysis by allowing us to abstract from the possibility of intertemporal substitution by consumers and to focus on firm’s adoption decisions. This issue is discussed in more detail at the end of Section 4.1.
Having access means that the consumer has the relevant equipment to buy at a virtual shop if desired. Access diffuses across consumers gradually according to an exogenous, deterministic process.²¹ All consumers gain access at the same rate, regardless of their type. Let \( a(t) \) denote the fraction of consumers with Internet access at time \( t \). We make the following assumption about \( a(t) \).

**Assumption 1** \( a(0) = 0 \), \( a(t) \) is differentiable on \( (0, \infty) \), \( a'(t) > 0 \) and \( a(t) < 1 \) for \( t \) in \( (0, \infty) \), and \( \lim_{t \to \infty} a(t) = 1 \). ■

Assumption 1 allows for many types of growth processes, including the common \( S \)-shaped diffusion of technology.²²

To summarize, a consumer’s preferences can be compactly stated as follows

\[
U = \begin{cases} 
1 - p - \gamma & \text{if purchase at the physical shop at price } p, \\
1 - v - p - \delta & \text{if purchase at a virtual shop at price } p, \\
0 & \text{if the consumer does not purchase at all,}
\end{cases}
\]

where \( \gamma = 0 \) for local switchers and for loyals, and \( \gamma = +\infty \) for distant switchers; \( \delta = 0 \) for local and distant switchers and loyals who buy at the old firm’s virtual shop, and \( \delta = +\infty \) for consumers without access to the Internet or for loyals that buy from the new firm’s virtual shop.²³

### 3.3 Firms

Let \( i = "n" \) or "o" index the new and the old firm, respectively. Similarly, we use the index \( j = "p", "vo", \) or "vn", to refer to the physical shop, the old firm’s virtual shop, and the new firm’s virtual shop, respectively. We allow for the physical and virtual shops of the old firm to charge different prices, i.e. price discrimination by the old firm is permitted because not only this practice is legal, but also it is indeed observed for multi-channel firms.²⁴

Opening a virtual shop entails an entry cost of \( K > 0 \). This cost includes any investment to implement the e-commerce technology, including the costs of web-site design and new distribution and warehousing systems. The physical shop has a marginal cost of \( c \) in \((0, 1)\). The marginal costs of

²¹For simplicity, we assume that the adoption of e-commerce by firms has no effect on the diffusion of access across consumers. Since we focus on a single industry among many others in which e-commerce diffuses, the assumption that firms in that industry takes the overall diffusion process as exogenous is plausible.

²²The assumption that the access diffuses completely in the limit can be relaxed without altering our main conclusions, at the expense of complicating the notation.

²³The assumptions that local switchers have no alternative other than the physical shop, and that distant switchers have no outside option, are made for simplicity. Reserve prices can be allowed to differ across consumer types.

²⁴For instance, Friberg, Ganslandt and Sandström (2001) document that book and CD retailers which have operations both on-line and off-line do indeed charge different prices in their physical and virtual shops.
virtual shops are both equal to \( c - \Delta \), where \( \Delta \) in \((0,c]\) is the reduction in marginal cost made possible by the e-commerce technology.\(^{25}\)

Let \( \rho \equiv \Delta - v \) be the incremental per consumer profit of a virtual shop relative to the physical shop.\(^{26}\) If \( \rho > 0 \), the virtual shop is more profitable per consumer compared to the physical shop. Case \( \rho > 0 \) applies to digital goods by definition because \( v < 0 \) for such goods, and to non-digital goods if \( v < \Delta \), i.e. if the disutility is small compared to the cost reduction. An example of the latter case could be books and CD's, for which there is little inconvenience to the consumer from shopping on-line and there may be important reductions in costs. For the time being, we assume that \( \rho > 0 \). However, some e-commerce operations can be relatively less profitable per consumer. We will analyze case \( \rho < 0 \) in the extensions to the model.

**Assumption 2** \( \rho > 0 \).

Let \( s_{it} \) be the indicator of adoption by firm \( i \), i.e. \( s_{it} = 1 \), if firm \( i \) has adopted the technology at or before time \( t \), and \( s_{it} = 0 \) otherwise. The state of adoption at time \( t \) is given by the pair \( s_t = (s_{nt},s_{ot}) \). A firm’s pricing strategy, \( d_{si}(t) \), is a rule that specifies the cumulative distribution function, \( G^j_{st}(p) \), according to which each of its open shops chooses prices at any point in time given \( s_t \).\(^{27}\) The cumulative distribution of prices charged by shop \( j \) at any point in time given \( s_t \) is denoted by \( G^j_{st}(p) \). The lowest and highest price in its support are \( p^j_{st} \) and \( p^j_{st} \), respectively. \( G^j_{st}(p) \) can be degenerate at some price, in which case the firm has a pure strategy for shop \( j \).

Let \( D^j_{st}(p) \) be the demand function for shop \( j \), given \( s_t \). Define \( \Pi^j_{st}(p) \) as the expected instantaneous profit for shop \( j \) when it charges \( p \), given the pricing strategies of both firms for their shops. Also, let \( V^j_{st}(t) \) be the maximum instantaneous profit of firm \( i \), given a pricing strategy by its rival. This profit is simply the sum of the maximum instantaneous profits of a firm’s shops.

An open-loop adoption strategy, \( A_i \), is simply a choice of an adoption time for firm \( i = n,o \). Denote firm \( i \)'s adoption time by \( t_i \). Let \( r > 0 \) be the market interest rate. The total payoff for the old firm as a leader and a follower are given by

\(^{25}\)Our analysis can accommodate differences across firms in entry cost and marginal cost. These differences would easily generate different adoption times. However, we want to maintain our focus on the impact of other parameters on the adoption decision.

\(^{26}\)The definition of \( \rho \) implies that the parameters \( \Delta \) and \( v \) are not separately identified. A high value of \( \rho \) can be attributed to high cost reduction (high \( \Delta \)), or high utility gain (high \( v < 0 \)), or both. As will be clear, the adoption decisions depend on \( \Delta \) and \( v \) only through \( \rho \).

\(^{27}\)For simplicity, we do not allow firm’s pricing strategies to depend on past history of prices. Ruling out such dependence rules out other equilibria, such as collusive ones. While this is restrictive, we believe that there is no overwhelming empirical evidence yet that points us to consider collusive equilibria in on-line markets.
\[ L^o(t_o, t_n) = \int_0^{t_o} V_{00}^o(t)e^{-rt}dt + \int_{t_o}^{t_n} V_{01}^o(t)e^{-rt}dt + \int_{t_n}^{\infty} V_{11}^o(t)e^{-rt}dt - Ke^{-rt_o}, \]
\[ F^o(t_o, t_n) = \int_0^{t_o} V_{00}^o(t)e^{-rt}dt + \int_{t_o}^{t_n} V_{01}^o(t)e^{-rt}dt + \int_{t_n}^{\infty} V_{11}^o(t)e^{-rt}dt - Ke^{-rt_o}. \]

Similarly, the payoff functions for the new firm are

\[ L^n(t_n, t_o) = \int_{t_n}^{t_o} V_{10}^n(t)e^{-rt}dt + \int_{t_o}^{\infty} V_{11}^n(t)e^{-rt}dt - Ke^{-rt_n}, \]
\[ F^n(t_n, t_o) = \int_{t_n}^{\infty} V_{11}^n(t)e^{-rt}dt - Ke^{-rt_n}. \]

### 3.4 Equilibrium

We now formally define the equilibrium of the game. A (\*) denotes equilibrium quantities and functions.

**Definition (Equilibrium).** A subgame perfect Nash equilibrium is a pair of adoption strategies \( \{A^*_i, A^*_n\} \) and a pair of pricing strategies \( \{d^*_i(t), d^*_n(t)\} \) for \( t \in [0, \infty) \) such that

i) For all \( t \in [0, \infty) \), given pricing strategies \( \{d^*_i(t), d^*_n(t)\} \) and \( A^*_i \), \( i' \neq i \), firm \( i \) chooses \( A^*_i \) to maximize its total payoff,

ii) For all \( t \in [0, \infty) \) and \( s_t \), and given \( d^*_{i'}(t) \), \( i' \neq i \), firm \( i \) chooses \( d^*_{i'}(t) \) to maximize its instantaneous profit.

The definition implies that we consider pure open-loop adoption strategies.\(^{28}\) Firms can commit to an adoption date, or, alternatively, firms cannot react to their rivals’ adoption.\(^{29}\) While equilibria in open-loop strategies have a very simple structure, they are in general not subgame perfect, especially in the case of identical firms, e.g. Reinganum (1981). However, because in our case firms are asymmetric, we are able to impose conditions and identify parameter sets for which those strategies are subgame perfect, as we show in Section 4.2.\(^{30}\) There, we also argue that the pure strategy equilibria we consider are capable of explaining the adoption patterns summarized in Table 2. Nevertheless, we cannot rule out the possibility that adoption patterns result from firms’ actions that involve strategic pre-emption or waiting associated with closed-loop (or feedback) strategies. Previous studies have focused on such closed-loop strategies and mostly on adoption equilibria in mixed strategies, as in Fudenberg and Tirole

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\(^{28}\) Note also that since \( a(t) < 1 \) for all \( t < \infty \), firms never face the same environment in successive periods, and therefore do not play a repeated game.

\(^{29}\) Our definition implicitly accounts for the effects of prices on adoption decisions. In fact, period profits are what matters for the adoption decision. To the extent that deviations from equilibrium prices affect the period profits, they affect the adoption policy as can be seen from equations (9)-(12).

\(^{30}\) Outside of those parameter sets the open-loop strategies are not subgame perfect.
(1985). In the extensions, we consider such strategies and discuss why they may also be relevant in explaining e-commerce adoption patterns.

4 Characterization of equilibrium

We first characterize the price equilibria at a given time for a given state of adoption. Then, using the instantaneous profits in these equilibria, we characterize the equilibrium adoption times.

4.1 Price equilibria

From the definition of consumer’s utility function, equilibrium behavior of different consumer types can be described as follows. Loyals buy from the old firm’s shop that offers the highest utility. Local switchers buy from the shop that offers the highest utility. Finally, distant switchers buy from the cheapest of the virtual shops. Given this behavior, we can proceed to characterize price equilibria for firms.

