The Diffusion of Cellular Telephony in Portugal before UMTS: A Time Series Approach

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In this paper, we propose a methodology to estimate diffusion processes that differs from the standard practice in two ways. First, we model the nonlinear long-run trend through the *Richards* curve, which is more flexible than the standard alternatives. Second, we propose a dynamic specification that accounts for: short-run dynamics, and a tendency to correct deviations from the nonlinear long-run trend. We apply the model to the diffusion of cellular telephony in Portugal. Statistical tests show that our model outperforms the standard diffusion models. We also use the model to characterize the diffusion process of the Portuguese cellular telephone industry, and test several hypothesis about the recent evolution of the industry.

1. INTRODUCTION

In the introduction we do two things. First, we give an overview of the paper's methodology and results. Second, we review the literature.

1.1. Overview of the Paper

It is common practice in empirical studies on the diffusion of innovations over time, to assume that the number of individuals that adopt an innovation follows an S-shaped curve.[1] After the introduction of the innovation, there is a initial stage where few individuals adopt the innovation. This early stage is followed by a period of rapid growth of the number of adopters. Finally, the diffusion process reaches a maturity phase, in which the growth of the number of adopters slows down, and the total number of adopters gradually stabilizes.

The most used S-shaped growth curves in the empirical literature on diffusion are the logistic and Gompertz curves.[2] In spite of their "good fit", these models display at least two serious shortcomings. The first and more important problem of these curves, is their lack of flexibility with respect to two important characteristics of the diffusion process: (i) the inflection point, and (ii) the asymmetry around the inflection point. For the logistic and Gompertz curves, the inflection point always occurs at a fixed fraction of the saturation level; 50% for the logistic and, approximately, 37% for Gompertz. The logistic curve is symmetric around the time at which the inflection point occurs. The Gompertz curve is asymmetric, and the speed of adoption is larger in the later stages of the diffusion process. The second

1. Geroski (2000) discusses the theoretical arguments used to justify the S-shaped adoption curve.
2. Both models have a long tradition in the literature of product and technology diffusion. Griliches (1957) is an early example of the use of the logistic curve (see also Dixon, 1980). Chow (1967) uses the Gompertz curve.

problem of these curves is the difficulty of discriminating empirically between them. Typically, the comparison of goodness of fit measures is inconclusive. Thus, in many cases, the model selection is done on a subjective basis. This is specially problematic, since the predictions from the two models tend to diverge drastically, as the diffusion process evolves.

In this paper, we propose a methodology to estimate diffusion processes that differs from the standard practice in two ways. First, we use the *Richards* curve, also known as generalized logistic curve, to model the diffusion trend. This slightly more complex, but still parsimonious model, does not impose rigid constraints on the location of the inflection point, and allows the estimation of the degree of asymmetry around the inflection point. Besides, it includes as a special case the logistic curve, and can approximate arbitrarily well the Gompertz curve. Thus, one can test explicitly those specifications against each other, without having resort to subjective considerations.[3] The second way in which our approach differs from the standard practice, is that we propose a dynamic model, following the standard practice in modern time series analysis, that accounts for both short-run and long-run related effects, and that uses exhaustive specification testing for model selection (see Hendry, 1995).

The Portuguese cellular telephony industry provides a suitable application for the framework we have in mind. In Portugal, the firm associated with the incumbent, *TMN*, started its activity in 1989 with the analogue technology *C-450*.[4] In 1991, the sectoral regulator, *ICP-ANACOM*, assigned two licenses to operate the digital technology *GSM 900*.[5] One of the licenses was assigned to *TMN*. The other license was assigned to the entrant *VODAFONE*.[6] In 1995, *TMN* pioneered internationally the introduction of pre-paid cards. The proportion of subscribers that hold a pre-paid card increased quickly, and reached what seems to be its steady state level of 79% in 1999. In 1997, the regulator assigned three licenses to operate the digital technology *GSM 1800*.[7] Two licenses were assigned to *TMN* and *VODAFONE*. A third license was assigned to the entrant *OPTIMUS*.[8] In 1999, *TMN* discontinued the analogue service. The legislation of the *E.U.* imposed the full liberalization of the telecommunications industry at the end of the nineties.[9] This meant that any firm licensed by the sectoral regulator could offer its services. In particular, any firm could offer fixed telephony services, either through direct access based on their own infrastructures, or through indirect access, available for all types of calls. In Portugal the liberalization took effect in 2000.[10] Finally, in 2001 *ICP-ANACOM* assigned licenses to operate the *3G* technology *IMT2000/UMTS*, and service began in 2004.

The Portuguese cellular telephony industry had a fast diffusion, measured by the number of subscribers.[11] Between 1992, the date of the introduction of *GSM*, and 2003, the number of subscribers increased at an average rate of 65% a year (Figure 1a). The number of subscribers duplicated, on aver-

3. There are other flexible models that have been used in diffusion studies, such as the FLOG model of Bewley and Fiebig (1988). Mahajan et al. (1990), Meade and Islam (1998) and Tsoularis and Wallace (2002) review several related models.

4. Initially, the service was provided by a consortium of two firms of the group of the telecommunications incumbent. In 1991, the consortium became one single firm called *TELECOMUNICAÇÕES MÓVEIS NACIONAIS* (*TMN*).

5. The licenses were assigned through a public tender, following the EU Directive 91/287 instructing member states to adopt the *GSM* standard. System *GSM 900* operates on the 900 MHz frequency. System *GSM 1800* operates on the the 1800 MHz frequency, and is compatible with the former. The upstream and downstream velocity is 9.6-14 kbps.

6. Initially the firm's name was *TELECEL-COMUNICAÇÕES PESSOAIS, S.A.* and was later renamed *TELECEL-VODAFONE*, following changes in the shareholder structure.

7. The licenses were assigned through a public tender, following the EU Directive 96/2, instructing member states to grant at least two *GSM 900* licenses, and to allow additional firms to use *GSM 1800*.

8. *OPTIMUS-TELECOMUNICAÇÕES, S.A.* was also granted a license to operate *GSM 900*.

9. Directive 90/388/EEC and Directive 96/19/EC

10. The date for the European liberalization was 1998. Portugal, like other countries, benefited from a derogation (Decision 97/310/EC of the Commission). Note that the entry of *OPTIMUS* in 1998 was independent from the liberalization process.

11. A *Subscriber* is a user with a contractual relationship with a national provider of the land cellular service, namely in the form of a subscription plan, or an active prepaid card. Hence, a subscriber is an "account", rather than an "individual".

age, every 16.6 months. The speed of the diffusion lead to high and rising penetration rates.[12] In 1999, the penetration rate of cellular telephony overtook the penetration rate of fixed telephony (Figure 1b). In 2003 the penetration rate of fixed telephony was 40% and decreasing, and the rate of penetration of cellular telephony was 89% and increasing. The deployment of *UMTS*, conceivably, will give an additional impulse to cellular telephony.

Our objective is twofold. First, we want to evaluate the performance of our framework, compared with the standard approach. Second, we want to characterize the diffusion process of cellular telephony in Portugal before *UMTS*. We pay particular attention to three aspects. First, we are interested in whether the entry of *OPTIMUS* in 1998 increased the speed of the diffusion. The effects of competition on diffusion processes have received some attention on the literature.[13] Conceivably, the entry of *OPTIMUS* in 1998, led the other two firms to lower their prices, and increase promotion. Either of these policies should have increased the speed of the diffusion. Second, we are interested in whether the full liberalization of the telecommunications market in 2000 affected the speed of the diffusion of cellular telephony in Portugal. It is unclear what should have been the impact of the full liberalization of the telecommunications market in Portugal. On the one hand, more competition in fixed telephony should have pushed the prices of this service down, and reduced the substitution between fixed and cellular telephony.[14] On the other hand, the liberalization involved a tariff rebalancing which increased the telephone subscription fee and the price of local calls. Third, we are interested whether there are network economies in the cellular telephone industry. The existence of network economies is one of the distinctive aspects of telecommunication services. Network interconnection obligations mitigate, but do not eliminate this effect. Differences between intra and inter network calls resurface the value for a consumer of belonging to a large network. Empirical analysis confirms this perspective.[15]

Using quarterly data ranging from 1994 to 2003, we estimated several time series models, for the aggregate data, and for each of the three firms operating in Portugal. We used a battery of specification tests in order to asses the statistical properties of our regression models. The analysis shows that simpler models that only add a disturbance term to a S-shaped trend present serious symptoms of misspecification. The dynamic effects we consider are important characteristics of the diffusion process. Including them in diffusion models enhances the statistical properties of the estimations. We think that our estimates provide a useful benchmark, with respect to which more structural models can be evaluated.

