

# Access to Bottleneck Inputs under Oligopoly: a Prisoners' Dilemma?\*

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## Abstract

In this article, we analyze the incentives of vertically integrated oligopolists to concede access to their bottleneck inputs to an entrant in the downstream market. We develop a two-stage game, where in the first stage a downstream entrant negotiates an access price with two vertically integrated incumbents, and in the second stage firms compete on Salop's circle. The firms may be asymmetrically located on the circle, to reflect differences in consumer shares. For some levels of asymmetry, the incumbents face a prisoners' dilemma with respect to conceding access to their bottleneck inputs. Entry by a downstream firm may lead to lower retail prices. However, entry may also lead to higher retail prices for the access provider and for the entrant. We also consider the cases where there are three incumbents, where the entrant makes the access price offers, and where the incumbents collude.

**Key Words:** *Bottleneck Input, Vertical Integration, Oligopoly, Entry.*

**JEL Classification:** L43, L96.

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# 1 Introduction

As it is well known, the monopolist owner of a bottleneck production factor, which is also present in the downstream market, may have the incentive and the ability to restrict access to the bottleneck production factor, to limit competition in the downstream market. Examples of this are telecommunications, electricity, or railways incumbents, which may restrict access to their local networks, to limit competition in the retail markets.

In some industries, more than one vertically integrated firm owns a bottleneck production factor, to which they can give access to downstream entrants. In mobile telephony, more than one firm own a license to use the radio-electric spectrum and a mobile telecommunications network, to which they can give access to downstream entrants, like mobile virtual network operators. In broadband access to the Internet, in some countries, both the telecommunications incumbent and cable television firms own local access networks capable of delivering telecommunications services, to which they can give access to downstream entrants, like Internet service providers. In the film industry, several studios produce movies that can be exhibited at their own theaters or at independent theaters. In the airline industry, in some markets, there are several national or local carriers that can sell their handling services to foreign carriers.

In those industries, there are at least three reasons to suspect that vertically integrated oligopolists have different incentives than a vertically integrated monopolist, to concede access to their bottleneck production factors to a downstream entrant. First, even if an incumbent denies access to his bottleneck production factor, there is no guarantee that the entrant will not obtain access elsewhere. Second, an incumbent that provides access to an entrant shares with the other incumbents the downstream revenue loss caused by the entrant. Third, if entry cannot be blocked, then it is probably better for each incumbent to be the access provider. This allows the incumbent to earn additional wholesale revenues, that at least partially compensate any loss in retail revenues caused by the entrant. Altogether, this suggests that incumbents may face a prisoners' dilemma. They would be better off if entry did not occur. However, individually they have incentives to be the one who gives access to the entrant.

We develop a two-stage game to analyze these issues. In the first stage, an entrant negotiates the access price with two incumbent firms. In the second stage, firms compete on retail prices. We model the industry as a horizontally differentiated product oligopoly on Salop's circle (Salop, 1979). A distinguishing characteristic of our approach is that the firms may be asymmetrically located on the circle, with the degree of asymmetry captured by a single parameter. The varying relative location of firms on the loop captures differences in consumer shares.

The model confirms our intuition. For some values of the asymmetry parameter, it is a dominant strategy for the incumbents to concede access to the entrant, although they would be better off without entry. However, this intuition requires two qualifications. First, for some values of the asymmetry parameter, although entry is the equilibrium of the bargaining stage, the incumbents do not face a prisoners' dilemma. For some incumbents, conceding access is not a dominant strategy. Second, for other values of the asymmetry parameter, the equilibrium of the bargaining stage is for the incumbents to deny access to their bottleneck production factor. These results highlight the importance of product differentiation and firm asymmetry in the outcome of the bargaining stage over the access price.

As expected, entry may lead to lower retail prices. Interestingly, however, entry may also lead the retail prices of the access provider and of the entrant to rise above pre-entry levels. If the access provider is a direct rival of the entrant, by hiking his retail price he increases the sales of the entrant, and thereby his own wholesale revenues. If access is sold above the marginal cost, and the entrant is otherwise equally efficient than the incumbents, overall he has higher costs than the incumbents. As a consequence, the entrant may charge a retail price higher than the retail prices the incumbents charged prior to entry.