Consider first case \( s_t = (0, 0) \), where no firm has a virtual shop yet. Let \( \Gamma(\cdot) \) be the degenerate cumulative distribution function which assigns a probability of one to its argument. Since the physical shop is a monopoly, the old firm’s equilibrium pricing strategy is

\[
G_{00}^p(p) = \Gamma(1).
\]

The old firm sells to all consumers but distant switchers, making a profit of

\[
V_{00}^o(t) = (1 - c). \tag{3}
\]

Next, consider case \( s_t = (0, 1) \), where only the old firm has a virtual shop. Denote by \( p_j \) the price charged by shop \( j = p, vn, vo \). Assume that, when indifferent between the two shops, a consumer buys from the virtual shop. The demand functions for the two shops are

\[
D_{01}^p(p) = \begin{cases} 
1 & \text{if } p < p_{vo} + v, \ p \leq 1 \\
1 - a(t) & \text{if } p_{vo} + v \leq p \leq 1, \\
0 & \text{if } 1 < p.
\end{cases}
\]

\[
D_{01}^{vo}(p) = \begin{cases} 
a(t)(1 + \alpha) & \text{if } p < p_p - v, \\
a(t)(1 + \alpha) & \text{if } p < p_p - v, p \leq 1 - v, \\
0 & \text{if } \min\{1, p_p\} - v < p.
\end{cases}
\]

The old firm chooses its prices so that it serves all consumers with Internet access through its virtual shop, and all consumers without Internet access through the physical shop. The equilibrium pricing strategies are

\[
G_{00}^p(p) = \Gamma(1), \ G_{01}^{vo}(p) = \Gamma(1 - v),
\]
and the profit is given by

$$V_{01}^*(t) = (1 - c) + a(t)[\alpha(1 - c) + (1 + \alpha)\rho].$$

(4)

The old firm’s incremental instantaneous profit from operating a virtual shop is then

$$V_{01}^*(t) - V_{00}^*(t) = a(t)[\alpha(1 - c) + (1 + \alpha)\rho].$$

(5)

Next, turn to case $$s_t = (1, 0)$$, where only the new firm has adopted. The demand functions are

$$D_{10}^p(p) = \begin{cases} 1 & \text{if } p < p_{vn} + v, \ p \leq 1, \\ 1 - a(t)\sigma & \text{if } p_{vn} + v \leq p \leq 1, \\ 0 & \text{if } 1 < p. \end{cases}$$

$$D_{10}^v(p) = \begin{cases} a(t)(\sigma + \alpha) & \text{if } p + v \leq 1, \ p + v \leq p_p, \\ a(t)\alpha & \text{if } p + v \leq 1, \ p + v > p_p, \\ 0 & \text{if } \min\{1, p_p\} - v < p. \end{cases}$$

In this case, there exists no Nash equilibrium in pure strategies.\textsuperscript{31} To characterize the equilibrium mixed strategies, we proceed as follows. Ignoring ties, the expected profit of the new firm when it charges $$p \leq 1 - v$$ is

$$\Pi_{10}^v(p) = (p - c + \Delta)a(t)\{\alpha + \sigma[1 - G_{10}^p(p + v)]\}.$$ 

Similarly, the expected profit of the old firm when it charges $$p \leq 1$$ is

$$\Pi_{10}^o(p) = (p - c)\{(1 - a(t)\sigma) + a(t)\sigma[1 - G_{10}^v(p + v)]\}.$$ 

Let $$b_p$$ be the lowest price the old firm is willing to charge in its physical shop to sell to all consumers but distant switchers. For $$b_p$$ to be in the support of the price distribution, the profit from charging $$b_p$$ must be equal to the profit from charging $$p = 1$$. This implies

$$(b_p - c) - (1 - c)[(1 - a(t)\sigma + \lambda) \equiv 0,$$

from which we obtain

$$b_p = c + (1 - c)[1 - a(t)\sigma].$$

\textsuperscript{31}To see this, first note that the old firm’s price is always in $$[c, 1]$$, and the new firm’s price is always in $$[c - \Delta, 1 - v]$$.

Consider the different scenarios. If $$p_o \leq 1$$ and $$p_p < p_{vn} + v$$, the new firm makes no profit on local switchers, whereas by charging $$p_{vn} < p_p - v$$ it can make positive profits. If $$p_p = p_{vn} + v \leq 1$$, one of the shops can undercut the other slightly and obtain a discrete gain in profit. If $$p_{vn} + v < p_p \leq 1$$, the new firm captures all local switchers with access and can increase its profits by raising its price slightly.
Similarly, denote by $b_{10}^{vn}$, the lowest price the new firm is willing to charge to sell to all switchers with Internet access. For $b_{10}^{vn}$ to be in the support of the price distribution, the profit from charging $b_{10}^{vn}$ must be equal to the profit from charging $p = 1 - v$. This implies

$$(b_{10}^{vn} - c + \Delta)a(t)(\alpha + \sigma) - (1 - c + \rho)a(t) = 0,$$

from which we obtain

$$b_{10}^{vn} = c - \Delta + \frac{\alpha}{\alpha + \sigma}(1 - c + \rho).$$

Note that $b_{10}^{vn}$ is increasing in the measure of distant switchers, $\alpha$. The new firm can sell to distant switchers at price $1 - v$. Larger $\alpha$ implies that the opportunity cost of charging a price lower than $1 - v$ to sell to local switchers is greater. Let $\rho^c \equiv - [1 - a(t) (\alpha + \sigma)] (1 - c)$ be the lowest value of $\rho$ for which $b_{10}^{vn} \leq b^p - v$. In this section, we focus on case $b_{10}^{vn} < b^p - v$. This holds if $\alpha + \sigma < 1$, i.e. the total measure of switchers is relatively small. We discuss the other case, $b_{10}^{vn} \geq b^p - v$, at the end of this section.

If the new firm charges $b^p - v$ with probability 1, it sells to all switchers with Internet access and earns

$$\Pi_{10}^{vn} = a(t)(b^p - c + \rho)(\alpha + \sigma).$$

If the physical shop charges 1 with probability 1, it sells to all consumers without Internet access and to loyals with Internet access and earns

$$\Pi_{10}^{p} = [1 - a(t)\sigma](1 - c).$$

Using the expressions for the equilibrium expected profits for the two firms, the equilibrium price distributions can be characterized as follows.

**Proposition 1** When only the new firm has a virtual shop, the equilibrium prices are given by

$$G_{10}^{p}(p) = \begin{cases} 0 & \text{if } p < b^p, \\ \left(\frac{\alpha + \sigma}{\sigma}\right) \left(1 - \frac{(1 - c)[1 - a(t)\sigma] + \rho}{p - c + \rho}\right) & \text{if } b^p \leq p < 1, \\ 1 & \text{if } 1 \leq p, \end{cases}$$

$$G_{10}^{vn}(p) = \begin{cases} 0 & \text{if } p < b^p - v, \\ 1 - \left(\frac{1 - a(t)\sigma}{a(t)\sigma}\right) \left(\frac{1 - v - p}{p + v - c}\right) & \text{if } b^p - v \leq p < 1 - v, \\ 1 & \text{if } 1 - v \leq p. \end{cases}$$

The intuition behind Proposition 1 is clear. Both shops compete for local switchers with Internet access. Only the physical shop sells to consumers with no Internet access. For the new firm, charging a price lower than $1 - v$ entails both an expected marginal benefit associated with more sales to local
switchers with Internet access, and a marginal loss due to smaller per consumer profit on distant switchers with Internet access. Similarly, for the old firm charging a price lower than 1 entails an expected marginal benefit associated with increased sales to local switchers with Internet access, and a marginal cost associated with a smaller per consumer profit on loyals and local switchers without Internet access. Since the measure of distant switchers is smaller than the measure of loyals ($\alpha < \lambda$), the opportunity cost of charging lower prices is higher for the old firm than the new firm. As a consequence the physical shop’s price is stochastically higher. The physical shop charges $p = 1$ with positive probability, i.e. $G_{10}^{p}(p)$ has a mass point at $p = 1$. Price $p = 1$ can be interpreted as its regular price, and lower prices can be viewed as discounts to attract local switchers.

As the on-line market size $a(t)$ increases, both prices stochastically decrease. For the physical shop, a higher Internet access rate for local switchers implies a higher opportunity cost of charging high prices. Therefore, it has more incentive to charge lower prices. As a consequence, the new firm’s virtual shop also has to charge a lower price to attract these consumers. On the other hand, higher $\lambda$ leads to stochastically higher prices for both firms. When the fraction of loyals is higher, the old firm has less incentive to lower its price and compete with the new firm for local switchers with Internet access. This leads to higher prices by the new firm as well. An increase in $\alpha$ implies stochastically lower prices for the old firm, but does not change the new firm’s prices.

The equilibrium profits in this case are

$$V_{10}^{o}(t) = [1 - a(t)](1 - c),$$

$$V_{10}^{n}(t) = (\sigma + \alpha)a(t)[(1 - c + \rho) - \sigma(1 - c)a(t)].$$

An important observation is that the new firm’s profit in this case is strictly concave in $a(t)$. By charging the price $b_{10}^{o} - v$, the virtual shop of the new firm can sell to all switchers with Internet access. Note that $b_{10}^{p}$ is decreasing in $a(t)$. Higher $a(t)$ implies higher market size for the new firm’s virtual shop, but also stochastically lower prices.

Finally, consider case $s_{t} = (1, 1)$, where both firms have virtual shops. The demand functions in this case can be obtained as before and are omitted. In this case, there exists no equilibrium in which the virtual shops play pure pricing strategies. First, note that the old firm has a dominant strategy of charging the monopoly price $p = 1$ in its physical shop.\textsuperscript{32} At this price, the physical shop sells to all local consumers without Internet access. Furthermore, firms do not have pure pricing strategies for their virtual shops.\textsuperscript{33}

\textsuperscript{32}Since the virtual shop is more profitable than the physical shop, the old firm would like to induce all consumers with Internet access to buy from its virtual shop. The old firm thus never charges a price at or below $1 - v$ in its physical shop, and sells only to local consumers with no access to the Internet. But since it has monopoly power over these consumers, it charges the highest possible price of 1.