Our results indicate that the logistic curve is a valid statistical model for the diffusion process of cellular telephony in Portugal, but only as a long-term trend, from which data significantly and persistently deviates in the short-run. We found that the saturation level of the Portuguese cellular telephony market for *GSM*, implies a penetration rate around to 100%.[16] Our results also provide an estimate of the period at which the maturity phase began. Interestingly, the slowdown of the adoption of cellular telephony began in the year 2001, shortly after the full liberalization of the Portuguese telecommunications market. We found no evidence of a structural change in the diffusion process as a consequence of the liberalization process. However, the entrance of the third operator increased the speed of the diffusion process on the short run, with little or no long-run effects. Finally, we investigated the cross-impacts of the diffusion processes of each firm in the market.

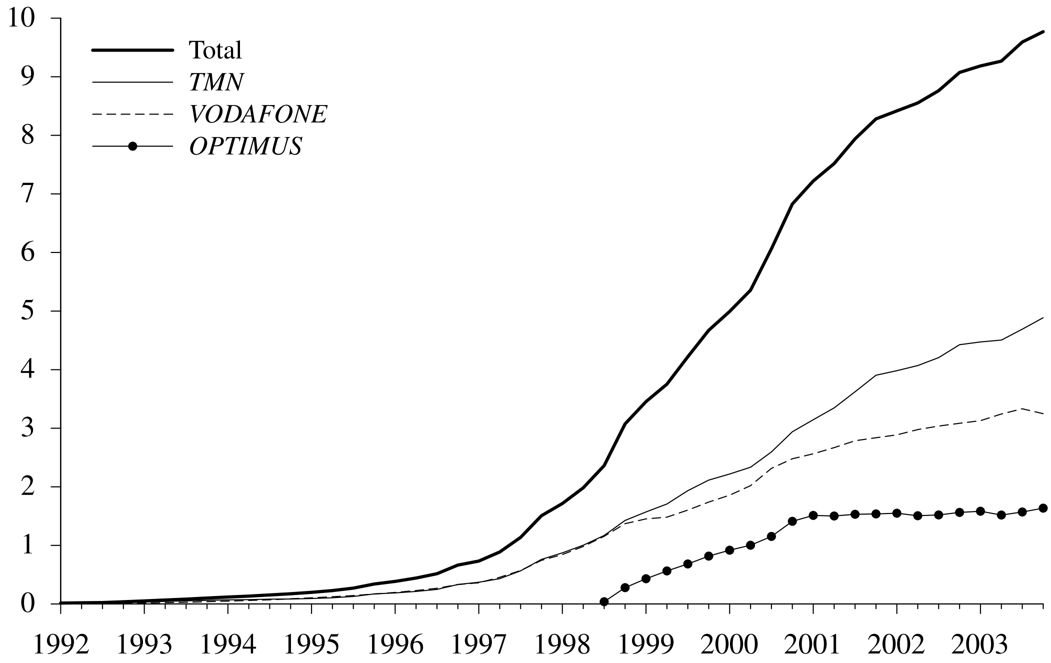
12. The *Penetration Rate* is the number of subscribers per 100 inhabitants.

13. See Gruber and Verboven (2001a), Gruber and Verboven (2001b) and Gruber (2001) for recent contributions related to the diffusion of cellular telephony. See Geroski (2000) and the papers cited therein for a more general discussion.

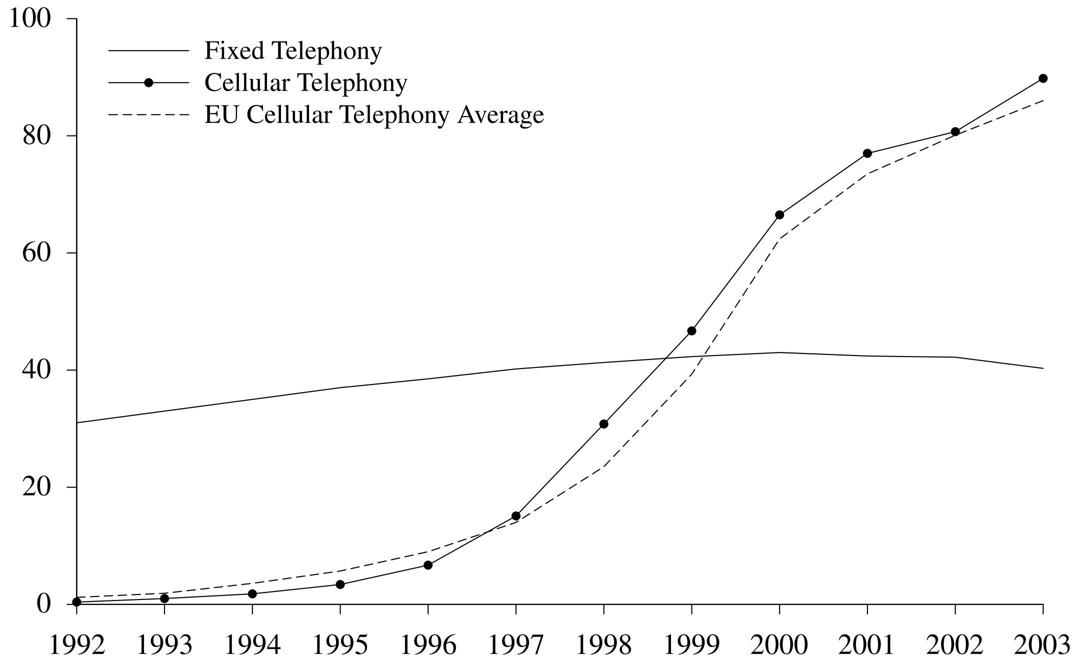
14. Barros and Cadima (2000) document the substitution between cellular and fixed telephony in Portugal. See Rodini et al. (2002) for the case of the US.

15. See, e.g., Doganoglu and Grzybowski (2003).

16. It has been argued that the industry is a natural oligopoly (Valletti, 2003). If true, this will depend on both the technology, and the number of potential users. This gives a special importance to estimating the saturation level of the market.



(a) Total Number of Subscribers (in millions).



(b) Cellular Telephony Penetration Rates (%).

FIGURE 1

Cellular Telephony Diffusion in Portugal

1.2. *Literature Review*

From a methodological point of view, our research draws from the literature on generalizations of the logistic curve (Mahajan et al., 1990; Meade and Islam, 1998; Tsoularis and Wallace, 2002), and the modern econometric approach to time series models (Hendry, 1995). Geroski (2000) surveys the literature on new technology diffusion. Here we focus on the research directly related to our work.

Barros and Cadima (2000) estimated jointly the diffusion curves for cellular and fixed telephony for Portugal. They adopted the logistic curve for the diffusion of fixed telephony, and the Gompertz curve for cellular telephony. In addition, they imposed a saturation level for cellular telephony that implied a penetration rate of 70%, and used as regressors variables like the prices of the services and gross domestic product per capita. They found a negative impact of the diffusion of cellular telephony on the fixed telephony penetration rate. But no effect in the reverse direction. The data ranged from 1993 to the third quarter of 1999. Botelho and Pinto (2000) fitted several curves to the Portuguese data. They concluded that the logistic provided the better fit, and estimated a saturation level that implies a penetration rate of 67.4%. The data ranged from 1989 to the second quarter of 2000.

Doganoglu and Grzybowski (2003) analyzed the cellular telephony industry in Germany from January 1998 to June 2003. Their results suggest that network effects played a significant role in the diffusion of cellular services in Germany. Gruber and Verboven (2001a) analyzed the technological and regulatory determinants of the diffusion of cellular telephony in the *E.U.*, using a logistic model of diffusion. They found that the transition from the analogue to the digital technology during the early nineties, and the corresponding increase in spectrum capacity, had a major impact on the diffusion of cellular telephony. The impact of introducing competition has also been significant, during both the analogue and the digital period, though the effect was small compared to the technology effect. Gruber (2001) analyzed the diffusion of cellular telephony in Central and Eastern Europe. They found that about 20% of the population will adopt cellular telephony, and that the speed of diffusion increased with the number of firms.

The remainder of the paper is organized as follows. Section 2 describes the Richards' curve. Section 2 discusses a dynamic diffusion model that it is used to estimate the main characteristics of the diffusion process. In Section 3, we apply an extended dynamic model to each of the firms operating in the Portuguese market. Section 5 concludes.

2. THE RICHARDS CURVE

2.1. *The Model*

The Richards curve is an S-shaped deterministic function of time.[17] If $N(t)$ follows a *Richards* curve, then:

$$N(t) = \kappa\left[1+\mu e^{-\gamma(1+\mu)(t-\tau)}\right]^{-1/\mu}, \tag{1}$$

The parametrization of the Richards curve given in equation (1) simplifies the interpretation of the parameters. Parameter $\kappa > 0$ is the *saturation level*, i.e., $\lim_{t\to\infty} N(t) = \kappa$; parameter τ is the time period at which $N(t)$ has an *inflection point* , i.e., the period at which maximal absolute growth occurs; parameter $\gamma > 0$ is the *relative growth rate* at time $t = \tau$; and $\mu \geq 0$ is a *shape* parameter related to the degree of asymmetry of the adoption process.

