

Should alternative mergers be considered by antitrust authorities?*

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Abstract

This paper illustrates that taking alternative mergers into consideration when analyzing the effects of a proposed merger may provide some information to the antitrust authorities. In particular, the use of revealed preference may allow the authorities to establish an expected upper limit on the efficiency gains obtained in a given merger that also increases the participants' market power. Such limit can then be compared to the lower threshold necessary for merger approval. The policy implications of this result are discussed.

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JEL Codes: L41, L10

1 Introduction

There is much ongoing debate regarding the present way antitrust authorities intervene in the markets. Their objectives (should the authorities aim at increasing consumer surplus or welfare?), the scope of their analysis (should an efficiency defense be admitted or not?) as well as the “long term” consequences of a decision (what are the effects of a rejection: the no-merger case or an alternative merger? what is expected to happen after the approval of a merger?) are being increasingly questioned and open to debate. The issue of the authorities' relevant objectives was the subject of recent work by Lyons (2002), Fridolfsson (2001) and Neven and Roller (2000) while, for instance, Nilssen and Sorgard (1998) discussed the possibility of sequential mergers, where a second merger depended on the authorities decision regarding an earlier one. Some of these aspects are also briefly discussed in Horn and Stennek (2002).

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Although theoretically satisfactory, the view that the authorities should foresee the consequences of approving or rejecting a given merger or acquisition in terms of its effect on other concentration operations (that might be prevented or induced) is hard to defend. The economic theory hardly presents a consensual way of anticipating which merger will occur and, even if it did, the informational requirements would be prohibiting.¹ However, this does not necessarily mean that alternative mergers or acquisitions should not be considered at all. In fact, looking at other mergers that might have taken place instead of the merger(s) under analysis may reveal some information, previously unknown to the authorities. This paper explores a way of obtaining such additional information using revealed preference. The revealed preference for a merger that leads to an increase in market power may enable the establishment of conditions that will narrow the admissible range for some parameters that are unknown to the authorities.² We are especially concerned with the case in which these unknown variables relate to the extent of the cost reductions generated by the merger, typically considered as unobservable by the authorities (see, for instance, Farrell and Shapiro (1990) and also the 1997 revisions to section 4 of the US Merger Guidelines, where it is stated that “Efficiencies are difficult to verify and quantify in part because much of the information relating to efficiencies is uniquely in the possession of the merging firms. Moreover, efficiencies projected reasonably and in good faith by the merging firms may not be realized”).

We will assume that the proposed concentration operation, which is under analysis, is the most profitable one among those considered admissible.³ We use the fact that a merger for market power is more likely to be more profitable than an alternative merger (where market power plays a lesser role) when efficiencies are not very significant, to establish an upper bound on the magnitude of cost reductions. This happens because large unit cost reductions tend to be more profitable when firms produce larger quantities, which is more conceivable after the occurrence of a merger that does not substantially increase market power. One crucial underlying assumption is that the cost reductions under scrutiny are not merger specific, that is, they may be obtained in any merger and do not depend on the identity of insiders.

The revealed limit on the magnitude of these cost reductions may be sufficient for the authorities to reject the merger, depending on the authorities’ objectives. For simplicity purposes we will assume that the authorities are concerned with consumer surplus alone and also that they admit an efficiency defense (otherwise,

¹For instance, if one were to adopt the core concept (as in Horn and Persson (2001) or (2001a)), one would need to have information on the cost reductions obtained in any conceivable alternative merger. The full extent of the externalities involved in the formation of any possible coalition(s) would also have to be addressed. Alternatively, mergers may result from some non-cooperative game. Such endogenous mergers have been studied, for instance by Bloch (1995) and (1996), but, in most cases, the results depend crucially on the protocol, that is, for instance, on the order firms move.

²Revealed preference has been previously applied to analyze the consistency of merger policy by Nilssen (1997).

³This is a reasonable assumption, for instance, when one firm is willing to acquire any one of its symmetric rivals and firms are willing to sell if they are paid more than their current profits. Other criteria might be used to establish which merger is likely to be proposed, such as the notion of core of a partition function form game (see Horn and Persson (2001) and (2001)) or the equilibrium of an extensive form merging game (see, for instance, Bloch (1995) and (1996)).

horizontal mergers should not be approved). Section 2 presents a well known model that, in section 3, will be used to illustrate how some information can be obtained after the authorities are notified of a merger for market power when alternative mergers are equally plausible. Finally, section 4 concludes.