\textsuperscript{33}Any fixed price $p$ in $(c - \Delta, 1 - v]$ charged by one of the virtual shops can be undercut by the other for a discrete
We can characterize the equilibrium mixed strategies in this case following a similar approach as in the case of \( s_t = (0, 1) \). Ignoring ties, the expected instantaneous profits for the virtual shops when they charge \( p \leq 1 - v \) are

\[
\Pi_{11}^{vn}(p) = (p - c + \Delta)a(t)(\sigma + \alpha)[1 - G_{11}^{vn}(p)],
\]
\[
\Pi_{11}^{vo}(p) = (p - c + \Delta)a(t)\{\lambda + (\sigma + \alpha)[1 - G_{11}^{vo}(p)]\}.
\]

The lowest price the new firm is willing to charge is \( b_{11}^{vn} = c - \Delta \). Let \( b_{11}^{vo} \) be the lowest price the old firm is willing to charge in its virtual shop to sell to all consumers with Internet access. For \( b_{11}^{vo} \) to be in the support of the price distribution, the profit at price \( b_{11}^{vo} \) must be equal to the profit at price \( 1 - v \). In other words,

\[
(b_{11}^{vo} - c + \Delta)a(t)(1 + \alpha) - (1 - c + \rho)a(t)\lambda = 0,
\]

which yields

\[
b_{11}^{vo} = c - \Delta + \frac{\lambda}{1 + \alpha}(1 - c + \rho).
\]

Note that \( b_{11}^{vo} > b_{11}^{vn} \). If the new firm’s virtual shop charges \( b_{11}^{vo} \) with probability 1, it sells to all switchers with Internet access and earns

\[
\Pi_{11}^{vn} = a(t)(b_{11}^{vo} - c + \Delta)(\alpha + \sigma).
\]

If the old firm’s virtual shop charges \( 1 - v \) with probability 1, it sells to all loyals with Internet access and earns

\[
\Pi_{11}^{vo} = a(t)(1 - c + \rho)\lambda.
\]

Given these equilibrium profits, we can characterize the equilibrium price distributions as follows.

**Proposition 2** When both firms have virtual shops, the equilibrium prices are given by

\[
G_{11}^{p}(p) = \Gamma(1),
\]
\[
G_{11}^{vo}(p) = \begin{cases} 
0 & \text{if } p < b_{11}^{vo}, \\
1 - \left( \frac{\lambda}{1 + \alpha} \right) \left( \frac{1 - c + \rho}{p - c + \Delta} \right) & \text{if } b_{11}^{vo} \leq p < 1 - v, \\
1 & \text{if } 1 - v \leq p,
\end{cases}
\]
\[
G_{11}^{vn}(p) = \begin{cases} 
0 & \text{if } p < b_{11}^{vo}, \\
1 - \left( \frac{\lambda}{\sigma + \alpha} \right) \left( \frac{1 - v - p}{p - c + \Delta} \right) & \text{if } b_{11}^{vo} \leq p < 1 - v, \\
1 & \text{if } 1 - v \leq p.
\end{cases}
\]

Gain in profit. The case in which both virtual shops charge a price equal to the marginal cost \( c - \Delta \) is not an equilibrium, either: the old firm’s virtual shop does not make any profit on switchers, but can increase its profit on loyals with Internet access by raising its price above the marginal cost.
We note some important features of the firm’s pricing strategies. First, prices do not depend on $a(t)$, unlike in the case of $s_t = (1, 0)$. The old firm serves consumers with and without Internet access through two different shops. Therefore, the physical shop does not directly compete with the new firm’s virtual shop for local switchers with Internet access. Only the virtual shops compete for local and distant switchers with Internet access, and their prices depend only on the relative measure of loyal and switchers. Second, the old firm’s virtual shop charges stochastically higher prices compared to the new firm’s virtual shop. This follows because the loyal with Internet access always buy from the old firm’s virtual shop and this reduces the incentives for the old firm to charge lower prices to attract the switchers with Internet access. As a consequence, $G_{1v}^{vo}(p)$ has a mass point at $p = 1 - v$. Third, an increase in $\lambda$ leads to stochastically higher prices for both virtual shops, as in Proposition 1. Fourth, the prices for both virtual shops are stochastically lower for higher $\alpha$. The larger the measure of distant switchers with Internet access, the larger is the marginal return for both virtual shops to charging a price lower than $1 - v$.

The equilibrium instantaneous profits are

\begin{align}
V_{11}^o(t) &= [1 - a(t)\sigma](1 - c) + a(t)\lambda \rho, \\
V_{11}^n(t) &= a(t)\lambda \left( \frac{\sigma + \alpha}{1 + \alpha} \right) (1 - c + \rho).
\end{align}

Note here that, in contrast to the case $s_t = (1, 0)$, the new firm’s profit is linear, not strictly concave, in $a(t)$. This follows because the new firm’s pricing strategy is now independent of $a(t)$. The old firm’s incremental instantaneous profit from opening a virtual shop is

\begin{equation}
V_{11}^o(t) - V_{10}^o(t) = a(t)\lambda \rho.
\end{equation}

Observe that the physical shop’s profit decreases to zero in the limit as $a(t)$ increases, because the physical shop only sells to local consumers without Internet access. In the presence of fixed costs, the old firm shuts down the physical shop eventually. However, if the diffusion of access is never complete, and/or if fixed costs are not too high, the physical shop always remains open.\textsuperscript{34}

The analysis so far was carried out under the assumption that $b_{10}^{vo} < b^p - v$, i.e. consumer surplus at the new firm’s minimum price is higher than the surplus at the old firm’s minimum price, or equivalently, the mass of switchers is not too large, i.e. $\alpha + \sigma < 1$. When $b_{10}^{vo} \geq b^p - v$, the functional forms of both the price distributions and the profits change as $a(t)$ increases over time. Let $\rho^m$ be the upper bound for $\rho^c$. When $\rho$ is in $(0, \rho^m)$, the functional form of instantaneous profits change over time for the case $s_t = (1, 0)$. Due to this complication, it is possible, but tedious, to analyze this case. Since the additional insight for adoption patterns is not substantial, we omit this case for brevity.\textsuperscript{34}

\textsuperscript{34}Consumers’ preferences for traditional shopping experience can also lead to the same result. If there are enough consumers who always prefer buying at the physical, the physical shop remains open.
Before closing this section, we make two remarks. First, to keep the model tractable, we abstracted away from such dynamics by assuming full consumer turnover each period. In a dynamic setting like ours, consumers can engage in intertemporal substitution if they are long-lived. A consumer can wait to buy at possibly lower future prices if the current lowest price is too high. Such tendency is reinforced in the presence of mixed strategy equilibria, as prices change from one period to the other. However, if consumers’ discount factor is high, or if there is a high penalty for postponing consumption, this concern becomes less of an issue.\textsuperscript{35} Second, the nature of the mixed-strategy price equilibria and the resulting price distributions clearly depend on the exogenously specified discrete consumer types. While we believe that our specification of three consumer types (loyals, local switchers, and distant switchers) captures the mix of consumers in a general way, alternative specifications can be made and the structure of the price distributions would be different.

4.2 Adoption equilibria

To analyze adoption equilibria, we first make the following assumption about the parameters of the model.

**Assumption 3** $K < \frac{1}{r} \min\{\lambda \left( \frac{\sigma + \alpha}{1 + \alpha} \right) (1 - c + \rho), \lambda \rho \}$. ■

Assumption 3 states that the entry cost is low enough so that a firm opens a virtual shop eventually, regardless of whether it is a leader or a follower. This prevents the uninteresting case where a firm never adopts because the entry cost is very high. Note also that the assumptions $K > 0$ and $a(0) = 0$ rule out the possibility of immediate adoption by firms at time $t = 0$.

Let $\theta = (r, K, c, \rho, \lambda, \alpha)$ be the vector of parameters. It is straightforward to show that, under Assumption 1, the functions in (1) and (2) are strictly quasi-concave in their first arguments, and admit unique interior maxima, independent of their second arguments. Denote these maximizers by $t^L_i(\theta)$ and $t^F_i(\theta)$, $i = n, o$. For the old firm, $t^L_o$ and $t^F_o$ are the solutions, respectively, to the first order conditions

\begin{align*}
- V^o_{01}(t^L_o) + V^o_{00}(t^L_o) + rK & = 0, \quad (9) \\
- V^o_{11}(t^F_o) + V^o_{10}(t^F_o) + rK & = 0. \quad (10)
\end{align*}

\textsuperscript{35}Introducing dynamic decisions for consumers is promising but challenging. It is well-known that (see, e.g., Sargent and Ljungqvist (2000)), in such a dynamic environment, a consumer faces a buy-or-wait problem and holds a reservation price in deciding whether to buy. Here, the reservation price itself depends on time, as price distributions change over time. Firms would also adjust their strategies accordingly in the presence of such dynamic consumer behavior.
For the new firm, $t^L_n$ and $t^F_n$ are characterized, respectively, by \(^{36}\)

\[
-V^{n}_{10}(t^L_n) + rK = 0, \quad (11)
\]

\[
-V^{n}_{11}(t^F_n) + rK = 0. \quad (12)
\]

Given the ranking of the functions $V^{i}_{n}$, it is easy to see that $t^L_o < t^F_o$ and $t^L_n < t^F_n$.

Let $F^{\ast}(\theta) \equiv F^{i}(t^F_i, t^F_o)$. Let $t^i_\theta \equiv t^i_\theta(\theta)$ be the earliest time for which firm $i$ is indifferent between being a leader and a follower, i.e., $L^{i}(t^L_i, t^F_o) = F^{\ast}(\theta)$. Value $t^c_\theta$ exists if $L^{i}(0, t^F_o) \leq F^{\ast}(\theta)$, which we assume from now on. \(^{37}\) Obviously, $t^c_\theta < t^L_i$. Next we will define 3 sets required for the characterization of adoption equilibria. Let $\Phi^n$ be the set of parameter values for which $t^L_n < t^c_	heta$:

\[
\Phi^n \equiv \{ \theta : L^n(t^L_n, t^F_n) < L^n(t^c, t^F_o) \}.
\]

The inequality that defines $\Phi^n$ states that the old firm does not gain from pre-empting the new firm. Similarly, let $\Phi^o$ be the set for parameter values for which $t^L_o < t^c_	heta$:

\[
\Phi^o \equiv \{ \theta : L^o(t^L_o, t^F_o) < L^n(t^c, t^F_o) \}.
\]

Finally, let $\Phi^i$ be the set of parameter values for which $t^L_n \geq t^c_	heta$ or $t^L_o \geq t^c_	heta$:

\[
\Phi^i \equiv \{ \theta : L^o(t^L_n, t^F_n) \geq L^o(t^c_\theta, t^F_o) \text{ or } L^n(t^L_o, t^F_o) \geq L^n(t^c_\theta, t^F_o) \} \quad (13)
\]

Using the sets just defined, one can characterize the pure strategy equilibrium adoption dates as follows.