Equation (1) encompasses several special cases. We will only discuss the two cases commonly used in the economics literature on the diffusion of innovations: (i) the logistic curve, and (ii) the Gompertz curve.

17. Richards (1959) first introduced a generalization of the logistic curve (see also Nelder, 1962), and Nelder (1961) used it for estimation purposes.

If $\mu = 1$ then

$$N(t) = \kappa\left[1 + e^{-2\gamma(t-\tau)}\right]^{-1}, \tag{2}$$

which is the *Logistic* curve.

As μ tends to 0 the Richards curve (1) tends to the *Gompertz* curve. Rewrite (1) as:

$$\frac{(N(t)/\kappa)^{\mu} - 1}{\mu} = -(e^{\gamma(1+\mu)(t-\tau)} + \mu)^{-1}. \tag{3}$$

The left side of equation (3) is the Box-Cox transformation of $N(t)/\kappa$. Thus, when μ tends to 0, both sides of equation (3) tend to:

$$\ln(N(t)/\kappa) = -e^{-\gamma(t-\tau)}. \tag{4}$$

Solving for $N(t)$ yields

$$N(t) = \kappa e^{-e^{-\gamma(t-\tau)}}, \tag{5}$$

which is the Gompertz curve.

Figure 2a plots the Richards curve for three values of parameter μ. The logistic curve, $\mu = 1$, imposes the constraint that the inflection point occurs when half of the diffusion took place, i.e., $N(\tau) = \kappa/2$ if $\mu = 1$. Values of μ larger than 1 imply that the inflection point occurs at higher levels of adoption, i.e., $N(\tau) > \kappa/2$ if $\mu > 1$. And, $\lim_{\mu\to\infty} N(\tau) = \kappa$. Conversely, $N(\tau) < \kappa/2$ if $\mu < 1$. In this case, $\lim_{\mu\to 0} N(\tau) = e^{-1}\kappa$, which correspond to the inflection point of the Gompertz curve. In the general case, the inflection point is $N(\tau) = \kappa/(1+\mu)^{1/\mu}$.

Figure 2b plots the number of new subscribers per period, $dN(t)/dt$, for various values of parameter μ. The maximum number of adoptions occurs at period $t = \tau$, at which the inflection point occurs. An important feature of logistic curve is that the number of adoptions is symmetric around τ. Values of μ larger than 1 imply a fatter left tail than the logistic curve, and values of μ smaller 1 imply a fatter right tail. Hence, as μ increases, the slowdown in the adoption process after period $t = \tau$ is increasingly pronounced.

Taking logarithms and differentiating (1), one obtain the growth rate of $N(t)$ as:

$$g(t) = \frac{d\ln N(t)}{dt} = \frac{\gamma(1+\mu)}{e^{\gamma(1+\mu)(t-\tau)} + \mu} = \frac{\gamma(1+\mu)}{\mu}\left[1 - \left(\frac{N(t)}{\kappa}\right)^{\mu}\right] \tag{6}$$

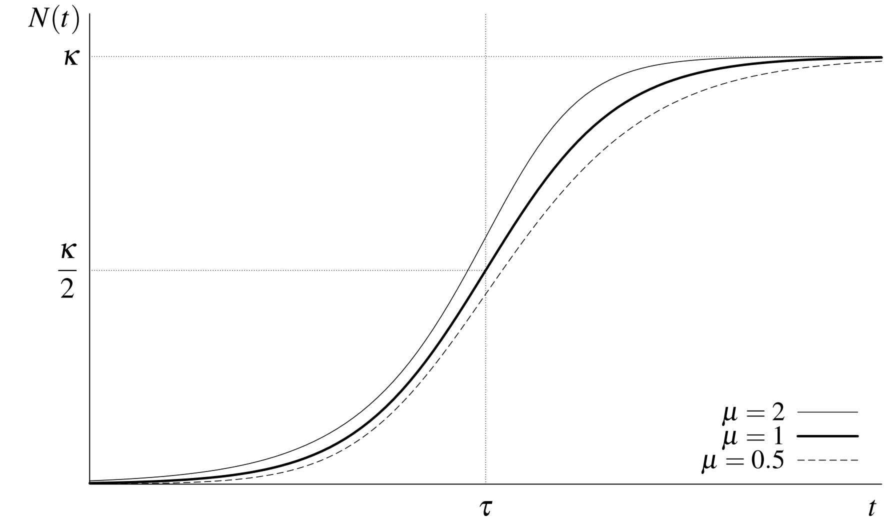
Figure 2c plots the growth rate of $N(t)$ for different values of parameter μ. The last equality of Equation (6) is very informative. Ratio $\gamma(1+\mu)/\mu$ is called the "natural rate of growth". It can be interpreted as the rate of growth that would occur, if there were no constraints limiting the adoption process, i.e., no saturation level, and the diffusion process could grow without limits. The term in square brackets, $\left[1 - (N(t)/\kappa)^{\mu}\right]$, shows the decay of the growth rate over time, as the number of adopters increases. In the case of the logistic curve, $\mu = 1$, the decay is linear in the cumulative number of subscribers. In other cases where $\mu \neq 1$, the decay is a nonlinear function of $N(t)$. As an example, in the limiting case of the Gompertz curve, i.e., as μ tends to 0, $g(t) = -\gamma\ln(N(t)/\kappa)$.

2.2. *An Illustration*

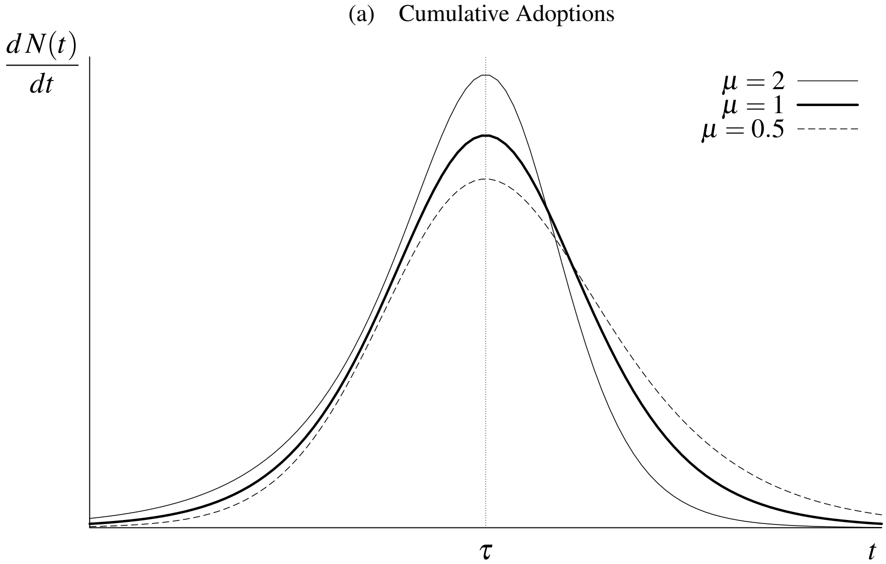
Next we use the Richards curve to characterize the adoption of cellular telephones in Portugal. We use the following empirical specification:

$$y_t = N(t)e^{u_t}, \tag{7}$$

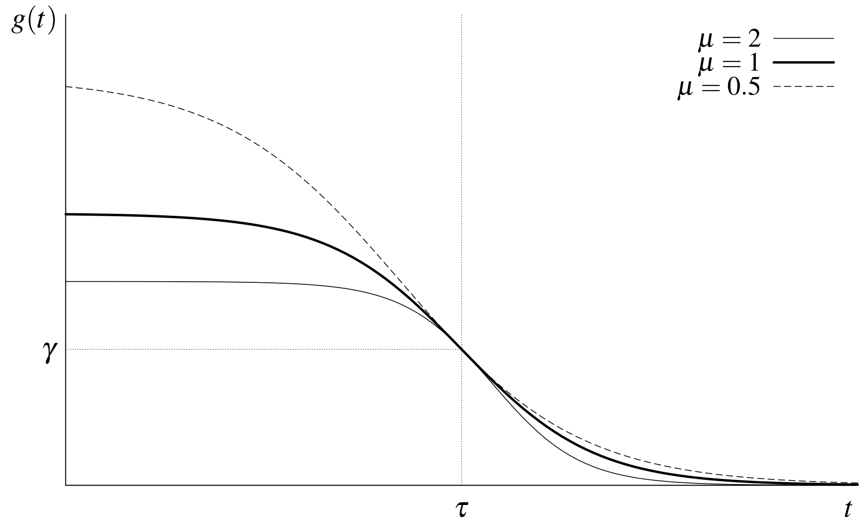
where y_t is the number of subscribers in millions, $N(t)$ is the deterministic trend given in (1), and u_t is a regression disturbance, which is supposed to have the usual properties of no serial dependence,