2 The model

To exemplify how revealed preference can be used to obtain information regarding non merger-specific efficiencies, let us consider the circular city model of Vickrey (1964) and (1999), also referred to as the model of Salop (1979). With n firms symmetrically distributed on a loop, this model has the nice property that mergers between firms that are, *ex-ante*, symmetric may have very different effects. Naturally, if the study of alternative mergers is supposed to reveal some information, then these cannot have precisely the same effects as the merger under analysis. With this type of model, and assuming n is odd, we will have $(n-1)/2$ different two-firm mergers each with its impact on prices, profits and consumer surplus. However, other models could be used to obtain qualitatively similar results, such as a model of Cournot competition between sellers of non symmetrically differentiated products.

As mentioned above, any merger is assumed to reduce the insider firms' marginal costs, i.e., efficiencies are not merger specific. More particularly, we assume that the marginal cost of each firm is given by $D + d/k_i$ where k_i is the amount of some cost reducing asset that firm $i = 1, \dots, n$ owns.⁴ After a merger between firms i and j , the marginal cost of the firm resulting from the merger will be $D + d/(k_i + k_j)$. Assuming firms are initially endowed with the same amount k of the cost reducing asset, insiders' marginal costs will change from $D + d/k$ to $D + d/(2k)$ due to the merger. Let us define $c \equiv D + d/k$ as the pre-merger marginal cost and let ε represent the post-merger reduction in marginal costs, that is, $\varepsilon \equiv D + d/k - D - d/(2k) = d/(2k)$. This reduction in marginal costs is a function of the parameter d which is assumed to be unobservable to the authorities (alternatively, the authorities could be uncertain about the true extent of cost reductions due to the fact that the cost reducing asset, k , might also be unobservable).

As usual, consumers are assumed to be uniformly distributed along the loop (with unit length) and face quadratic transportation costs, with ty^2 measuring the cost of travelling distance y . Consumers are assumed to have a valuation V that is high enough to guarantee that they always purchase one unit of the good. The no-merger equilibrium has each firm setting a price $P_i = c + t/n^2$ and having individual profits of $\Pi_i = t/n^3$. Note that this is precisely the profit any firm will have when there is a merger involving non-neighboring firms

⁴This type of cost function is very similar to the one by Perry and Porter (1985). This particular function has been previously used in merger analysis by Persson (1999).

and when mergers do not lead to any cost savings, that is, when $d = 0$.

The following subsections present the equilibrium prices and profits for mergers between any pair of non-neighboring firms and between any two neighboring firms. These mergers differ because the latter type of merger will also have a market power motive while the former type is motivated by cost reductions alone. Recall that we assume that the authorities have been notified of a merger involving neighbors and are presently concerned with its impact on consumers. Note that, in this setting, only such merger is likely to concern the authorities, especially if cost reductions are inexpressive.

2.1 Merger between non consecutive firms

In what follows, and without loss of generality, it will be assumed that one of the insiders is firm 1. In the appendix we show that a merger between firm 1 and firm l (with $2 < l < n$) will lead to the following equilibrium prices

$$P_i = \frac{(a^{n-l+i} + a^{i-1} + a^{n+1-i} + a^{l-i})}{(1 - a^n)} x + \frac{t}{n^2} + c, i = 1, \dots, l \quad (1)$$

where $x = \varepsilon/\sqrt{3}$ and $a = (2 + \sqrt{3})$. The way to obtain the prices set by firms $l + 1$ to n is simply to redefine l as firm $(n + 2 - l)$ thus obtaining

$$P_{n+2-i} = \frac{(a^{l-2+i} + a^{i-1} + a^{n+1-i} + a^{n-l+2-i})}{(1 - a^n)} x + \frac{t}{n^2} + c, i = 2, \dots, n + 2 - l. \quad (2)$$

It is easy to check that $P_1 = P_l$, for all l , and that $P_{1+j} = P_{l-j}$, for $1 < j < l$.