**Proposition 3** The pure strategy equilibrium adoption dates are given by

\[
\{t^*_n, t^*_o\} = \begin{cases} 
\{t^L_n, t^F_o\} & \text{if } \theta \text{ in } \Phi^n \\
\{t^c_\theta, t^L_o\} & \text{if } \theta \text{ in } \Phi^o.
\end{cases}
\]

For sets $\Phi^n$ and $\Phi^o$, firms adopt at different dates. For set $\Phi^i$, adoption dates also differ, but the equilibrium involves mixed strategies, as in Fudenberg and Tirole (1985), in which a follower always has an incentive to preempt its rival and become a leader in adoption. The empirical evidence reviewed in section 2 suggests, however, that in most cases new firms adopted first. We therefore focus mainly on pure strategy equilibria, and particularly the equilibria that arise under set $\Phi^n$, because such equilibria are empirically relevant. As noted earlier, the pure strategies we consider do not involve

\(^{36}\)Note that $t^L_n$ is the smaller root of the quadratic $V^{n}_{10}(t) - rK$. In other words, at $t = t^L_n$, $V^{n}_{10}(t)$ is strictly increasing in $t$.

\(^{37}\)Inequality $L'(0, t^F_o; \theta) \leq F^{\ast}(\theta)$ holds, e.g., if $L'(0, t^F_o; \theta) \leq 0$. 

20
strategic pre-emption or delay by either firm. Such actions can also explain some of the patterns observed in Table 2. For instance, the adoption gap between the leaders and the followers tend to be “short” in a relative sense, e.g. 1 or 2 years, which may result from the new firm pre-empting the old-firm. We discuss the relevance of this type of strategies in the extensions.

Before closing this section, we give some examples of adoption equilibria and show that the sets $\Phi^n$ and $\Phi^o$ are in general non-empty. The inequality that defines $\Phi^n$ can be written as $R^n(\theta) < 0$, where $R^n(\theta) = L^n(t^n_L, t^n_F) - F^n(t^n_o, t^n_o)$. The value of $R^n(\theta)$ can be easily calculated for any $\theta$. The following is an example of a parameter configuration for which the new firm always adopts first.

**Example 1** Let $\rho = 0.1$, $\alpha = 0.2$, $\lambda = 0.4$, $c = 0.1$, $K = 0.75$ and $r = 0.05$, which implies a discount rate of $1 + r = 0.952$. Note that, for this choice of parameters, we have $\alpha < \lambda$, and $rK = 0.0375 < \min\{\lambda \left(\frac{\alpha}{\alpha + c}\right) (1 - c + \rho), \lambda \rho\} = \min\{0.267, 0.04\} = 0.04$, so that Assumption 2 is satisfied.

Let $a(t) = \frac{e^t - 1}{e^t}$. This function satisfies Assumption 1. Adoption times are $t^n_L = 0.049$, $t^n_F = 0.151$, $t^o_L = 0.470$, $t^o_F = 2.772$. In this case, $R^n(\theta) = -0.013 < 0$ and $t^n_L < t^n_o = 0.136$. Thus, the parameter configuration is an element of $\Phi^n$. ■

Similarly, we can write the inequality that defines $\Phi^o$ as $R^o(\theta) < 0$, where $R^o(\theta) = L^o(t^o_L, t^o_F) - F^n(t^n_o, t^n_o)$. In the following example, the old firm always adopts first.

**Example 2** Let $\rho = 0.5$, $\alpha = 0.1$, $\lambda = 0.9$, $c = 0.5$. All other parameter values are as in Example 1. Adoption times are $t^n_L = 0.209$, $t^n_F = 0.260$, $t^o_L = 0.078$, $t^o_F = 0.087$. We then obtain $R^o(\theta) = -0.0022 < 0$ and $t^o_L < t^o_o = 0.144$. Thus, the parameter configuration is an element of $\Phi^o$. ■

5 Analysis

5.1 Timing of adoption

We first analyze how individual adoption dates change in response to changes in parameters, given a pure strategy adoption equilibrium. The next proposition summarizes the comparative statics for important parameters.\(^{38}\)

**Proposition 4.**

\(^{38}\)An increase in $r$ or $K$ delays adoption by both firms. Since these effects are straightforward, we do not elaborate on them.
(i) Consider the equilibrium where the new firm is a leader in adoption:

(i.1) An increase in the per consumer incremental profit, $\rho$, decreases both adoption dates,

(i.2) An increase in the proportion of loyals, $\lambda$, decreases the old firm’s adoption date,

(i.3) An increase in the marginal cost, $c$, increases the new firm’s adoption date, and does not change the old firm’s adoption date,

(i.4) An increase in the measure of distant switchers, $\alpha$, decreases the new firm’s adoption date and does not change the old firm’s adoption date,

(ii) Consider the equilibrium where the old firm is a leader in adoption:

(ii.1) An increase in the per consumer incremental profit, $\rho$, decreases both adoption dates,

(ii.2) An increase in the proportion of loyals, $\lambda$, does not change the old firm’s adoption date,

(ii.3) An increase in the marginal cost, $c$, increases both adoption dates,

(ii.4) An increase in the measure of distant switchers, $\alpha$, decreases both adoption dates.

Table 3 contains the results of Proposition 4 in a compact form.

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>$\rho$</th>
<th>$\lambda$</th>
<th>$c$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>New firm</td>
<td>$t_n^L$</td>
<td>↓</td>
<td>↑↓</td>
<td>↑</td>
</tr>
<tr>
<td>leads:</td>
<td>$t_n^F$</td>
<td>↓</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Old firm</td>
<td>$t_o^L$</td>
<td>↓</td>
<td>0</td>
<td>↑</td>
</tr>
<tr>
<td>leads:</td>
<td>$t_o^F$</td>
<td>↑↓</td>
<td>↑</td>
<td>↑</td>
</tr>
</tbody>
</table>

Table 3. The effect of parameters on adoption dates

Note that changes in $\rho$ affect the adoption dates of both firms in a similar way regardless of the nature of the equilibrium, whereas changes in other parameters ($c$, $\alpha$ and $\lambda$) affect adoption dates asymmetrically in at least one of the equilibria. Clearly, the cases where there are asymmetric effects are more important for the question of which firm adopts first. Consider first the equilibrium where the new firm is a leader. If incremental profitability, $\rho$, increases, then both firms adopt sooner. An increase in the measure of loyals, $\lambda$, has a potentially ambiguous effect on the new firm’s adoption date, but always leads to earlier adoption by the old firm. Higher $\lambda$ implies a higher measure of captive customers for the old firm’s virtual shop and allows the old firm’s virtual shop to charge higher prices, implying higher profit and earlier adoption. Higher $\lambda$ has two effects on the new firm’s profit. First, it implies a decrease in the measure of local switchers. Second, it leads the old firm to charge higher prices, allowing the new firm to charge higher prices. These two effects work in opposite directions. For low levels of $\lambda$, the net effect of an increase in $\lambda$ is an increase in profit, whereas for high levels of $\lambda$ the net effect is a decrease in profit. When the measure of distant switchers, $\alpha$, is small, the first effect dominates the second effect, and the new firm adopts later.
An increase in the marginal cost, $c$, increases the new firm’s adoption date, but has no effect on the old firm’s adoption date. Except for distant switchers, the old firm can sell either through its virtual shop or its physical shop. Thus, it only cares about the per consumer incremental profit, $\rho$, whereas the new firm cares about the total per consumer profit, $1 - c + \rho$. Finally, an increase in the measure of distant switchers, $\alpha$, decreases the new firm’s adoption date, but has no effect on the old firm’s adoption date. When only the new firm has a virtual shop, it is the only firm that sells to distant switchers with Internet access. An increase in the measure of distant switchers means an increase in the measure of its captive consumers, which increases the new firm’s profit, and leads the new firm to adopt sooner. When both firms have adopted, the old firm can also sell to distant switchers through its virtual shop. However, since the old firm’s virtual shop charges stochastically higher prices than the new firm’s virtual shop, the old firm, on average, does not sell to distant consumers, hence the independence of its adoption date from $\alpha$.

Next, consider the equilibrium where the old firm is a leader. An increase in $\rho$ decreases both firms’ adoption dates, as before. A change in $\lambda$ has no effect on the old firm’s adoption date. The old firm makes the same per consumer profit on loyals with Internet access and local switchers with Internet access. Hence, a change in their relative fractions does not matter. An increase in $\lambda$, raises the profit of the new firm when $\lambda$ is low, and decreases the profit of the new firm when $\lambda$ is high. An increase in $c$ increases the new firm’s adoption date, as higher $c$ implies lower profit per consumer. The old firm cares about both the incremental profit per consumer for loyals and local switchers, and the absolute profit per consumer for distant switchers. The former does not depend on $c$, and the latter decreases as $c$ increases. The overall effect is an increase in the old firm’s adoption date. Finally, an increase in $\alpha$ decreases both firms’ adoption dates. Unlike in the previous equilibrium, the old firm sells to distant switchers when it is the leader, and its profit increases as $\alpha$ does.

### 5.2 Sequence of adoption

Since the data suggest that new firms usually adopted earlier, it is important to know the conditions under which this happens. The set of parameters for which the new firm is always a leader was defined earlier by $\Phi^n$. When does the parameter vector fall in this set?

Example 1 shows that the set $\Phi^n$ is non-empty. A full analytical characterization of $\Phi^n$ is difficult because the condition that defines this set, $R^n(\theta) < 0$, is complex. However, we can characterize the sets $\Phi^n$ and $\Phi^o$ graphically, since the conditions $R^n(\theta) < 0$ and $R^o(\theta) < 0$ are prone to numerical analysis. In doing so, we will focus on the effects of relatively more interesting parameters, $\lambda$ and $\rho$.

Figure 1 plots the sets $\Phi^n$ and $\Phi^o$ in the $(\rho, \lambda)$ plane assuming that $rK = 0.01$ and all other parameter values are as in Example 1. While $\rho$ is limited to the range $[0,1]$ to keep the graph
simple, we consider all $\rho > 0$. Recall that the parameters must satisfy the constraint $\alpha < \lambda$ as well as Assumption 2. The points to the north of the line $\lambda = \alpha$ satisfy $\alpha < \lambda$. The dashed curve in Figure 1 represents the boundary $\lambda \rho = rK$. Assumption 2 requires $\lambda \rho > rK$, so the right side of this boundary is admissible. The other part of Assumption 2 requires $\lambda \left( \frac{\lambda + \alpha}{1 + \alpha} \right) (1 - c + \rho) > rK$. This is always satisfied for all $\lambda > \alpha$ in this case, regardless of the value of $\rho$.