(a) Cumulative Adoptions



(b) Adoptions



(c) Growth Rate of Adoptions

FIGURE 2

Richards Curve for Different Values of Parameter μ.

homoskedasticity, and normal distribution.[18] Taking logarithms in equation (7), we obtain:

$$\ln y_t = \ln N(t) + u_t = \ln \kappa - \frac{1}{\mu} \ln \left[1 + \mu e^{-\gamma(1+\mu)(t-\tau)} \right] + u_t. \tag{8}$$

We estimate the parameters of the above nonlinear regression model through the maximum-likelihood method.[19] The selection of starting values for the parameters of equation (8) was the only practical difficulty encountered when estimating. The preceding discussion on the interpretation of parameters helped us in this task. For parameter κ we used the maximum of y_t. For parameter μ we used a starting value of 1, the logistic curve case. For parameter τ we plotted the first difference of y_t, and guessed visually at which time period it attained the maximum.[20] And finally, for parameter γ, we examined the growth rate of y_t around the guessed value for τ. With these starting values, convergence was achieved in a few iterations.

The results of the estimation are presented in Table 1.[21] We report three sets of estimates for: (i) the Richards curve, (ii) the logistic curve, obtained by restricting μ to equal 1, and (iii) the Gompertz curve, obtained by restricting μ to equal 0.

From Table 1, the parameter estimates vary considerably across the three models. For example, the estimated saturation level is around 9.3 millions subscribers for the Richards curve, but around 47 millions for the Gompertz curve. Observe that, in spite of the large differences in the estimates of the logistic and Gompertz specifications, they provide a similar fit to the data, as measured by the $\bar{R}^2$. We used the Wald tests, reported in the last row of Table 1, to discriminate between these three models. Both the logistic and the Gompertz curves were strongly rejected in favor of the more general Richards model.

A standard 95% confidence interval estimator of the saturation level for the Richards curve ranges from 8.7 to 10.0 millions. The estimate of τ is not significantly different from 0. According to the normalization we adopted for t, this is compatible with the inflection point of $N(t)$ being located at the first quarter of year 2000. From the estimated value of μ, the implied inflection point took place when the cumulative adoptions reached the 56% of the saturation level.

The goodness of fit, measured by the $\bar{R}^2$, is very close to 1. However, this is not surprising given the strong trend showed by the data. There are also clear symptoms that equation (7) is misspecified. The low value of the Durbin-Watson statistic indicates the presence of a strong pattern of positive autocorrelation in the residuals confirmed by visual inspection of Figure 3. A complete set of statistical tests, not reported here, points to the existence of: (i) a strong pattern of serial dependence of u_t, both in the form of residual autocorrelation, and in the form of autoregressive conditional heteroskedasticity; (ii) heteroskedasticity of the error term; and (iii) functional form misspecification. These observations cast doubts on the validity of the estimations presented in Table 1, and, in particular, on the validity of the Wald tests of Table 1, used to discriminate among the three models.

18. From (7), we use a multiplicative error term. Three reasons favor our specification over the commonly used additive error term. First, the estimation residuals are better behaved. Second, the multiplicative specification fits better with the dynamic model discussed in the following section. And third, the multiplicative model is logically coherent, in the sense that it allows a unbounded normally distributed disturbance term, while at the same time, the dependent variable is bounded.

19. All econometric results were obtained with Eviews 3.1.

20. Parameter τ is measured in the same units used for t. Thus, for ease of interpretation of parameter τ, t was normalized to be 0 on the first quarter of 2000, 1 on the second quarter of 2000, and so on.

21. Our data consists in the number of subscribers per firm, and was obtained from the firms. For *TMN* we have annual observations from 1989 to 1991, and quarterly observations from 1992 to the end of 2003. For *VODAFONE*, we have an annual observation for 1992, and quarterly observations from 1993 to the end of 2003. And for *OPTIMUS*, we have quarterly observations from the third quarter of 1998 to the end of 2003. From this data we have constructed the total number of subscribers covering the period from the first quarter of 1993 to the last quarter of 2003. The sample used in the regressions of Table 1 covers the period 1994.I to 2003.IV to ease the comparison with the dynamic specifications presented in sections 3 and 4.

TABLE 1

Static Model Estimates

	Richards	Logistic	Gompertz
κ	9.334 (0.318)***	10.716 (0.394)***	47.691 (15.127)***
γ	0.111 (0.004)***	0.095 (0.002)***	0.040 (0.004)***
τ	0.017 (0.280)	0.870 (0.437)*	21.952 (5.565)***
μ	1.729 (0.253)***	1.000	0.000
$\bar{R}^2$	0.998	0.997	0.985
$\hat{\sigma}$	0.070	0.086	0.182
$\ln L$	51.798	43.159	13.029
DW	0.435	0.303	0.123
Wald		8.291***	46.636***

Maximum-likelihood estimates and standard errors in parentheses. Sample period: 1994.I–2003.IV (40 observations). Coefficients that can be considered different from zero at the 1%, 5% and 10% significance levels are marked with ***, ** and *, respectively. $\bar{R}^2$ is the adjusted coefficient of determination. $\hat{\sigma}$ is the estimated standard deviation of the error term. *DW* is the Durbin-Watson statistic. $\ln L$ is the value of the logarithm of the likelihood function at the estimates. The *Wald* tests in the last row compare the Richards model, μ unrestricted, with the logistic, $\mu = 1$, and the Gompertz, $\mu = 0$, growth curves. These Wald statistics are distributed under the null hypotheses as χ^2 with one degree of freedom. Values of the Wald test greater than the critical values at the 1%, 5% and 10% significance levels are marked with ***, ** and *, respectively.

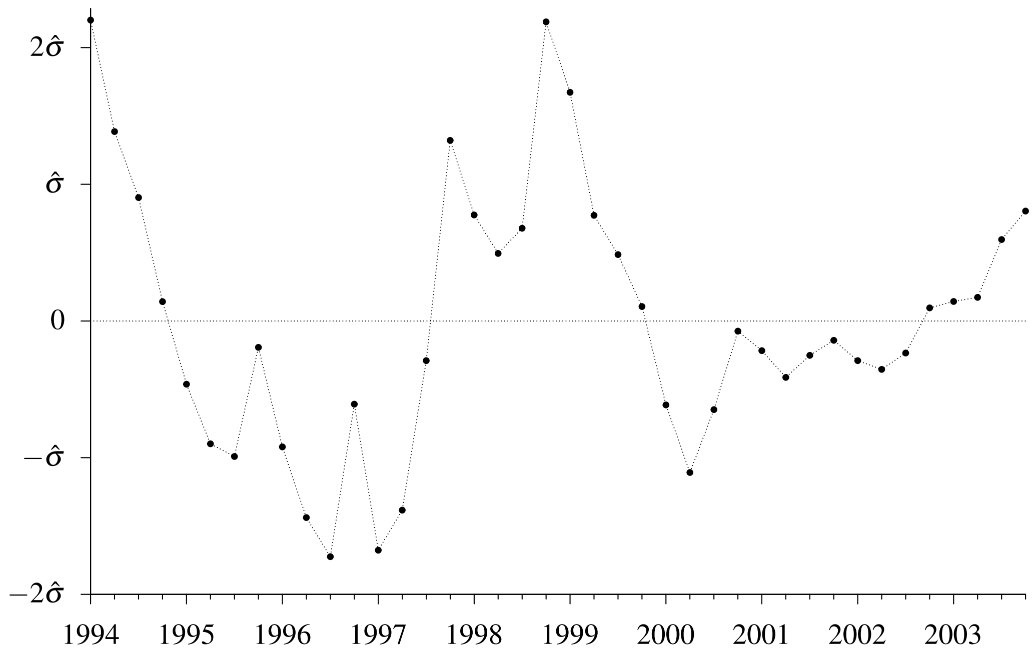


FIGURE 3

Residuals from the Richards Static Model.