The price differences must be sufficiently small in order to have an interior solution, i.e., in order for all firms to have a positive demand from both their left and right sides. Let $\Delta_{i,i+1}$ denote the absolute value of the difference in the price set by two consecutive firms, i and $i + 1$, with $i + 1 \leq (l + 1)/2$. It is easy to check that P_i is increasing in i for $i \leq (l + 1)/2$ and therefore $\Delta_{i,i+1} = P_{i+1} - P_i$. Using the equilibrium prices above, we have:

$$\Delta_{i,i+1} = ((a^{n-l+i+1} + a^i + a^{n-i} + a^{l-i-1}) - (a^{n-l+i} + a^{i-1} + a^{n+1-i} + a^{l-i})) \frac{x}{(1 - a^n)} \quad (3)$$

It is straightforward to show that,

$$\begin{aligned}\frac{\partial \Delta_{i,i+1}}{\partial i} &= (a^{n+1-i} - a^{n-i} + a^{l-i} - a^{l-i-1} + a^{n-l+i+1} - a^{n-l+i} + a^i - a^{i-1}) \frac{x(\ln a)}{1-a^n} < 0 \text{ for } i+1 \leq \frac{l+1}{2} \quad (4) \\ \frac{\partial \Delta_{i,i+1}}{\partial l} &= (a^{n-l+i+1} - a^{n-l+i} + a^{l-i} - a^{l-i-1}) \frac{-x(\ln a)}{1-a^n} > 0 \text{ for } i+1 \leq \frac{l+1}{2} < n. \quad (5)\end{aligned}$$

Hence, the largest price difference occurs between one of the insiders and the nearest outsider located to the side of the circumference where the distance to the other insider is the largest. This price difference is therefore maximized when $l = n - 1$ and the firms in question are firm 1 and firm 2 (or firm $n - 2$ and firm $n - 1$). We make the assumption that the largest equilibrium price difference is smaller than the transportation cost between any two consecutive firms. This is equivalent to stating that the insiders cost advantage cannot be very large:

$$P_2 - P_1 < \frac{t}{n^2} \Leftrightarrow x < \bar{x}_1 \equiv \frac{t}{n^2} \left(\frac{1-a^n}{4(5-3\sqrt{3})(a^n-a^3)} \right). \quad (6)$$

The merged firm, firm $1 + l$, will have joint profits given by (see appendix A):

$$\Pi_{1+l} = 2 \left(\left(\frac{(a^{n-l+1} + 1 + a^n + a^{l-1})}{(1-a^n)} + \sqrt{3} \right) x + \frac{t}{n^2} \right)^2 \frac{n}{t} \quad (7)$$

Insiders' profits increase with P_1 which is a function of l and is maximized for $l^* = \frac{1}{2}n + 1$ when n is even or $l^* = (n + 1)/2$ when n is odd (in this case a merger between firm 1 and firm $(n + 3)/2$ would also yield the same profits).

The most profitable merger involves firms at the highest possible distance. The intuition is the following: a firm with lower costs will set a lower price. This will affect directly only the two neighboring firms that will also set a lower price. The lower the neighbor's price is, the less will the firm with the lower costs benefit. But this happens if the neighboring firm has an insider on each side. In that case it will lower its prices substantially, thus hurting both insiders. Contrary to the mergers between neighbors (analyzed below), all outsiders will see their profits decline after the merger between firm 1 and firm l , especially those located near the insiders.

The insiders' aggregate profits after the optimal merger (that is the one that increases profits the most) are

$$\Pi_{1+l^*} = 2 \left(\left(\frac{1+a^n+2a^{\frac{n}{2}}}{1-a^n} + \sqrt{3} \right) x + \frac{t}{n^2} \right)^2 \frac{n}{t} \quad (8)$$

if the number of firms is even or

$$\Pi_{1+t^*} = 2 \left(\left(\frac{1 + a^n + 2a^{\frac{n-1}{2}}}{1 - a^n} + \sqrt{3} \right) x + \frac{t}{n^2} \right)^2 \frac{n}{t} \quad (9)$$

if the number of firms is odd.