In the region defined by the corner points A, B, C and D, we have $R^o(\theta) < 0$ and only the pure strategy equilibrium in which the new firm always adopts first applies. This region is labelled as $\Phi^o$.\(^{39}\) Example 1 is represented by the pair $(\rho, \lambda) = (0.1, 0.4)$ in this region. Note that the contour $R^o(\theta) = 0$ has a negative slope. By the shape of the contour $R^o(\theta) = 0$, for any given $\rho$ the fraction of loyals, $\lambda$, must be sufficiently low for the parameter vector to be an element of $\Phi^o$. Similarly, for any given $\lambda$, $\rho$ must be sufficiently low for the parameter vector to be in the set $\Phi^o$. Note also that when $\rho$ is sufficiently high, the pure strategy equilibrium in which the new firm always adopts first does not exist.

Next, consider the contour $R^n(\theta) = 0$, indicated by the curve between points D and E. In the region to the northwest of this contour, we have $R^n(\theta) < 0$. This region is labelled $\Phi^n$. In this region, only the pure strategy equilibrium in which the old firm always adopts first applies. Example 2 is represented by the pair $(\rho, \lambda) = (0.5, 0.9)$ in this region. The contour $R^n(\theta) = 0$ also has a negative slope, implying that, for any given $\rho$, higher values of $\lambda$ result in parameter vectors that fall in $\Phi^n$. Similarly, for any given $\lambda$, $\rho$ must be sufficiently high for the parameter vector to be in $\Phi^n$.

While Figure 1 applies for a certain choice of the parameters $\alpha$ and $c$, the shapes of the sets $\Phi^n$ and $\Phi^o$ are similar for different choices. Figure 2 investigates the change in these sets as the measure of distant switchers, $\alpha$, changes. We increase $\alpha$ from its Example 1 level of 0.2 to 0.25. This shifts the line $\lambda = \alpha$ upwards, making the constraint $\lambda > \alpha$ tighter. The other constraints on the parameters do not bind in this case. The contour $R^n(\theta) = 0$ shifts left, implying that the set $\Phi^n$ becomes smaller. The contour $R^o(\theta) = 0$ also shifts left, making the set $\Phi^o$ larger. As a result, the set of parameters for which the old firm is always a leader in adoption gets larger, whereas the set of parameters for which the new firm is always a leader shrinks. Thus, a larger distant market helps the old firm become a leader in adoption.

A qualitatively similar result is obtained for a change in the marginal cost, $c$. A decrease in $c$ shifts both contours $R^n(\theta) = 0$ and $R^o(\theta) = 0$ out to right. Since these effects are similar to the effects in the case of $\alpha$, we omit comparative statics on the marginal cost. Overall, a decrease in the marginal cost, $c$, enlarges the set of parameters for which the pure strategy equilibrium in which the new firm always adopts first. In contrast, the set of parameters for which the old firm always adopts first gets

\(^{39}\)Here, we use $\Phi^n$, in a slight abuse of its original definition, to refer to the part of $\Phi^n$ that is projected onto the $(\rho, \lambda)$ plane for given $c$ and $\alpha$. 

24
smaller. Therefore, in terms of being a leader in adoption, the new firm actually benefits from facing an established firm with lower marginal cost.

The effects of $\alpha$ and $c$ on the sets $\Phi^n$ and $\Phi^o$ are intuitive. Both firms can benefit, in general, from a larger distant market, represented by a larger $\alpha$. However, unless the old firm adopts first, it has no benefit from the distant market, as the new firm is aggressive enough to capture all distant consumers when it adopts first, as discussed earlier. Therefore, the old firm, by adopting earlier than its rival, can serve distant switchers for a while before the new firm does, and the benefit from doing so is bigger for higher $\alpha$. The old firm benefits more from opening a virtual shop when $c$ is high, because it can use its virtual shop to serve, at least, its loyal customers with Internet access at a much higher per consumer profit compared to its physical shop than in the case when $c$ is low. Therefore an increase in $c$ enlarges the set of parameters for which the old firm is always a leader in adoption.

To summarize, our analysis suggests that the new firm is always a leader in adoption when it faces an established firm with low levels of loyalty in an industry where the physical shop has low marginal cost, the e-commerce technology brings low incremental profit, and there is little opportunity for market expansion. Similarly, the established firm always leads in adoption if it enjoys relatively high loyalty, its physical shop has relatively high marginal cost, the incremental profit from e-commerce is relatively high, and the market expansion effect is relatively high.

### 5.3 Behavior of prices

The model has a number of implications for the behavior of prices. A common finding in studies of on-line prices is significant and persistent price dispersion, even for homogenous goods (see, e.g., Brynjolfsson and Smith (2000a)). In the model here, on-line price dispersion across firms emerges in both types of pure strategy adoption equilibria. The dispersion, as measured by the range of prices in the market increases with entry, and remains stable after both firms adopt, despite continuing on-line market size growth – unless there is further entry, which is not allowed in the model. An important question is whether on-line prices are always lower than off-line prices. Bailey (1998) found that on-line prices can be higher for off-line prices for the case of books, CD’s and software. Brynjolfsson and Smith (2000a), and Ancarani and Shankar (2002) concluded the reverse for such homogenous goods. In the model on-line prices are lower as long as buying on-line reduces utility, i.e. $v > 0$. Consumers must be compensated for this loss of utility. If there is utility gain, i.e. $v < 0$, prices can actually be higher on-line, as in the case of digital goods. Note also that the market-share weighted average price exhibits different behavior compared to the unweighted average price. The former considers the actual volume of transactions that takes place at each price. Brynjolfsson and Smith (2000a) and Chevalier and Goolsbee (2002) emphasize the importance of the behavior of weighted prices in understanding the
competitiveness of on-line markets. As Internet access increases and adoption occurs, market shares of the firms change, affecting the evolution of both on-line and off-line weighted average prices.

Another empirical observation is that higher on-line market share does not necessarily imply a lower price. Brynjolfsson and Smith (2000a) found that during their sample period retailers with high on-line market shares actually charged higher prices. The model suggests loyalty as one possible explanation for this observation. Higher loyalty implies a higher on-line market share and a higher price for the old firm, consistent with the evidence that retailers with traditional market presence charge higher prices on-line compared to pure Internet-based retailers (see, e.g., Ancarani and Shankar (2002) and Pan, Shankar and Ratchford (2002)). The patterns of pricing in the early stages of e-commerce thus depended on the types of firms in the market.

Another issue is the relevance of mixed pricing strategies in on-line markets. Baye, Morgan and Scholten (2001) find that observed pattern of pricing in on-line markets can be reconciled with basic models of price competition involving mixed strategies. Iyer and Pazgal (2001) observe that the identity of the minimum priced retailer changes from one period to the other in their sample of CD retailers, consistent with mixed strategy pricing in their model. Similarly, Arbatskaya and Baye (2002) find that maximum and minimum prices as well as the identity of the firms charging these prices change frequently in on-line mortgage markets. In accordance with these empirical findings, our model suggests that the price rank of retailers can change from one period to the other.

6 Extensions

6.1 Other types of adoption equilibria

So far we discussed open-loop adoption strategies that lead to equilibria in pure strategies. Open-loop strategies are functions of calendar-time only. Firms precommit to their adoption dates and do not change them in response to their rivals’ actions. Such strategies are plausible when firms do not observe each other’s adoption dates or observe them with a lag, or when implementing the technology takes time. It is well-known that equilibrium in open-loop strategies is not subgame perfect in the case of a symmetric duopoly (see Fudenberg and Tirole (1985)). In our case, however, the asymmetry between firms allowed us to identify subsets of the parameter space where the pure strategy open-loop equilibria we investigated are indeed subgame perfect. As a result, each firm’s adoption time can be characterized as a simple break-even point: adoption occurs at the instant the marginal benefit of adoption equals its marginal cost. We used the pure strategy open-loop equilibria to explain the observed adoption patterns because the structure of these equilibria are very simple and comparative statics with respect to the parameters can be performed easily. Moreover, as we argue in Section 7,
these equilibria seem to describe the e-commerce adoption process well. Nevertheless, there are also equilibria in closed-loop strategies, which may explain some of the adoption patterns in Table 2.

Closed-loop strategies allow firms to condition their adoption decisions on their rivals’ actions, not just on calendar time. Closed-loop strategies emerge when firms can easily observe each others’ actions and when firms can implement the technology instantaneously. These features are relevant for e-commerce, because Internet allows firms to easily monitor each other’s adoption, and implementing e-commerce technology is usually much faster than setting up a traditional retail business.

Closed-loop strategies were studied by Fudenberg and Tirole (1985), Hoppe (2000), and Götz (2000). These strategies prescribe an action, “adopt” or “do not adopt”, for the old and the new firm at any point in time \( t \), conditional on the history of the game upto time \( t \). Information and reaction lags are assumed to be negligible. So if one firm adopts, the other may follow consecutively, but at the same instant of time. To accommodate both mixed and pure strategies, define a pair of closed-loop strategies \( \{H_o(t),H_n(t)\} \) as two cumulative probability distributions over \( t \in [0, \infty) \), where \( H_i(t) \) is the probability that firm \( i = o,n \) has adopted at or before time \( t \), conditional on whether its rival has adopted before time \( t \). A closed-loop equilibrium in adoption strategies is defined by a pair \( \{H_o^*(t),H_n^*(t)\} \) such that each firm’s strategy is a best response to the other’s for every subgame starting at each \( t \in [0, \infty) \). As in Fudenberg and Tirole (1985) and Hoppe (2000), two different types of closed-loop equilibria emerge here: one involving pre-emptive strategies where one or both firms pre-empt their rival, and the other involving strategic waiting, where one or both firms delay adoption so as not to invoke a quick response from their rival.