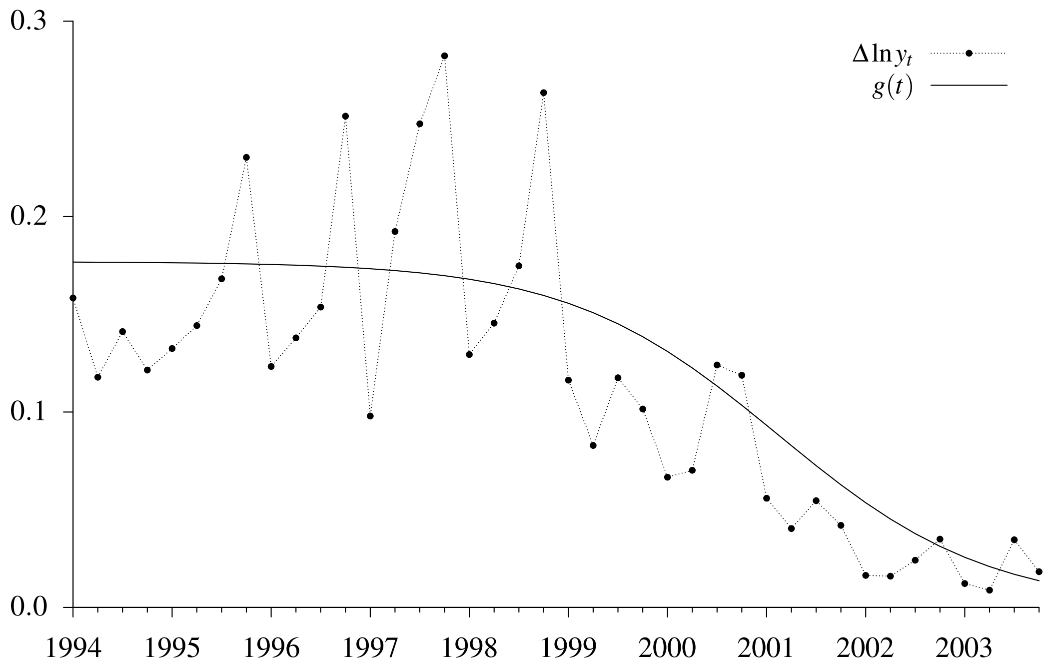


FIGURE 4
Growth Rates of Number of Subscribers: The connected dots are the growth rates of subscribers, calculated as $\Delta \ln y_t$. The line is a nonlinear least squares estimation the $g(t)$ function defined in (6).

3. A DYNAMIC AGGREGATE APPROACH

In this section, we extend our basic static model, by developing a dynamic model that simultaneously accounts for long-term and short-term adjustments.

3.1. The Model

We continue to assume that, at least in the long run, equation (7) describes the adoption process. With this interpretation in hand, u_t is the logarithmic discrepancy between y_t, and its long-run value, $N(t)$, as explicitly shown in equation (8). In order to incorporate short-run effects, we apply first differences to equation (8),

$$\Delta \ln y_t = \Delta \ln N(t) + \Delta u_t \approx g(t) + \Delta u_t, \tag{9}$$

where we have approximated $\Delta \ln N(t)$ by the growth rate $g(t)$ from equation (6). As shown in Figure 4, equation (9) expresses the relative growth rate of y_t, as random fluctuations, Δu_t, around a non-linear trend term, $g(t)$.

Assume that Δu_t can be written as

$$\Delta u_t = v_t - \alpha u_{t-1} \tag{10}$$

With this specification, we allow persistent deviations from the long-term trend. Variable u_t is a long-run error term, and each period, a fixed fraction $0 < \alpha \leq 1$ of the long term error in the previous period, u_{t-1}, is corrected. This is in contrast with the static model of Section 2, in which an instantaneous adjustment to $N(t)$, i.e., $\alpha = 1$, was implicitly assumed.

In addition to this adjustment towards the long-run trend, there is also a contemporaneous error term, v_t. This last component is decomposed as:

$$v_t = \sum_{j=1}^{L} \delta_j \Delta \ln y_{t-j} + \sum_{m=1}^{M} \phi_m D_{mt} + w_t, \tag{11}$$

where we account for short-run dynamics by means of L lags of the dependent variable, and we account for seasonal and other irregular deterministic effects through M dummy regressors D_{mt}. Finally, w_t is an is a non-autocorrelated, homoskedastic, and normally distributed innovation term.

Substituting equations (8), (10), and (11) in equation (9):

$$\Delta \ln y_t = g(t) - \alpha \left[\ln y_{t-1} - \ln N(t-1) \right] + \sum_{j=1}^{L} \delta_j \Delta \ln y_{t-j} + \sum_{m=1}^{M} \phi_m D_{mt} + w_t. \tag{12}$$

Apart from the deterministic variables D_{kt}, equation (12) decomposes the changes in the dependent variable in three terms: (i) a deterministic non-linear trend, $g(t)$, (ii) an adjustment mechanism driving $\ln y_t$ towards the nonlinear long-run trend $N(t)$, i.e., $\alpha[\ln y_{t-1} - \ln N(t-1)]$, and (iii) terms capturing the persistence across time of temporary shocks, $\sum_j \delta_j \Delta \ln y_{t-j}$. Equation (12) describes the stochastic process of y_t in a way similar to error correction models. In this kind of models the adjustments of the dependent variable are related to the extent to which the dependent variable deviates from an equilibrium relationship with other explanatory variables (Banerjee et al., 1993). Equation (12) directly relates changes in $\ln y_t$, with the size of the long-run disequilibrium captured by $u_{t-1} = \ln y_{t-1} - \ln N(t-1)$. Also note that equation (12) models both the short-run and the long-run behavior of $\ln y_t$: the growth rate of y_t depends on the short-run parameters, $(\alpha, \delta_j, \phi_m)$, and the long-run parameters, $(\kappa, \gamma, \tau, \mu)$, through functions $N(t)$ and $g(t)$, defined in equations (1) and (6). In fact, the model of equation (12) encompasses the static model of the previous section, which is obtained when $\alpha = 1$, $\delta_j = 0$, and $\phi_m = 0$ for all j and m.

3.2. Estimation

As illustrated by Figure 4, the data is characterized by a strong seasonal pattern. Thus, our baseline model includes four lags of the quarterly growth rates, $\Delta \ln y_t$, and deterministic seasonal dummies, $Seas_{st}$:

$$\Delta \ln y_t = g(t) - \alpha \left[\ln y_{t-1} - \ln N(t-1) \right] + \sum_{j=1}^{4} \delta_j \Delta \ln y_{t-j} + \sum_{s=1}^{3} \phi_s Seas_{st} + w_t. \tag{13}$$

We imposed the normalization that the sum of the seasonal effects over a year is equal to 0. This is accomplished by proper definition of the seasonal dummies. Our dummies $Seas_{st}, s = 1,2,3$, take the value 1 on the s-th quarter of every year and take the value -1 on the 4th quarter of every year. The parameters ϕ_s on equation (13) measure the seasonal effect on quarter $s = 1,2,3$. The seasonal effect on the 4th quarter is $\phi_4 = -(\phi_1 + \phi_2 + \phi_3)$.

As before, all the parameters of equation (13) were jointly estimated through the maximum-likelihood method. Starting values for the long-run parameters were selected as described in a Section 2.2. The starting value for parameter α was set to $\alpha = 1$. The remaining parameters were initially set to 0.

The estimation of model in equation (13) produced good results at the expense of a slightly overparametrized specification. Further experimentation lead us to favor the more parsimonious model:

$$\Delta \ln y_t = g(t) - \alpha \left[\ln y_{t-1} - \ln N(t-1) \right] + \delta' \Delta_4 \ln y_{t-1} + \delta_3' \Delta \ln y_{t-3} + \sum_{s=1}^{3} \phi_s Seas_{st} + w_t, \tag{14}$$

where $\Delta_4 \ln y_t = \sum_{j=1}^{4} \Delta \ln y_{t-j} = \ln y_t - \ln y_{t-4}$ is the yearly growth rate of y_t. Equation (14) imposes two restrictions on the parameters of equation (13). These restrictions are supported by a Wald test and the statistical properties of the model do not deteriorate when they are imposed. The maximum-likelihood estimates of equation (14) are shown in Table 2 under the heading of Model I. Also, in Table 3 we report several statistical misspecification tests. As can be seen, there is no signal of serial dependence, heteroskedasticity or non-normality on the residuals of Model I. However, the general RESET tests reject the null hypothesis of correct specification.