2.2 Merger between consecutive firms

After a merger between firm 1 and firm 2 the insiders' aggregate profit function is:

$$\Pi_{1+2} = (P_1 - c_1) \left(\frac{1}{n} + n \frac{P_n - P_1}{2t} + n \frac{P_2 - P_1}{2t} \right) + (P_2 - c_2) \left(\frac{1}{n} + n \frac{P_1 - P_2}{2t} + n \frac{P_3 - P_2}{2t} \right). \quad (10)$$

In the appendix B we show that the equilibrium prices are the following:

$$P_1 = P_2 = c + \frac{t}{n^2} + \frac{1}{3} \frac{t}{n^2} \frac{-3 + \sqrt{3}a^n - \sqrt{3} + 3a}{a^n - a} - \frac{a + a^n}{a^n - a} x, \quad (11)$$

$$P_i = \frac{a^{i-1} + a^{2+n-i}}{a - a^n} \left(x - \frac{t}{n^2} \frac{\sqrt{3}}{3} \right) + \frac{t}{n^2} + c, i = 3, \dots, n. \quad (12)$$

These results extend those obtained by Levy and Reitzes (1992) to include the possibility of insiders' marginal cost reductions after a merger. The effect this merger has on prices is not straightforward. On the one hand each insider has a clear incentive to raise its price because part of the consumers will shift to the other insider firm. On the other hand, the lower costs compel insiders to lower their prices.

As compared to the pre-merger prices, post-merger prices decrease if (note that this is a condition for the insiders' prices to decrease that implies that average prices will also decrease)

$$x > \underline{x} \equiv \frac{1}{\sqrt{3}} \frac{t}{n^2} \quad (13)$$

As no merger decreases transportation costs (and these are minimized at the pre-merger equilibrium where demand between any two firms is shared equally amongst them), a necessary condition for the merger to be allowed is that prices decrease, that is, $x > \underline{x} \equiv t/\sqrt{3}n^2$.

As in the first case, the largest price difference involves firm 2 and firm 3 (or firm 1 and firm n):

$$\Delta_{2,3} = |P_2 - P_3| = \left| \frac{1}{3} \frac{t}{n^2} \frac{-3 + \sqrt{3}a^n - \sqrt{3} + 3a}{a^n - a} - \frac{a + a^n}{a^n - a} x - \frac{a^2 + a^{n-1}}{a - a^n} \left(x - \frac{t}{n^2} \frac{\sqrt{3}}{3} \right) \right| \Leftrightarrow$$

$$\Delta_{2,3} = \left| \frac{-t(3 - \sqrt{3}a^n + \sqrt{3} - 3a + a^2\sqrt{3} + a^{n-1}\sqrt{3}) + 3xn^2(-a - a^n + a^2 + a^{n-1})}{3n^2(a^n - a)} \right|$$

This price difference is positive (negative) if the insiders increase (decrease) their prices, that is, if cost reductions are sufficiently small (large). If $x < t/(n^2\sqrt{3})$ then we must have

$$P_2 - P_3 < \frac{t}{n^2} \Leftrightarrow x > -\frac{t}{n^2} \frac{a^n + a}{(3 - \sqrt{3})(a^n - a^2)} \quad (14)$$

which is always true given that x is positive.

Otherwise, when insiders lower their prices after the merger, we must have that

$$P_3 - P_2 < \frac{t}{n^2} \Leftrightarrow \frac{t}{n^2} \frac{\sqrt{3}(a^n - a) - (1 - \sqrt{3})(a^n - a^2)}{(3 - \sqrt{3})(a^n - a^2)} \equiv \bar{x}_2 > x \quad (15)$$

Consequently, in order to have non-negative demands in any conceivable circumstance we assume that $x < \min\{\bar{x}_1, \bar{x}_2\} = \bar{x}_1$ (for $n > 3$). Equilibrium profits for the insiders can be shown to be

$$\Pi_{1+2} = \left(\left(1 + \frac{\sqrt{3}}{3} \right) \frac{a^n - 1}{a^n - a} \frac{t}{n^2} - \left(\frac{a + a^n}{a^n - a} - \sqrt{3} \right) x \right)^2 \frac{n}{t}. \quad (16)$$