Consider first the pre-emption equilibria. As described in section 4.2, when the parameters are in the set \( \Phi^f \) defined in (13), one or both of the firms want to pre-empt the rival firm. If only one of the inequalities in the definition of \( \Phi^f \) holds, then the only equilibrium is in pure strategies. Depending on which inequality holds, either the new firm pre-empts the old firm and the adoption dates are \( \{t^*_n,t^*_o\} = \{t^L_o,t^F_o\} \), or the old firm pre-empts the new firm and the adoption dates are \( \{t^*_n,t^*_o\} = \{t^F_n,t^L_n\} \). If both inequalities hold at the same time, each firm wants to pre-empt the other. There exists then no pure-strategy equilibria and the equilibrium is in mixed-strategies.\(^{40}\) There are

\(^{40}\)To see this, assume that \( t^F_o < t^L_n \), which implies \( \max\{t^c_o,t^c_n\} \leq t^L_n \). The other case, \( t^L_o < t^F_n \), is similar. Clearly, it is in each firm’s interest to adopt exactly at \( t^c_i \), \( i = n,o \), if the rival has not adopted at or before that time. Suppose that the new firm’s pure strategy is to not adopt until \( t^c_n \) and adopt at \( t^c_n \) with probability one, if its rival has not adopted yet. Anticipating this strategy, the old firm’s best response is to pre-empt the new firm by adopting at \( t^L_n - \varepsilon \) with probability one, by the definition of \( \Phi^f \). But if the old firm plans to adopt at \( t = t^L_n - \varepsilon \) with probability one, the new firm can improve its payoff by adopting at time \( t - \varepsilon \) with probability one, again by the definition of \( \Phi^f \). Reasoning backwards, for any \( t \in [\max\{t^c_n,t^c_o\},t^L_n] \), a firm wants to pre-empt its rival to prevent being pre-empted later on. Before \( t^c_o \), it is a dominant strategy not to adopt for \( i = n,o \). Thus, there is no pure adoption strategy. The equilibrium involves one of the firms adopting at \( \max\{t^c_n,t^c_o\} \) with some positive probability and the other following at \( t^c_o \) or \( t^c_n \), depending on which
two possible equilibrium outcomes: either the new firm pre-empts the old firm or the old firm pre-empts the new firm. The probability with which each outcome is observed depends on the parameters. The threat of pre-emption does not lead to equalization of payoffs, unlike in the case of Fudenberg and Tirole (1985), because firms are asymmetric and their profit functions differ. Firms’ equilibrium mixed strategies are also not identical.

Consider next the closed-loop equilibria with waiting. Waiting strategies emerge when the parameters are such that when a firm adopts early, its rival wants to follow quickly, dissipating the leader’s gain from early adoption. Therefore, one or both firms firm hold back adoption so as not to trigger an adoption by the rival firm too soon. As we argue below, however, the equilibria that involve waiting by one or both of the firms do not seem to be relevant for the adoption patterns we observe.

What are the empirical implications of the different types of equilibria? If the open-loop pure-strategy equilibria are relevant, either the new firm or the old firm should always lead. Similarly, pure-strategy closed-loop equilibria with pre-emption imply that one of the firms should always lead, but the adoption date of the pre-empting firm is earlier than that firm’s pure-strategy open-loop adoption date as a leader. Mixed-strategy closed-loop equilibria with pre-emption imply that the new firm should adopt in some, but not all, cases and the old firm should adopt in others, depending on the probability with which each firm pre-empts the other. In the case of waiting equilibria, technology should not be adopted very early by either firm, where the “earliness” of adoption is judged relative to the time the new technology becomes available for adoption by both firms. In our case, it is reasonable to assume that the e-commerce technology became available in a form that can be readily implemented by firms around early to mid-1990’s, although the technology existed in a more primitive form before 1990.

To assess the relevance of different types of equilibria, consider again two key observations from Table 2: (i) new firms adopted first in almost all cases, and (ii) new firms adopted sufficiently early during mid-1990’s when the e-commerce technology was fairly new. Observation (i) rules out closed-loop pure-strategy equilibrium in which the old firm pre-empts the new firm. It is also hard to reconcile this observation with closed-loop mixed-strategy pre-emption equilibria. If firms were indeed using mixed strategies, we should have observed at least in some cases the old firm adopting first. However, in only one (clothing and apparel) of the 10 industries in Table 2 old firms were clear leaders in adoption. One cause for this pattern may be a non-representative sample biased heavily in favor of industries where new firms tended to adopt first. Our documentation covers many industries and firms, suggesting that such bias is not likely. Therefore, if the equilibrium indeed involves mixed strategies, it must be the case that at a given period the likelihood that the new firm adopts, given that the old firm has not adopted yet, is very high, and the likelihood that the old firm adopts, given that the

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firm is the follower.
new firm has not adopted yet is very low. In other words, the probability that the new firm preempts the old firm must be very high. In any case, the adoption patterns in clothing and apparel do not seem to result from a random outcome of mixed strategies where the probability that the old firm pre-empts its rival is very low. Rather, the nature and structure of this industry seem to be conducive for established firms to be always leaders. As discussed in Section 7, when the parameters are chosen to describe the environment of this industry, a pure-strategy equilibrium is more plausible.

In view of observation (ii), there seems to be no delay in adoption by new firms. Delay by traditional firms does also not appear to be relevant. If that was the case, we would have observed more industries where the old firm was the leader, but adopted sufficiently late, probably in the late 1990’s or early 2000’s, as opposed to early to mid-1990’s. Such a pattern is not observed in Table 2.

The discussion so far leaves room for pure-strategy closed-loop equilibria in which the new firm always pre-empts the old firm. In these equilibria, new firms adopt at time $t^L_0 < t^L_n$, whereas in open-loop pure-strategy equilibrium in which the new firm always leads, the new firm’s adoption occurs at $t^L_L < t^L_0$. This difference between these timings of adoption can, in principle, be used to distinguish between the two equilibria. However, there does not seem to be a straightforward way to estimate $t^L_n$ and $t^L_0$ separately. It is therefore difficult to distinguish between the two equilibria. Our preference for open-loop equilibria reflects their simple structure and tractability.

6.2 Digital goods

We argued earlier that for the class of digital goods the convenience of on-line delivery and return, as well as easier access to product information through on-line demos, can result in higher consumer utility compared to the physical market, i.e. $v < 0$. Moreover, selling these products on-line can entail substantial cost savings, i.e. high $\Delta$, as they require neither storage space nor transportation. Therefore, for these products the Internet is likely to become the dominant channel of sale. Our analysis for non-digital goods for the case of positive $\rho$ then applies to the case of digital goods with one exception: For these goods, consumers hold a higher reservation price for the product sold by a virtual shop, i.e., $1 - v > 1$. Consequently, virtual shops charge stochastically higher prices than the physical shop to extract some of this extra surplus. Therefore, digital product prices can be higher in the virtual market.

6.3 Low on-line profitability and cannibalization

We now consider the equilibrium for the case when a virtual shop is less profitable than the physical shop, i.e. Assumption 2 now reads $\rho < 0$. For this analysis, we need to modify Assumption 3 such that $K < \frac{1}{\lambda} \left( \frac{\pi + \alpha}{1 + \alpha} \right) (1 - c + \rho)$, because the old firm’s incremental per period profit, $\lambda \rho$, is
negative when $\rho < 0$. A virtual shop is still profitable in absolute sense, i.e. per consumer profit is $1 - c + \rho > 0$. However, the old firm’s total profit depends on the loss from selling to loyals and local switchers at a negative incremental per consumer profit, and the gain from selling to distant switchers at a positive absolute per consumer profit. The old firm thus faces some degree of cannibalization and it can even give up adoption if $\rho$ is sufficiently low.

The prices and profits for case $\rho < 0$ are in Appendix B. When the new firm adopts earlier than the old firm, its pricing strategy in the virtual market is so aggressive that it captures all distant switchers, because it has no loyal customers to rely on. Consequently, the old firm has no incentive to adopt as a follower when $\rho < 0$, otherwise it makes lower profit on its loyal customers. Does the old firm also give up adoption as a leader? From Appendix B, the critical value of $\rho$ below which the old firm never adopts as a leader is given by $\rho_L = -\frac{\alpha}{1 + \alpha}(1 - c)$, which is decreasing in the share of distant switchers, $\alpha$, and in the profitability of the physical shop, $(1 - c)$. When $\rho$ is in $(\rho_L, 0)$, the old firm can open a virtual shop. In this case, it gives up a per consumer profit of $\rho < 0$ on local switchers with Internet access to be able to sell to distant switchers with Internet access at a per consumer profit of $1 - c + \rho > 0$, partially “cannibalizing” its local customers. Thus, a negative on-line per consumer profitability does not necessarily imply that the old firm is never a leader in adoption. However, if there is no market expansion effect, i.e. $\alpha = 0$, then the old firm never adopts when $\rho < 0$.

What are the implications of these results on the view that cannibalization was an important deterrent of adoption? When the market expansion effect is large the virtual market has to be highly unprofitable compared to the physical market for the established firm not to adopt. A delay in adoption by an established firm is thus more likely to occur when market expansion is small and the virtual market is relatively unprofitable.

### 6.4 Loyalty for the new firm

The basic model considers loyalty for the old firm only. However, Internet-based firms also invest heavily in loyalty programs and some firms, such as Amazon.com, already seem to enjoy significant loyalty. We now consider briefly the implications of gradually developing loyalty for the new firm. We specify a relatively simple process by which loyalty develops: A constant fraction $\beta$ of switchers that have just gained Internet access at time $t$ become loyal to the new firm. The mass of loyals for the new firm is then $\beta(1 - \lambda + \alpha)a(t)$ at time $t$, and it grows at a rate $\beta(1 - \lambda + \alpha)a'(t)$. The model can be easily worked out with this specification. Instead of repeating the full analysis, we provide a summary of the results. The cases of interest are $s_t = (1, 0)$ and $s_t = (1, 1)$. In case $s_t = (1, 0)$, a higher $\beta$ implies a higher mass of captive consumers for the new firm. As a result, the opportunity cost of the new firm from charging a low price to sell to switchers with Internet access increases. In contrast, the
marginal benefit of the old firm from charging a low price to sell to switchers with Internet access is lower. Both of these effects lead to stochastically higher prices by both firms. The value of the new firm, $V_{10}^n$, is strictly increasing in $\beta$, implying a higher profit for the new firm and a sooner adoption.

When $s_t = (1, 1)$, the prices and profits depend on how large the loyal customer bases for the two firms are. If the virtual shop of the old firm has a larger base of loyal customers than the virtual shop of the new firm, then the virtual shop of the old firm is more reluctant to cut its price. Otherwise, it is the virtual shop of the new firm that has less incentive to lower its price. The value of the new firm’s virtual shop $V_{11}^n$ is strictly increasing in $\beta$, and the new firm adopts sooner when it is a follower.