TABLE 2

Diffusion Models for the Aggregate Number of Subscribers

	Model I	Model II	Model III	Model IV	Model V
κ	9.440 (0.582)***	9.152 (0.284)***	9.553 (0.570)***	10.077 (0.507)***	10.183 (0.470)***
γ	0.101 (0.007)***	0.109 (0.003)***	0.100 (0.007)***	0.094 (0.002)***	0.093 (0.002)***
τ	2.849 (0.961)***	2.129 (0.387)***	3.209 (0.922)***	3.726 (0.955)***	3.647 (0.900)***
μ	1.268 (0.325)***	1.689 (0.185)***	1.270 (0.293)***	1.000	1.000
α	0.321 (0.110)***	1.000	0.334 (0.100)***	0.279 (0.059)***	0.283 (0.057)***
$\Delta_4 \ln y_{t-1}$	0.294 (0.092)***	0.594 (0.123)***	0.291 (0.086)***	0.197 (0.055)***	0.184 (0.053)***
$\Delta \ln y_{t-3}$	−0.352 (0.193)*	−0.327 (0.310)	−0.254 (0.185)		
$Seas_{1t}$	−0.028 (0.009)***	−0.011 (0.015)	−0.037 (0.011)***	−0.041 (0.012)***	−0.034 (0.009)***
$Seas_{2t}$	−0.017 (0.008)**	−0.017 (0.014)	−0.023 (0.011)**	−0.018 (0.007)**	−0.016 (0.007)**
$Seas_{3t}$	0.020 (0.011)*	0.007 (0.018)	0.015 (0.014)		
$Seas_{1t} \times S1999_t$			0.030 (0.016)*	0.039 (0.013)***	0.033 (0.012)**
$Seas_{2t} \times S1999_t$			0.009 (0.015)		
$Seas_{3t} \times S1999_t$			0.003 (0.016)		
Opt_{t-1}					0.003 (0.016)
$\bar{R}^2$	0.837	0.573	0.860	0.864	0.878
$\hat{\sigma}$	0.030	0.048	0.028	0.027	0.026
DW	1.924	0.629	1.737	1.716	1.653
$\ln L$	89.285	69.412	94.537	91.646	94.457

Maximum-likelihood estimates and standard errors in parentheses. Dependent variable id $\Delta \ln y_t$. Sample period: 1994.I–2003.IV (40 observations). Coefficients that can be considered different from zero at the 1%, 5% and 10% significance levels are marked with ***, ** and *, respectively. $\bar{R}^2$ is the adjusted coefficient of determination. $\hat{\sigma}$ is the estimated standard deviation of the error term. DW is the Durbin-Watson statistic. $\ln L$ is the value of the logarithm of the likelihood function at the estimates.

TABLE 3

Misspecification Tests for the Aggregate Number of Subscribers Models

	Model I	Model II	Model III	Model IV	Model V
AR(1)	0.863	0.000***	0.391	0.429	0.321
AR(2)	0.607	0.000***	0.638	0.570	0.600
AR(3)	0.311	0.000***	0.256	0.261	0.221
AR(4)	0.462	0.001***	0.278	0.290	0.299
AR(5)	0.361	0.001***	0.358	0.277	0.203
ARCH(1)	0.205	0.019**	0.915	0.910	0.533
ARCH(2)	0.209	0.072*	0.454	0.984	0.673
ARCH(3)	0.357	0.117	0.456	0.958	0.836
White	0.403	0.328	0.304	0.349	0.543
Jarque-Bera	0.890	0.873	0.906	0.887	0.776
RESET (up to 2nd power)	0.006***	0.000***	0.071*	0.105	0.089*
RESET (up to 3rd power)	0.025**	0.000***	0.127	0.261	0.131
Chow Forecast (from 2000.I)	0.832	0.728	0.988	0.992	0.976
Chow Forecast (from 2001.I)	0.966	0.642	0.995	0.973	0.922
Chow Forecast (from 2002.I)	0.832	0.344	0.962	0.998	0.998
Chow Forecast (from 2003.I)	0.704	0.485	0.924	0.940	0.952

The figures are p-values of specification tests for the models presented in Table 2. AR(l) is the Breusch-Godfrey Lagrange Multiplier test against autocorrelation up to order l. ARCH(l) is the Engle Lagrange Multiplier test against autoregressive conditional heteroskedasticity up to order l. These are followed by the White test against heteroskedasticity of unknown form, not including interaction terms. The Jarque-Bera statistics is used to test the null hypothesis of normality of the error term. The following two rows report general specification Ramsey's RESET tests including the squares and the squares and cubes of the predictions. The four last rows report Chow forecast tests. Test statistics greater than their corresponding 1%, 5%, 10% critical values are marked with ***, **, and *, respectively.

The estimates of Model I of the long run parameters κ and γ are similar to those obtained with the simpler static model of Table 1. But the estimates of the time at which inflection occurs, τ, and the shape parameter, μ are very different. The estimate of τ for Model I implies that the inflection occurred at the end of year 2000. Also, the estimate of parameter μ is not significantly different from 1, so we fail to reject the hypothesis of a symmetric long-run trend.

The poor performance of the static model of the previous section could be caused by the omission of short run dynamics and seasonal effects. Model II is designed to provide a fairer comparison, allowing for seasonal effects and short-run dynamics but imposing an instantaneous adjustment to the non-linear trend, $\alpha = 1$. Model II is clearly worse than Model I. The huge reduction in the fit rejects the hypothesis $\alpha = 1$, and the misspecification tests show a substantial amount of serial dependence in the residuals. Therefore, we conclude that slow adjustment to the nonlinear trend is an important feature of our data. Failing to account this, seriously bias the estimates of the parameters driving the diffusion process.

As said before, Model I fails to pass the RESET tests. This could be due to the varying pattern of seasonality in our data, clearly illustrated in Figure 4, but not accounted in Model I. We have extended Model I by adding the interactions of the seasonal dummies, $Seas_{st}$, with a step dummy, $S1999_t$, which takes value 0 for observations preceding year 1999 and 1 for year 1999 and on. The coefficients of these interaction variables measure the change in the seasonal pattern from 1999. The estimates of the extended model are reported under the heading Model III in Table 2. The estimates of Model III suggests a change in seasonality affecting especially to the effect of the first quarter. The statistical properties of Model III are better than those of Model I: only one RESET test rejects the null hypothesis at the 10% significance level.

Model IV imposes some restrictions on the parameters of Model III. First, we impose the hypothesis of a logistic long-run trend, $\mu = 1$. Second, we set to 0 the insignificant parameters of Model III. These restrictions are not rejected by a Wald test, and, in fact, the goodness of fit, measured by the $\bar{R}^2$, is slightly better. The misspecification tests of Table 3 do not signal any problem with Model IV. Also the Chow forecast tests do not signal problems of parameter instability. With respect to the short-run dynamics, it is worth noting that the estimate of the parameter associated with the lagged yearly growth rate, $\Delta_4 \ln y_{t-1}$, is positive and significantly different from 0. This could be interpreted as evidence in favor of the existence of network economies. An increase in the growth rate of the number of subscribers accelerates the adoption process in subsequent periods.

Model V of Table 2 is estimated to analyze the impact of the entrance of *OPTIMUS* in the market. This model extends Model IV by adding a dummy variable Opt_t which takes value 1 on 1998.III, coinciding with the *OPTIMUS* entrance, and 0 otherwise. Observe that this dummy enters in Model V lagged one period, so the effect of the entrance occurs one quarter after the entrance. We experimented with other specifications, but we detected no structural change in the parameters around the time the entrance occurred.

3.3. Analysis

Further inspection of Table 2 leads to the following observations.

Observation 3.1. *A logistic long-run trend is a valid model for the Portuguese cellular telephone industry.* ♦

From Table 2, one fails to reject the hypothesis that $\mu = 1$. This conclusion might come as a surprise after the build up of Section 2. But in fact what it does is highlight, once again, the inadequacy of the static approach.

Observation 3.2. *The saturation level for GSM ranges between* 9.2 *and* 11.1 *millions subscribers.*♦

This conclusion is supported by a 95% confidence interval estimate for the κ parameter constructed from Model V of Table 2. The diffusion of *GSM* had almost concluded by the end of 2003. Very likely, the deployment of *UMTS* will give an additional impulse to the diffusion process.

Observation 3.3. *The inflection point occurred in the fourth quarter of 2000.* ♦

This conclusion follows from the estimated value of τ parameter. This implies that when *OPTIMUS* entered the market the diffusion process was far from over yet.

Observation 3.4. *The data is driven towards a nonlinear long-run trend by an error correction mechanism, where each period, about 30% of the long-run disequilibrium of the previous period is corrected.* ♦

This observation comes from the estimate of parameter α in Model V of Table 2. This conclusion has two implications. First, the adjustment process exhibits a very large inertia. Second, the hypothesis of instantaneous adjustments to the long-run trend, which corresponds to $\alpha = 1$, underlying the static model, is strongly rejected.

Observation 3.5. *There are important, but decreasing over time, seasonal fluctuations.* ♦

Our estimates indicate that there is a strong seasonal pattern during 1994–1998. Afterwards, the seasonal fluctuations dampened. The strong negative effect of the first quarter vanishes after 1999. Consequently, the strong positive effect of the fourth quarter is dampened from that date on. Hence, in the last years of our sample, as the diffusion process progressed and the number of subscribers reached the saturation level, there was less leeway for strong seasonal fluctuations. Note that the dampening of seasonal effects started right after the entry of *OPTIMUS*.