3 Revealed Preference

In this section we use the assumption that the merger taking place is the one where insiders prices increase the most. The merger between any consecutive firms is more profitable than the most profitable alternative (with n even) if

$$\frac{1}{2} \left(\left(1 + \frac{\sqrt{3}}{3} \right) \frac{a^n - 1}{a^n - a} \frac{t}{n^2} - \left(\frac{a + a^n}{a^n - a} - \sqrt{3} \right) x \right)^2 \frac{n}{t} > \left(\left(\frac{1 + a^n + 2a^{\frac{n}{2}}}{1 - a^n} + \sqrt{3} \right) x + \frac{t}{n^2} \right)^2 \frac{n}{t} \quad (17)$$

which can be simplified to:

$$x < \overline{x}_{even} \equiv \frac{t}{n^2} \frac{\left(1 + \frac{\sqrt{3}}{3} \right) \frac{a^n - 1}{a^n - a} - \sqrt{2}}{\left(\frac{a + a^n}{a^n - a} - \sqrt{3} \right) + \sqrt{2} \left(\frac{1 + a^n + 2a^{\frac{n}{2}}}{1 - a^n} + \sqrt{3} \right)} \quad (18)$$

We have therefore established an upper bound on the magnitude of the marginal cost reductions obtainable through merger.⁵

The following lemma illustrates that considering alternative mergers may provide the authorities with sufficient information to reject a given merger between neighboring firms.

Lemma 1 *Provided that $n > 6$, the fact that the merger for market power (i.e. the merger between neighboring firms) is more profitable reveals that the cost reductions are insufficient to lower prices and, therefore, to raise consumer surplus.*

Proof. Recall from above that the merger reduces prices if and only if cost reductions are significant, that is, if and only if (13) is verified. Assume initially that n is even. Additionally, as shown above, the merger between consecutive firms is more profitable than any alternative if (18) holds true. These two conditions are impossible to verify simultaneously if

$$\frac{t}{n^2} \frac{\left(1 + \frac{\sqrt{3}}{3}\right) \frac{a^n - 1}{a^n - a} - \sqrt{2}}{\left(\frac{a + a^n}{a^n - a} - \sqrt{3}\right) + \sqrt{2} \left(\frac{1 + a^n + 2a^{\frac{n}{2}}}{1 - a^n} + \sqrt{3}\right)} < \frac{t}{\sqrt{3}n^2} \quad (19)$$

which is equivalent to

$$f(n) \equiv \left(1 + \sqrt{3}\right) \frac{a^n - 1}{a^n - a} - \sqrt{6} - \left(\frac{a + a^n}{a^n - a} - \sqrt{3}\right) - \sqrt{2} \left(\frac{1 + a^n + 2a^{n/2}}{1 - a^n} + \sqrt{3}\right) < 0 \quad (20)$$

Note that

$$\frac{\partial f(n)}{\partial n} = -\sqrt{2} \frac{a^{2n} (a^n - \sqrt{3}a) + a^2 + a^{n+1} \sqrt{3} + 2\sqrt{a^n} (a^n - a)^2}{(a^n - a)^2 \sqrt{a^n} (a^n - 1)^2} \frac{\partial(a^n)}{\partial n} < 0 \quad (21)$$

As $f(6) \approx 3.4816 \times 10^{-2}$ and $f(8) \approx -6.0089 \times 10^{-3}$ this proves the result: $f(n) < 0$, $\forall n > 6$ and n even.

When n is odd a similar proof establishes that $f(n) < 0$, $\forall n > 6$ and n odd. ■

The intuition for this result is the following: marginal cost reductions are more profitable when firms produce larger amounts of output. This is more likely to happen when insider firms sell products that do not compete directly. Otherwise, there is an incentive to increase prices that ultimately leads to a lower output level. As a consequence, when the merger for market power is more profitable than the best alternative (which is, in this case, a merger with a distant competitor) it must be that cost reductions do not play a major role.

⁵ If n is odd the same condition boils down to: $x < x_{odd} \equiv \frac{\left(\left(1 + \frac{\sqrt{3}}{3}\right) \frac{a^n - 1}{a^n - a} - \sqrt{2}\right)}{\left(\frac{a + a^n}{a^n - a} + \sqrt{2} \frac{1 + a^n + 2a^{\frac{n-1}{2}}}{1 - a^n} - \sqrt{3} + \sqrt{6}\right)} \frac{t}{n^2}$

Figure 1: Revealed upper bounds ($\overline{x_{even}}$ and $\overline{x_{odd}}$) and minimum threshold for decrease in prices (\underline{x})

Consider the limit case where there exist no cost reductions: only a merger with a neighbor is profitable. But as cost reductions increase, profits increase more when firms produce a larger output which is more likely to happen when insiders are not competitors. For the cases of $4 \leq n \leq 6$ one will also get an upper bound on the extent of cost reductions. Despite the fact it is not incompatible with a decrease in prices, this bound could still be used to update the authorities prior beliefs regarding the true extent of ε and eventually correct the prior decision. Figure 1 presents the revealed upper bounds ($\overline{x_{even}}$ or $\overline{x_{odd}}$) as well as the thresholds (\underline{x}) below which the merger under analysis increases prices. The upper limit on x that ensures that we have interior solutions ($\overline{x_1}$) is also plotted.