In summary, the effect of growing loyalty for the new firm is to speed up its adoption both when it is a leader and a follower. Interestingly, the adoption time of the old firm is not affected neither as a leader nor a follower.

### 6.5 The case with many firms

The extension of the model to the case of more than two firms is non-trivial and can be carried out in many ways. We focus on the case with one established firm and several new firms. This specification can represent an environment where there is a dominant old firm in the traditional market with a loyal base of customers. If potential entrants are identical, then as soon as any two of them are in the market, competition between them for switchers with on-line access derives their profits to zero. There exists no equilibrium in which more than one new firm adopts. A reasonable scenario that prevents this outcome is to assume that each new firm starts to acquire loyal customers at a constant rate as soon as it enters, as in Section 6.3. For simplicity, assume that the rate of loyalty development is identical across all new firms. In this case, it can be shown that when $k > 1$ new firms have entered, their instantaneous profits can be ranked in order of entry: the earliest entrant has the largest share of loyals and makes the highest profit, and the later entrants make successively lower profits. This property makes early adoption attractive for new firms and increases the incentives for a new firm to preempt the other new firms. A complete characterization of equilibria for this case is beyond the scope of this paper and is left for future work. We conjecture that competition between new firms to adopt early will enlarge the set of parameters under which one of the new firms leads in adoption.

### 7 Reconciling theory with empirical evidence

In this section, we relate the implication of the model to adoption patterns in various industries. We start with book retailing. In this industry, homogeneity of the product, easy verification of product features, and convenient on-line search for titles may have resulted in some utility gain for consumers when shopping on-line, i.e. a low $v < 0$, or little utility loss, i.e. low $v > 0$. Inventory, real estate,
and personnel costs are likely to be less for an on-line book retailer resulting in high cost savings, i.e. a high $\Delta$. Together, these imply a moderate-to-high level of incremental profitability, $\rho > 0$, in the virtual market for books. Furthermore, the relative homogeneity of these products may have rendered any advantages of loyalty for traditional firms less influential. Cassidy (2002) reports that, shortly before Amazon.com was founded, Barnes and Noble and Borders together shared close to a quarter of the market before Amazon.com moved in. How much of that market share was made up of loyal customers is unknown, but the combined market share of the two firms, which is an upper bound on the extent of loyalty, was not too high. Indeed, the book retailing industry remains relatively fragmented in the traditional market, and there are “no eight-hundred-pound gorillas in book publishing and distribution” (see Martin (1996)). Such an environment is conducive for early adoption by new firms. The adoption patterns for books can be generated by parameter configurations that involve moderate or high $\rho$ and low $\lambda$.

In clothing and apparel, most early adopters were established firms. The importance of loyalty in this product category may have given an advantage to traditional retailers such as Gap. Furthermore, for traditional firms, synergies between on-line and in-store operations may have resulted in lower marginal cost, i.e. a high $\Delta$, and a high utility for consumers, i.e. high $v < 0$. This implies a high, positive $\rho$ for these firms. This was not the case for new firms, which did not enjoy loyalty or synergies. Overall, the adoption patterns in apparel and clothing can be replicated by parameter configurations involving high $\lambda$ and high $\rho$ values.

Finally, consider the case of brokerage, as an example for digital goods. Stocks are convenient to trade on-line, which means that $v$ is negative and large in absolute value, and cost savings are likely to be significant, which means that $\Delta$ is large. The incremental profitability $\rho$ is thus likely to be positive and high. The Internet is also an ideal medium for searchers who look for cheap deals with little service. As discussed earlier, the presence of a large number of searchers in this market, i.e. a low loyalty, may have promoted entry by new firms. Early entry by the established firm Charles Schwab can also be explained within the model’s framework. Charles Schwab is a discount broker. Its clients are mostly interested in executing trades at a low cost, making on-line trading more convenient for consumers and cheaper for firms compared to traditional trading. In terms of the model, Charles Schwab had a high, positive $\rho$, which could explain why it adopted e-commerce very quickly. On the other hand, Merrill Lynch, a full-service broker, was a late adopter. Its clients demand a wide range of services that go beyond simply executing trades. For these services, the Internet is less convenient for consumers, and less cost-effective for firms. In terms of the model, Merrill Lynch had a smaller $\rho$ than Charles Schwab, which could explain its late adoption despite the loyalty exhibited by its customers who value full service.

Overall, early adoption by new firms can be explained by a combination of factors included in the
model, depending on product category and firm type. To what extent each factor plays a role in a given case is an empirical question. Lieberman (2002) is a recent step towards understanding on-line environments conducive for adoption by new firms.

8 Conclusion

We analyzed retailers’ incentives to adopt electronic commerce, emphasizing the effect of technology, preferences, consumer inertia, and market expansion on entry decisions and post-entry dynamics of prices. While other explanations, such as favorable financial markets, ample venture capital, and irrational behavior by entrepreneurs, can account for early adoption by new firms, the model here has focused on important differences between the traditional and virtual markets, and between established and new firms. The results provide a simple characterization of market environments conducive for adoption of e-commerce by new versus established firms and the observed adoption patterns can be explained by equilibria resulting from the model under different parameter configurations. The model provides guidance in assessing what might happen in other sectors that have not embraced e-commerce yet. Furthermore, the post-adoption price dynamics is broadly consistent with available empirical evidence. The analysis also demonstrates that the simple technology adoption framework can be extended to analyze entry decisions in more complicated market environments. Similar applications can be made in other settings where it is important to recognize the features of the market environment in the stage game. As an extension, it would be interesting to endogeneize the diffusion of Internet access among consumers by making the diffusion of the Internet among consumers a function of firms’ adoption and pricing decisions. This extension would allow firms to influence consumers’ adoption decisions using their adoption and pricing strategies. Another promising avenue is to introduce a process by which loyalty develops over time endogenously as a result of competition, in contrast to exogenous loyalty used here.

References


A Proofs

Proof of Proposition 1. Denote by $m_j(p) \geq 0$ the size of a mass point at price $p$ in shop $j$'s price distribution. Let

$$A_j = \begin{cases} 1 & \text{if } j = p, \\ 1 - v & \text{if } j = vn, \end{cases}$$

be the highest price for which shop $j$ has a strictly positive demand. We first prove the following lemma.

Lemma.

(i) $b^p \leq \bar{p}_{10}^j \leq \bar{p}_{10}^j \leq 1$ and $b^p_{10} - v \leq \bar{p}_{10}^m \leq \bar{p}_{10}^n \leq 1 - v$.

(ii) If $\bar{p}_{10}^j \leq \bar{p}_{10}^j$ and $m_j(\bar{p}_{10}^j) = 0$, then $\bar{p}_{10}^j = A_j$. If $\bar{p}_{10}^j \leq \bar{p}_{10}^j$, then $\lim_{p \uparrow A_j} G_{10}^j(p) = G_{10}^j(\bar{p}_{10}^j)$. If $\bar{p}_{10}^j \leq \bar{p}_{10}^j$ and $m_j(\bar{p}_{10}^j) = 0$ then $\lim_{p \uparrow A_j} G_{10}^j(p) = \lim_{p \uparrow \bar{p}_{10}^j} G_{10}^j(p)$.

(iii) $p_{10}^0 = 1$ and $\bar{p}_{10}^0 = 1 - v$.

(iv) $G_{10}^0 = \Pi_{10}^0 = 0$ and $G_{10}^m = (b^p - c)(\alpha + \sigma)a(t)$.

(v) $\Pi_{10}^0 = \Pi_{10}^0 = b^p - v$.

(vi) $G_{10}^0(\cdot)$ is continuous on $[b^p, 1]$ and $G_{10}^m(\cdot)$ is continuous on $[b^p - v, 1 - v]$.

Proof of Lemma.

(i) If $p > 1$, then $0 \leq \Pi_{10}^0(p; G_{10}^j) = 0$, whereas if $p > 1$, physical shop earns at least $\Pi_{10}^0(p; G_{10}^j) \geq \Pi_{10}^0(1; G_{10}^j) = [1 - \sigma a(t)](1 - c)$. Thus, $\bar{p}_{10}^j \leq 1$. If $p < b^p$, then by definition of $b^p$, $\Pi_{10}^0(p; G_{10}^j) < \Pi_{10}^0(1; G_{10}^j)$. Thus, $b^p \leq \bar{p}_{10}^j$. The proof of the second part is similar and is omitted.

(ii) Given the assumptions, $G_{10}^j(\bar{p}_{10}^j) = 1$. Thus, $\Pi_{10}^0(\bar{p}_{10}^j; G_{10}^j)$ is strictly increasing in $\bar{p}_{10}^j$ for $\bar{p}_{10}^j < A_j$, and has a maximum at $\bar{p}_{10}^j = A_j$. Furthermore, value $\Pi_{10}^0(p; G_{10}^j)$ is strictly increasing in $p$ for $p > \bar{p}_{10}^j$, and is strictly increasing for $p = \bar{p}_{10}^j$ if $m_j(\bar{p}_{10}^j) = 0$.

(iii) The first part follows from (i) and (ii). Suppose that $\bar{p}_{10}^j < 1 - v$. Then, from (iii) $\lim_{p \uparrow 1} G_{10}^j(p) = G_{10}^j(1 - v)$ and $\Pi_{10}^m(\bar{p}_{10}^m; G_{10}^j) < \lim_{p \uparrow 1 - v} \Pi_{10}^m(p; G_{10}^j)$.

(iv) The first part follows from (iii). For the second part, we have for $\rho > \rho^c$, $\Pi_{10}^m(\bar{p} - v; G_{10}^j) = (b^p - c + \rho)(\alpha + \sigma) > \Pi_{10}^m(1 - v; G_{10}^j) = (1 - c + \rho)(\alpha + \sigma)$.

(v) Suppose $b^p < \bar{p}_{10}^p$. Then for $p' \in [b^p - v, b^p - v)$, $\Pi_{10}^m(p'; G_{10}^j) > \Pi_{10}^m(b^p - v; G_{10}^j) = (b^p - c + \rho)(\alpha + \sigma)$, which contradicts (iv). The second part follows from the first part.