Observation 3.6. *The entry of* OPTIMUS *temporarily increased the speed of diffusion of cellular telephony in Portugal.* ♦

The parameter associated with the dummy Opt_{t-1} is significant and positive. This implies that the entrance of *OPTIMUS* was followed by a period of higher adoption rates. Figure 5 plots the impulse response function associated with the entrance. The year next to the entry was characterized by higher rates of adoption. But this effect is temporary and in the long run the increase in the number of firms has no effect in the diffusion process.

Observation 3.7. *The liberalization in 2000 had no effect on the speed of diffusion of cellular telephony in Portugal.* ♦

This conclusion can be gleaned from the Chow tests presented in Table 3. The data does not show evidence in favor of a structural break in the year 2000 nor in the following years. Given the discussion in Subsection 1.1 this should come as no surprise.

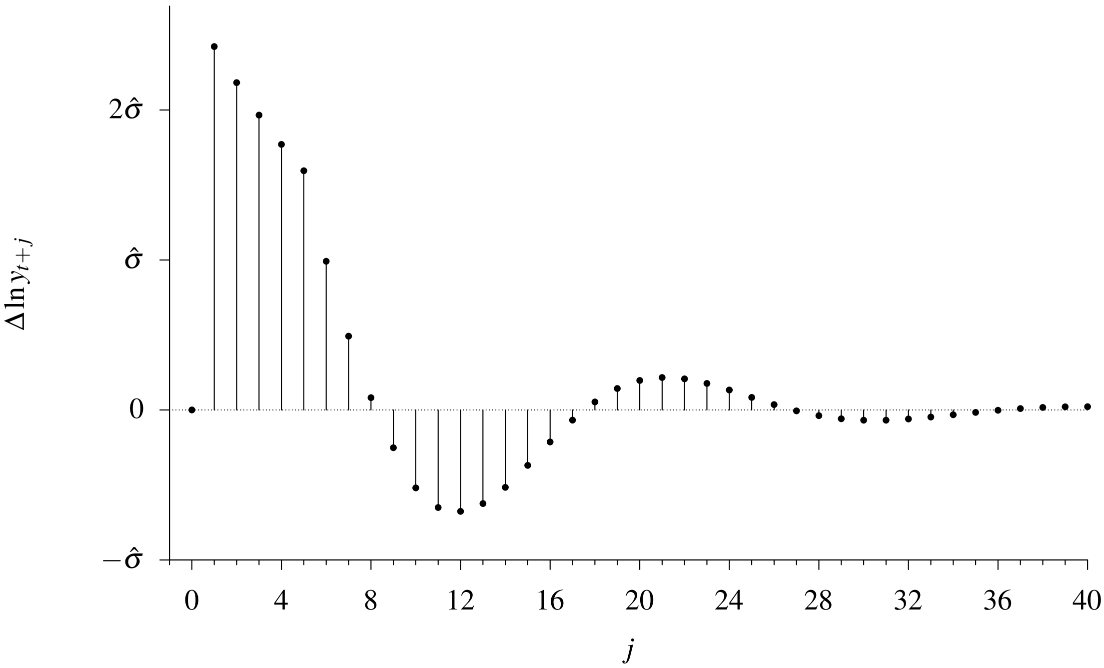


FIGURE 5
Impulse-response Function: variation of $\ln y_t$ as a consequence of the *OPTIMUS* entry, computed by simulation of Model V in Table 2.

4. A DYNAMIC DISAGGREGATE APPROACH

In this section we use dynamic specifications to analyze the evolution of the number of subscribers in each of the three firms operating in the Portuguese market.

4.1. The Model

Ideally, the effect of competition on the diffusion process of every firm in the market should be measured by estimating jointly a system of equations that account for cross effects. In our time-series framework, a VAR model incorporating non-linear trends, as those discussed in previous sections, could be used to unveil complex interactions among the three firms operating in the Portuguese market. However, this is not an option for us, given the late entry of *OPTIMUS* in our sample. Instead, we estimated separate equations for each firm, including: (i) the lagged growth rates of the firm's own number of subscribers, and (ii) the lagged growth rates of the number of subscribers of the other two rivals:

$$\Delta \ln y_t = g(t) - \alpha\left[\ln y_{t-1} - \ln N(t-1)\right] + \sum_{j=1}^{L} \delta_j \Delta \ln y_{t-j} + \sum_{k=1}^{K} \lambda_k \Delta \ln z_{t-k} + \sum_{m=1}^{M} \phi_m D_{mt} + w_t. \qquad (15)$$

Equation (15) extends equation (13) by adding K lags of the growth rate of subscribers in the other two rival firms, i. e., $\Delta \ln z_t$. Although far from ideal, these additional terms provide some indication of the effects of competition, and of complementarities or substitutabilities between the firms' products.

4.2. Estimation

Table 4 reports the estimates of models like equation (15) for each of the three operators. Those estimates were selected among several alternative models. They provide a satisfactory and parsimonious

TABLE 4

Diffusion Models for Each Operator in the Portuguese Market

	TMN	*VODAFONE*	*OPTIMUS*
κ	5.269 (0.139)***	3.274 (0.088)***	1.489 (0.026)***
γ	0.090 (0.001)***	0.104 (0.004)***	0.120 (0.011)***
τ	4.686 (0.445)***	−1.712 (0.627)**	1.481 (0.733)*
μ	1.000	1.000	3.396 (1.397)**
α	0.561 (0.072)***	0.390 (0.090)***	0.723 (0.147)***
$\Delta_4 \ln y_{t-1}$	0.160 (0.037)***	0.191 (0.046)***	
$\Delta_4 \ln z_{t-1}$	0.227 (0.051)***	−0.105 (0.050)**	
$\Delta \ln z_{t-1}$			1.427 (0.303)***
$Seas_{1t} \times S9598_t$	−0.039 (0.011)***		
$Seas_{2t} \times S9598_t$	−0.043 (0.011)***		
$Seas_{1t}$		−0.040 (0.009)***	
$Seas_{1t} \times S1999_t$		0.036 (0.013)**	
T	40	40	19
$\bar{R}^2$	0.884	0.865	0.936
$\hat{\sigma}$	0.026	0.029	0.022
DW	1.656	1.798	1.366
$\ln L$	93.577	89.707	48.834

Maximum-likelihood estimates and standard errors in parentheses. Dependent variable is $\Delta \ln y_t$. Sample period: 1994.I–2003.IV for *TMN* and *VODAFONE*; 1999.II–2003.IV for *OPTIMUS*. Coefficients that can be considered different from zero at the 1%, 5% and 10% significance levels are marked with ***, ** and *, respectively. $\bar{R}^2$ is the adjusted coefficient of determination. $\hat{\sigma}$ is the estimated standard deviation of the error term. *DW* is the Durbin-Watson statistic. $\ln L$ is the value of the logarithm of the likelihood function at the estimates.

explanation of the diffusion process for each firm. The misspecification tests shown in Table 5 do not signal strong statistical problems.

In general, the long run parameters are precisely estimated, with the exception of parameter μ in the case of *OPTIMUS*. This could indicate that precise estimation of Richards curve is not feasible in very small samples.

The estimated saturation levels for the three firms are compatible with the estimated aggregate saturation level. The sum of the κ estimates from the estimated models of Table 4 is 10.032, which it is nearly identical to the κ estimates of the parsimonious Model IV and V of Table 2. Using 95% confidence interval estimates, the saturation level ranges from 5.0 to 5.5 millions subscribers for *TMN*; from 3.1 to 3.5 millions subscribers for *VODAFONE*; and from 1.4 to 1.5 millions subscribers for *OPTIMUS*.

The short-term dynamics of growth rates do not follow a common pattern across firms. Two comments are in order. First, the role of own lagged growth rates varies considerably across firms. For *OPTIMUS*, no own lagged growth rates are significant. For the other firms, the effect of own lagged growth rates is positive. Second, the influence of rivals' lagged growth rates also vary considerably across firms. They are negative for *VODAFONE*, positive for *TMN*, and positive with a coefficient estimate greater than 1 for *OPTIMUS*.

The liberalization in 2000 had no effect on the speed of diffusion of cellular telephony in Portugal.