4 Conclusions

We showed, with a simple example, that taking other mergers into account may be relevant for the antitrust authorities. It has been argued in the literature that the authorities should not be myopic and that they should consider the future consequences of a merger in terms of leading the way for (or, instead, preempting) other mergers. A better understanding of the incentives firms have to merge is thus necessary to foresee the long term consequences of approving or rejecting the merger under analysis. We claim that such understanding may also enable the authorities to obtain information regarding the merger under scrutiny. By understanding why a merger was chosen over another, several conditions (for instance, regarding the extent of cost reductions) may be established. Using a simple circular city model where Perry and Porter (1985) type of cost reductions are

assumed to exist, we showed that the announcement of a merger for market power is enough to establish that such merger will increase prices, provided that there are more than six firms. The main assumptions needed for this result to hold are the following: (i) the criteria that leads to a merger being chosen over another is profitability (that is, the more profitable merger will take place), (ii) cost reductions are the same, regardless of the identity of insiders (this is a reasonable assumption in this model where firms are symmetric *ex-ante*). The qualitative results do not change substantially when other types of models are used. The results are driven by the fact that a merger for market power will lead to a lower aggregate output than an alternative merger (where insiders sell more differentiated products) and this does not depend on the type of model considered.

Naturally, this methodology does not apply to any merger. Those cases in which insider firms claim some cost savings and show that these are specific to the insiders involved leave no room for this type of reasoning. However, when insiders fail to substantiate these claims or when the authorities can expect similar marginal cost reductions in alternative mergers, then the revealed preference argument may be used. This may happen, for instance, when firms own complementary assets, (and the acquiring firm owns an asset that is different from the other firms') or when differentiation arises solely from advertising or location decisions by firms that use the same technology.

Having established whether the efficiencies are firm specific or not, the authorities should then determine the existence of alternative mergers. In some countries, the authorities presently conduct entry analysis, but this refers to entry into the market after a merger takes place. The proposed analysis should not be very different in terms of know-how, in the sense that alternative mergers can be interpreted as entry by the acquiring firm into another (although related) market.

If nothing else, this analysis highlights the relevance, for future research, of considering alternative mergers, of understanding the mechanisms that lead to one merger taking place instead of another and of establishing whether alleged cost reductions are indeed firm specific or not.

A Merger between non consecutive firms

A.1 Equilibrium prices

Firm i 's problem is to maximize its profit and its first-order conditions are:

$$\frac{\partial \left((P_i - c_i) * \left(\frac{P_{i+1} - P_i}{2t} n + \frac{P_{i-1} - P_i}{2t} n + \frac{1}{n} \right) \right)}{\partial P_i} = 0 \quad (22)$$

The n first-order conditions are

$$P_{i+1} - 4P_i + P_{i-1} = \frac{-2t}{n^2} - 2c_i, i = 1, \dots, n. \quad (23)$$

These can be simplified and written in matricial terms (after multiplying by n) as

$$\begin{bmatrix} -4 & 1 & 0 & \dots & 0 & 1 \\ 1 & -4 & 1 & 0 & \dots & 0 \\ 0 & 1 & -4 & 1 & 0 \dots & 0 \\ \vdots & & & & & \vdots \\ 0 & \dots & 0 & 1 & -4 & 1 \\ 1 & 0 & \dots & 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_{n-1} \\ P_n \end{bmatrix} = \begin{bmatrix} \frac{-2t}{n^2} - 2c_1 \\ \frac{-2t}{n^2} - 2c_2 \\ \frac{-2t}{n^2} - 2c_3 \\ \vdots \\ \frac{-2t}{n^2} - 2c_{n-1} \\ \frac{-2t}{n^2} - 2c_n \end{bmatrix}$$