(vi) Suppose that $G_{10}^0(\cdot)$ is discontinuous at $p' \in [b^p, 1)$. For $p' > b^p$, it pays for the new firm's virtual shop to transfer all mass from an $\epsilon$-neighborhood above $p' - v$ to some $\delta$-neighborhood below $p' - v$. For $p' = b^p$, it pays for the new firm's virtual shop to transfer mass from an $\epsilon$-neighborhood above $p' - v$ to $1 - v$. Thus, there is an $\epsilon$-neighborhood above $p' - v$ on which the new firm's virtual shop will put no mass. But then it cannot be an equilibrium strategy for the physical shop to put...
mass on \( p' \). For the second part, for \( p' \in \left[ b^p - v, 1 - v \right) \), the proof follows from the first part, replacing \( p' - v \) by \( p' + v \) and noting that the reasoning also applies for \( p' = 1 - v \).

Given the Lemma, the proof of Proposition 1 now follows from the equilibrium conditions \( \Pi_1^L(p) = \Pi_1^R(p) \), which can be solved to yield the expressions of the price distributions. The fact that the price distributions constitute a Nash Equilibrium and are unique follows from the arguments in Narasimhan (1988) and Osborne and Pitchik (1986).

**Proof of Proposition 2.** That the physical shop has a dominant strategy was shown in Footnote 29. The rest of the proof follows essentially the identical steps in the proof of Proposition 1.

**Proof of Proposition 3.** All we need to show is that each firm’s strategy is a best response to the other’s. By the definition of \( \Phi^n \), if \( \theta \) is in \( \Phi^n \) and the new firm adopts at \( t^L_n \), the old firm has no incentive to deviate and preempt the new firm. Therefore, it adopts at the unique date \( t^F_o \) that maximizes its payoff as a follower. Given that the old firm adopts at date \( t^F_o \), \( t^L_n \) is the unique optimal response by the new firm, because \( t^L_n \) maximizes its payoff as a leader. Hence, \( \{t^L_n, t^F_o\} \) is a pure strategy equilibrium in adoption dates. A similar argument can be made to show that \( \{t^F_o, t^L_n\} \) is also a pure strategy equilibrium.

**Proof of Proposition 4.** All the parts follow from applying the implicit function theorem to the first order conditions in (9) - (12). Let \( H \equiv V^o_{10} - V^o_{06} \) and \( J \equiv V^o_{11} - V^o_{10} \). For any given parameter \( \mu \), the derivatives of the adoption times are given by

\[
\frac{\partial t^L_n}{\partial \mu} = -\frac{\partial V_{10}^n}{\partial \theta} \frac{\partial \theta}{\partial \mu}, \quad \frac{\partial t^F_o}{\partial \mu} = -\frac{\partial V_{11}^n}{\partial \mu}, \quad \frac{\partial H}{\partial \mu} = -\frac{\partial H}{\partial \mu}, \quad \frac{\partial J}{\partial \mu} = -\frac{\partial J}{\partial \mu} \tag{14}
\]

It is easy to verify that \( \frac{\partial V_{11}^n}{\partial t} > 0 \), \( \frac{\partial H}{\partial t} > 0 \), and \( \frac{\partial J}{\partial t} > 0 \) for all \( t \). While \( \frac{\partial V_{10}^n}{\partial t} \) can be either positive or negative depending on \( t \), the new firm’s adoption as a leader always occurs for \( t \) that satisfies \( \frac{\partial V_{10}^n}{\partial t} > 0 \) (see also footnote 33). This region is defined as \( \{t : a(t) \in \left[ 0, \frac{1-c+c^2}{2(1-c)} \right] \} \) , where \( \frac{1-c+c^2}{2(1-c)} \) is the rate of access at which \( V^o_{10} \) is at its maximum. Therefore, each derivative in (14) has the opposite sign of the numerator on the right hand side. The signs of each derivative are

(i.1) \( -\frac{\partial J}{\partial \theta} = -\lambda a(t) < 0 \), \( -\frac{\partial V_{10}^n}{\partial \theta} = -(\sigma + \alpha) a(t) < 0 \),

(i.2) \( -\frac{\partial J}{\partial \alpha} = \rho a(t) < 0 \), \( -\frac{\partial V_{10}^n}{\partial \alpha} = a(t) [(1-c)(1-2\sigma+\alpha a(t)) + \rho] < 0 \),

(i.3) \( -\frac{\partial J}{\partial \sigma} = 0 \), \( -\frac{\partial V_{10}^n}{\partial \sigma} = (\sigma + \alpha)(1-\sigma a(t)) a(t) > 0 \),

(i.4) \( -\frac{\partial J}{\partial \mu} = 0 \), \( -\frac{\partial V_{10}^n}{\partial \mu} = -a(t)[(1-c)(1-\sigma a(t)) + \rho] < 0 \),

(ii.1) \( -\frac{\partial H}{\partial \theta} = -(1+\alpha) a(t) < 0 \), \( -\frac{\partial V_{11}^n}{\partial \theta} = -\lambda \left( \frac{\sigma+\alpha}{1+\alpha} \right) a(t) < 0 \),

(ii.2) \( -\frac{\partial H}{\partial \alpha} = 0 \), \( -\frac{\partial V_{11}^n}{\partial \alpha} = -(1+c+\rho) \left[ \frac{1-2\lambda+\alpha}{1+\alpha} \right] a(t) < 0 \),

(ii.3) \( -\frac{\partial H}{\partial \sigma} = \alpha a(t) > 0 \), \( -\frac{\partial V_{11}^n}{\partial \sigma} = \lambda \left( \frac{\sigma+\alpha}{1+\alpha} \right) a(t) > 0 \),

(ii.4) \( -\frac{\partial H}{\partial \mu} = -(1+c+\rho) a(t) < 0 \), \( -\frac{\partial V_{11}^n}{\partial \mu} = -\lambda \left( \frac{1-\sigma}{1+\alpha} \right) (1-c+\rho) a(t) < 0 \).
B Prices and profits for case $\rho < 0$

Prices and profits for case $\rho < 0$ are obtained following the basic steps in the proofs of Propositions 1 and 2. For $s_t = (0, 0)$, the price and profit of the old firm are the same as in the case of $\rho > 0$. The old firm does not adopt if

$$V_{01}^o(t) - V_{00}^o(t) = a(t)[(1 - c) + (1 + \alpha)\rho] < 0. \quad (15)$$

From (15), the lowest value of $\rho$ for which the old firm adopts can be obtained as $\rho_L = -\frac{c}{1 + \alpha}(1 - c)$. For $\rho < \rho_L$ the old firm’s virtual shop is inactive and the old firm makes a profit of $(1 - c)$. Therefore, for $s_t = (0, 1)$, the old firm’s profit is

$$V_{01}^o(t) = \begin{cases} a(t)[(1 - c) + (1 + \alpha)\rho] & \text{if } \rho \in (\rho_L, 0), \\ (1 - c) & \text{if } \rho \in (-1 - c, \rho_L). \end{cases}$$

Next, consider $s_t = (1, 0)$. For $\rho$ in $(\rho^c, 0)$, the prices and profits are the same as in the case of $\rho > 0$. If $\rho < \rho^c$, prices are given by

$$G_{10}^p(p) = \begin{cases} 0 & \text{if } p < b_{10}^v + v, \\ 1 - \left(\frac{\alpha}{\sigma}\right)\left(\frac{1-p}{p-c+p}\right) & \text{if } b_{10}^v + v \leq p < 1, \\ 1 & \text{if } 1 \leq p, \end{cases}$$

$$G_{10}^{cn}(p) = \begin{cases} 0 & \text{if } p < b_{10}^v, \\ 1 + \left(\frac{1-a(t)\sigma}{a(t)\sigma}\right) - \left(\frac{\alpha}{\sigma}\right)^{(1-c+p)} - \rho & \text{if } b_{10}^v \leq p < 1 - v, \\ 0 & \text{if } 1 - v \leq p. \end{cases}$$

The profits are given by

$$V_{10}^n(t) = \begin{cases} (\sigma + \alpha)a(t)[(1 - c + \rho) - \sigma(1 - c)a(t)] & \text{if } \rho \in (\rho^c, 0), \\ a(t)(1 - c + \rho)\alpha & \text{if } \rho \in (-1 - c, \rho^c), \end{cases}$$

$$V_{10}^o(t) = \begin{cases} (1 - \sigma a(t))(1 - c) & \text{if } \rho \in (\rho^c, 0), \\ \frac{(1-c+a(t))(1-c)}{\alpha + \sigma} - \rho & \text{if } \rho \in (-1 - c, \rho^c). \end{cases}$$

Finally, the old firm never opens a virtual shop when $\rho < 0$, so case $s_t = (1, 1)$ does not apply. Because the new firm has no loyal customers to rely on, it prices so aggressively that the old firm never gets to sell to any of the distant switchers.
<table>
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<th>Category</th>
<th>1999</th>
<th>2000</th>
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<th>Average</th>
<th>% Avg. Growth</th>
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<tr>
<td>Books and Magazines</td>
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<tr>
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<td>0.05</td>
<td>0.06</td>
<td>0.04</td>
<td>73.5</td>
</tr>
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</table>

Notes: The data are sorted by the average fraction. The data source is U.S. Census Bureau E-stats 2001. Average growth is simply the arithmetic mean of the percentage growth rates across the three years.

Table 1. Percentage of sales accounted by e-commerce in electronic and mail order houses (NAICS 454110)
<table>
<thead>
<tr>
<th>Category</th>
<th>Major early adopter(s)</th>
<th>Major late adopter(s)</th>
</tr>
</thead>
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<td>Books</td>
<td>Amazon.com (July 1995)</td>
<td>BarnesandNoble (May 1997), Borders (May 1998)</td>
</tr>
<tr>
<td>Movies</td>
<td>Netflix.com (1999)</td>
<td></td>
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<tr>
<td>Electronics</td>
<td>Value America Inc.com (February 1998)</td>
<td>Circuit City (July 1999), Radio Shack (May 1999), Best Buy (1999)</td>
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</table>

Source: Authors' own documentation based on phone calls and using company histories available on-line. Company names including ".com" refer to pure Internet-based companies. Bold indicates an established traditional firm that diversified into on-line retailing.

Table 2. Adoption dates of major early and late movers in on-line retailing
The sets $\Phi^n$ and $\Phi^o$

Figure 1: The sets $\Phi^n$ and $\Phi^o$ in the $(\rho, \lambda)$ plane
The effect of an increase in the mass of distant switchers, $\alpha$

![Graph showing the effect of an increase in the mass of switchers, $\alpha$, on the sets $\Phi^n$ and $\Phi^o$.](image)

Figure 2: The effect of an increase in the mass of switchers, $\alpha$, on the sets $\Phi^n$ and $\Phi^o$. 