TABLE 5
Misspecification Tests for the Individual Firms Models

	TMN	*VODAFONE*	*OPTIMUS*
AR(1)	0.288	0.574	0.113
AR(2)	0.551	0.711	0.255
AR(3)	0.743	0.686	0.326
AR(4)	0.645	0.564	0.299
AR(5)	0.559	0.496	0.426
ARCH(1)	0.919	0.686	0.712
ARCH(2)	0.969	0.623	0.952
ARCH(3)	0.840	0.327	0.665
White	0.424	0.469	0.785
Jarque-Bera	0.765	0.949	0.627
RESET (up to 2nd power)	0.260	0.527	0.668
RESET (up to 3rd power)	0.433	0.432	0.608
Chow Forecast (from 2000.I)	0.911	0.983	
Chow Forecast (from 2001.I)	0.964	0.998	
Chow Forecast (from 2002.I)	0.966	0.961	0.259
Chow Forecast (from 2003.I)	0.948	0.670	0.119

The figures are p-values of specification tests for the models presented in Table 4. AR(l) is the Breusch-Godfrey Lagrange Multiplier test against autocorrelation up to order l. ARCH(l) is the Engle Lagrange Multiplier test against autoregressive conditional heteroskedasticity up to order l. These are followed by the White test against heteroskedasticity of unknown form, not including interaction terms. The Jarque-Bera statistics is used to test the null hypothesis of normality of the error term. The following two rows report general specification Ramsey's RESET tests including the squares and the squares and cubes of the predictions. The four last rows report Chow forecast tests. Test statistics greater than their corresponding 1%, 5%, 10% critical values are marked with ***, **, and *, respectively.

Given the available data, we can not test this hypothesis for *OPTIMUS*. The Chow forecast tests in Table 5 do not provide evidence of a structural break in the period following the liberalization in the regressions of *TMN* or *VODAFONE*. This result is consistent with the aggregate regression.

4.3. Analysis

Inspection of Table 4 leads to the following observations:

Observation 4.1. *A long-run logistic trend is valid model for the number of subscribers of each firm.* ♦

The estimates for *TMN* and *VODAFONE* shown in Table 4 were obtained through a selection process starting with a more general model with μ unrestricted. In both cases, the Wald tests do not reject of hypothesis $\mu = 1$. The case of *OPTIMUS* needs some qualification. Although, a Wald test does not reject the logistic curve hypothesis, imposing the restriction $\mu = 1$ deteriorates the statistical properties of the model, and the misspecification tests signal the presence of residual autocorrelation.

Observation 4.2. *The diffusion processes differ on the time at which the inflection point is located.* ♦

The estimates in Table 4 imply that the inflection point occurred in mid 1999 for *VODAFONE*, in mid 2000 for *OPTIMUS*, and in mid 2001 for *TMN*. This implies that the slowdown in the growth rate in the number subscribers started first for *VODAFONE*, then for *OPTIMUS*, and finally for *TMN*.

Observation 4.3. *For each firm, the data is driven toward a nonlinear long-run trend by an error correction mechanism, where each period, a fraction of disequilibrium of the previous period, which differs across firms, is corrected.* ♦

The estimates of parameter α of Table 4 imply that *VODAFONE* has the slowest adjustment, 0.390, and is followed by *TMN*, 0.561. In either case, the hypothesis $\alpha = 1$, i.e., the hypothesis of instantaneous adjustment to the long-run trend, is strongly rejected. For *OPTIMUS* this hypothesis is marginally rejected at the 5% significance level, and can not be rejected at lower significance levels.

Observation 4.4. *There are important and varying over time seasonal patterns.* ♦

The seasonal patterns found for *TMN* and *VODAFONE* are roughly consistent with that encountered for the aggregate data.

For *TMN* we have controlled for seasonal variation with the inclusion of the interaction of seasonal dummies, $Seas_{st}$, and a step dummy taking the value of 1 in all quarters of the years 1995 to 1998. Our estimates show that seasonal fluctuations are only significant during this period. The sign of the estimates imply negative seasonal impacts in the first and second quarters, that are compensated by a strong positive seasonal effect in the fourth quarter.

In the case of *VODAFONE* we found a significant negative effect in the first quarter compensated by a positive effect in the fourth quarter. This pattern is considerably dampened from year 1999 and from that year seasonal variations practically vanish.

The short sample available for *OPTIMUS* precluded introducing seasonal dummies in the regressions. Nevertheless, the autocorrelation tests for the *OPTIMUS* estimations do not signal any seasonal pattern.

5. CONCLUSIONS

In this paper, we propose a dynamic approach to the empirical study of the processes of diffusion of innovations. Our model has two novel features. First, we model the long-run trend through the Richards curve, which is more flexible than the standard alternatives. Second, our model incorporates both short-run and long-run terms. We apply our framework to the diffusion of cellular telephony in Portugal, and find that our suggested dynamic approach outperforms the traditional static specifications. We also conduct exhaustive specification testing, in line with the modern econometric approach to time series models.

With the estimated models we can identify six interesting aspects of the diffusion process of the cellular telephony in Portugal. First, the logistic curve is a valid representation of the diffusion process, but only as a long-run trend. The data persistently deviates from its long-run trend in the short-run. Second, we identify significant effects of increased competition on the diffusion process. These effects are positive but short-lived, and they do not affect the long-run level of subscribers. Third, there are strong seasonal fluctuations that account for much of the variation in the early stages of diffusion. Interestingly, these fluctuations dampened coinciding with the entry of the third operator. Fourth, there are network economies. Fifth, we found no effects of the full liberalization of the telecommunications market on the diffusion process. Sixth, the long-term penetration rate for *GSM* is around 100%, and by now is over.

REFERENCES

BANERJEE, A., J. DOLADO, J. W. GALBRAITH, AND D. F. HENDRY (1993): *Co-Integration, Error-Correction, and the Econometric Analysis of Non-Stationary Data*. Oxford: Oxford University Press.

BARROS, P. AND N. CADIMA (2000): "The Impact of Mobile Phone Diffusion on the Fixed-Link Network." Discussion Paper 2598, CEPR.

BEWLEY, R. AND D. G. FIEBIG (1988): "A Flexible Logistic Growth Model with Applications in Telecommunications." *International Journal of Forecasting*, 4, 177–192.

BOTELHO, A. AND L. PINTO (2000): "Has Portugal Gone Wireless? Looking Back, Looking Ahead." Mimeo, Universidade do Minho.

CHOW, G. C. (1967): "Technological Change and the Demand for Computers." *American Economic Review*, 57, 1117–1130.

DIXON, R. (1980): "Hybrid Corn Revisited." *Econometrica*, 48, 1451–1461.

DOGANOGLU, T. AND L. GRZYBOWSKI (2003): "Diffusion of Mobile Telecommunications Services in Germany: a Network Effects Approach." Mimeo, Ludwig Maximilians Universitat, Munchen.

GEROSKI, P. A. (2000): "Models of Technology Diffusion." *Research Policy*, 29, 603–625.

GRILICHES, Z. (1957): "Hybrid Corn: An Exploration in the Economics of Technological Change." *Econometrica*, 25, 501–522.

GRUBER, H. (2001): "Competition and Innovation: The Diffusion of Mobile Telecommunications in Central and Eastern Europe." *Information Economics and Policy*, 13, 19–34.

GRUBER, H. AND F. VERBOVEN (2001a): "The Diffusion of Mobile Telecommunications Services in the European Union." *European Economic Review*, 45, 3, 577–588.

GRUBER, H. AND F. VERBOVEN (2001b): "The Evolution of Markets under Entry and Standards Regulation: The Case of Global Mobile Telecommunications." *International Journal of Industrial Organization*, 19, 7, 1189–1212.

HENDRY, D. F. (ed.) (1995): *Dynamic Econometrics*. Oxford: Oxford University Press.

MAHAJAN, V., E. MULLER, AND F. M. BASS (1990): "New Product Diffusion Models in Marketing: A Review and Directions for Research." *Journal of Marketing*, 54, 1–26.

MEADE, N. AND T. ISLAM (1998): "Technological Forecasting — Model Selection, Model Stability, and Combining Models." *Management Science*, 44, 1115–1130.

NELDER, J. A. (1961): "The Fitting of a Generalization of the Logistic Curve." *Biometrics*, 17, 89–110.

NELDER, J. A. (1962): "An Alternative Form of a Generalized Logistic Equation." *Biometrics*, 4, 614–616.

RICHARDS, F. J. (1959): "A Flexible Growth Function for Empirical Use." *Journal of Experimental Botany*, 10, 290–300.

RODINI, M., M. R. WARD, AND G. A. WOROCH (2002): "Going Mobile: Substitutability Between Fixed and Mobile Access." Working Paper CRTP-58, Center for Research on Telecommunications Policy, Haas School of Business.

TSOULARIS, A. AND J. WALLACE (2002): "Analysis of Logistic Growth Models." *Mathematical Biosciences*, 179, 21–55.

VALLETTI, T. (2003): "Is mobile telephony a natural oligopoly?" *Review of Industrial Organization*, 22, 47–65.