Denote the row concerning P_i by l_i . Performing the following operations over the rows

$$l_1 - \sum_{j=1}^{n-2} u_j l_{n-j} \text{ and } l_n + \sum_{j=1}^{n-2} u_{j+1} l_{n-j} \quad (24)$$

where each coefficient u_j can be obtained recursively from $u_0 = 0, u_1 = 1$ and $u_j = 4u_{j-1} - u_{j-2}$, results in the following equation in P_1 and P_2 :

$$-(4 + u_{n-2})P_1 + (1 + u_{n-1})P_2 = \frac{-2t}{n^2} - 2c_1 + \sum_{j=1}^{n-2} u_j \left(\frac{2t}{n^2} + 2c_{n-j} \right) \quad (25)$$

$$(1 + u_{n-1})P_1 - u_n P_2 = \frac{-2t}{n^2} - 2c_n - \sum_{j=1}^{n-2} u_{j+1} \left(\frac{2t}{n^2} + 2c_{n-j} \right) \quad (26)$$

Noting that $u_i = \frac{a^i - a^{-i}}{2\sqrt{3}}$ we can write these equations as:

$$-(4 + \frac{a^{n-2} - a^{-n+2}}{2\sqrt{3}})P_1 + (1 + \frac{a^{n-1} - a^{-n+1}}{2\sqrt{3}})P_2 = \frac{-2t}{n^2} - 2c_1 + \frac{t}{n^2} \sum_{j=1}^{n-2} \frac{a^j - a^{-j}}{\sqrt{3}} + \sum_{j=1}^{n-2} \frac{a^j - a^{-j}}{\sqrt{3}} c_{n-j} \quad (27)$$

$$(1 + \frac{a^{n-1} - a^{-n+1}}{2\sqrt{3}})P_1 - \frac{a^n - a^{-n}}{2\sqrt{3}}P_2 = \frac{-2t}{n^2} - 2c_n - \frac{t}{n^2} \sum_{j=1}^{n-2} \frac{a^{j+1} - a^{-j-1}}{\sqrt{3}} - \sum_{j=1}^{n-2} \frac{a^{j+1} - a^{-j-1}}{\sqrt{3}} c_{n-j} \quad (28)$$

It is straightforward to show that

$$\sum_{j=1}^{n-2} \frac{a^j - a^{-j}}{2\sqrt{3}} = \frac{1}{2\sqrt{3}} \frac{1 - a^{n-1} - a^{-n+2} + a}{1 - a} \quad (29)$$

$$\sum_{j=1}^{n-2} \frac{a^{j+1} - a^{-j-1}}{2\sqrt{3}} = \frac{1}{2\sqrt{3}} \frac{1 - a^{n+1} - a^{-n+2} + a^3}{a(1 - a)} \quad (30)$$

Let the insiders, firms 1 and l (by assumption nonconsecutive firms) see their costs decline by ε . All other firms will have costs c . Therefore the equations can be simplified and solved to obtain

$$P_1 = \left(\frac{a^{l-1} + a^{n-l+1} + a^n + 1}{1 - a^n} \right) x + \frac{t}{n^2} + c \quad (31)$$

$$P_2 = \left(\frac{a^{l-2} + a^{n-l+2} + a^{n-1} + a}{1 - a^n} \right) x + \frac{t}{n^2} + c \quad (32)$$

where $x = \varepsilon/\sqrt{3}$.

For all firms up to firm $l-1$ we can write

$$P_{i+1} = \frac{-2t}{n^2} - 2c + 4P_i - P_{i-1} \quad (33)$$

Solving this difference equation we obtain:

$$P_i = \frac{(a^{n-l+i} + a^{i-1} + a^{n+1-i} + a^{l-i})}{(1 - a^n)} x + \frac{t}{n^2} + c, i = 1, \dots, l \quad (34)$$

Redefining l as $n+2-l$ allows us to obtain the prices for the remaining firms.

A.2 Equilibrium profits

From the first-order conditions it is clear that individual profits are

$$\Pi_i = (P_i - c_i)^2 \frac{n}{t} \quad (35)$$

Thus, after the merger, insiders' aggregate equilibrium profits are given by

$$\Pi_{1+l} = \left(P_1 - c + x\sqrt{3} \right)^2 \frac{n}{t} + \left(P_l - c + x\sqrt{3} \right)^2 \frac{n}{t} = 2 \left(P_1 - c + x\sqrt{3} \right)^2 \frac{n}{t} \quad (36)$$

$$\Pi_{1+l} = 2 \left(\left(\frac{(a^{n-l+1} + 1 + a^n + a^{l-1})}{(1 - a^n)} + \sqrt{3} \right) x + \frac{t}{n^2} \right)^2 \frac{n}{t} \quad (37)$$

Insiders profits increase with P_1 which is a function of l . We will now establish which l maximizes P_1 . With this purpose the function $(a^{n-l+1} + a^{1-1} + a^{n+1-1} + a^{l-1})$ will be minimized:

$$\frac{\partial (a^{n-l+1} + a^{1-1} + a^{n+1-1} + a^{l-1})}{\partial l} = 0 \Leftrightarrow l = \frac{1}{2}n + 1 \quad (38)$$

The second order conditions for minimum are

$$(\ln(a))^2 (a^{n-l+1} + a^{l-1}) > 0 \quad (39)$$

which is always verified.

Thus P_1 (and insiders' profits) is maximized for $l^* = \frac{1}{2}n + 1$.

B Merger between consecutive firms

B.1 Equilibrium prices

After a merger between firm 1 and firm 2, insiders' aggregate profits are:

$$\Pi_I = (P_1 - c_1) \left(\frac{1}{n} + n \frac{P_n - P_1}{2t} + n \frac{P_2 - P_1}{2t} \right) + (P_2 - c_2) \left(\frac{1}{n} + n \frac{P_1 - P_2}{2t} + n \frac{P_3 - P_2}{2t} \right)$$

After a merger by firms 1 and 2 (when all firms are symmetrically located on the circumference) the first-order conditions can be simplified to and written as

$$\begin{bmatrix} -4 & 2 & 0 & \dots & 0 & 1 \\ 2 & -4 & 1 & 0 & \dots & 0 \\ 0 & 1 & -4 & 1 & 0 \dots & 0 \\ \vdots & & & & & \vdots \\ 0 & \dots & 0 & 1 & -4 & 1 \\ 1 & 0 & \dots & 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_{n-1} \\ P_n \end{bmatrix} = \begin{bmatrix} -t2/n^2 - 2c_1 + c_2 \\ -t2/n^2 - 2c_2 + c_1 \\ -t2/n^2 - 2c_3 \\ \vdots \\ -t2/n^2 - 2c_{n-1} \\ -t2/n^2 - 2c_n \end{bmatrix}$$

Denoting the i^{th} row by l_i , the following operations over the rows

$$l_1 - \sum_{j=1}^{n-2} u_j l_{n-j} \text{ and } l_2 - \sum_{j=1}^{n-3} u_j l_{j+3} - u_{n-2} l_1$$

will result in (following the same steps as above)

$$\begin{aligned} P_1 &= P_2 = c + \left(1 + \frac{\sqrt{3}}{3} \right) \frac{a^n - 1}{a^n - a} \frac{t}{n^2} - \frac{a + a^n}{a^n - a} x \\ P_i &= \frac{a^{i-1} + a^{2+n-i}}{a - a^n} \left(x - \frac{t}{n^2} \frac{\sqrt{3}}{3} \right) + \frac{t}{n^2} + c, i = 3, \dots, n \end{aligned}$$

where $x = \varepsilon/\sqrt{3}$ and $a = (2 + \sqrt{3})$.

B.2 Equilibrium profits

After such merger, insiders' profits are:

$$\Pi_{1+2} = (P_1 - c_1) \left(\frac{1}{n} + n \frac{P_n - P_1}{2t} \right) + (P_2 - c_2) \left(\frac{1}{n} + n \frac{P_3 - P_2}{2t} \right) \quad (40)$$

Using the first-order conditions, this boils down to:

$$(P_1 - c_1) \left(\frac{1}{n} + n \frac{(-2t/n^2 + P_1 - c_1)}{2t} \right) + (P_2 - c_2) \left(\frac{1}{n} + n \frac{-2t/n^2 + P_2 - c_2}{2t} \right) = (P_1 - c_1)^2 \frac{n}{t}. \quad (41)$$

Replacing P_1 and c_1 with the equilibrium values results in the expression (16).

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