We consider three variations of the model, to check the robustness of our results. The first variation introduces a third incumbent. The purpose of this variation is twofold. First, it allows us to analyze the impact of the number of incumbents on our results. Second, it allows us to analyze who has a larger incentive to give access, a direct or a distant rival of the entrant, and who the entrant prefers to get access from. The analysis shows that the main results hold. In addition, the analysis shows that the entrant prefers to get access from a direct rival, and that a direct rival has a larger incentive to give access than an distant rival. The second variation has the entrant, rather than the incumbents, making the access price offers. This variation captures the situation where the entrant has all the bargaining power. The analysis shows that entry occurs for the same parameter values as in the case where the incumbents make the access price offers, although at different access prices. The third variation has the incumbents colluding. The analysis shows that for some parameter values a cartel allows entry.

Our article relates to: **(i)** the literature on vertical foreclosure, **(ii)** the literature on raising rivals' costs and **(iii)** the literature on access pricing. The first literature strand addresses the question of whether a vertically integrated firm can increase or not its profit by foreclosing the downstream market. Rey and Tirole (2006) reviewed the case in which the upstream market is monopolized.<sup>1</sup> Ordover et al. (1990) and Hart and Tirole (1990) analyzed the case of an

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<sup>1</sup>For an article motivated by the telecommunications industry, see Biglaiser and DeGraba (2001).

oligopolistic upstream market, focusing on the profitability of vertical mergers. We depart from this literature by assuming that several incumbents are vertically integrated from the beginning, and focusing on their incentives to allow the entry of downstream competitors. Ordoover and Shaffer (2006) developed independently an approach similar to ours. The main difference between our articles resides on the demand model. The complete characterization of the equilibria in Ordoover and Shaffer (2006) is hard, due to the large number of parameters for the firms' baseline market shares, and for the consumers' cross-price sensitivities. Some of their results are obtained by numerical methods. As Ordoover and Shaffer (2006) acknowledge, an advantage of our model is that it has a closed form solution, which enables us to derive our results without resorting to numerical methods. Besides, our model allows us to derive various results in a unified way. The second literature strand, e.g., Krattenmaker and Salop (1986), Salop and Scheffman (1987), Vickers (1995), Sibley and Weisman (1998) or Economides (1998), analyzes the incentives of an upstream monopolist to raise his rivals' costs, inducing them to contract their market share.<sup>2</sup> In our model, setting a high access price is the only way of rising rivals' costs. However, since there are several incumbents, it unclear whether this practice prevents entry. Regarding the third literature strand on access price regulation, Laffont and Tirole (1993) and Masmoudi and Prothais (1994) analyzed Ramsey pricing, and Willig (1979), Baumol (1983), Baumol and Sidak (1994), and Armstrong *et al.* (1996) analyzed the efficient component pricing rule.<sup>3</sup> Armstrong and Vickers (1998) and Lewis and Sappington (1999) studied access pricing when the downstream market is not regulated. In these articles, the bottleneck input is controlled by a regulated monopolist. Our article differs from these by assuming that several incumbents own bottleneck inputs, and compete to provide access to the entrant. The access price is negotiated, rather than regulated.

The remainder of the article is organized as follows. In section 2, we present the model, whose equilibria we characterize in section 3. In sections 4, we present three variations to the model. Section 5 concludes.

## 2 Model

In this section, we develop a simple model that abstracts from many institutional details for three reasons: **(i)** first, to make the strategic interaction transparent, **(ii)** second, to broaden

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<sup>2</sup>For a dissenting view, see Bork (1954).

<sup>3</sup>For conditions on the equivalence of ECPR and Ramsey prices, see Laffont and Tirole (1994). For a survey of the literature on access regulation focusing on telecommunications, see Vogelsang (2003).

the applicability of the results, and (iii) third, for tractability. In section 5, we consider three variations of the model, to check the robustness of our results.

## 2.1 Environment

Consider an industry where firms sell products horizontally differentiated on Salop's circle (Salop, 1979).<sup>4</sup> Initially there are two firms, the *incumbents*. A third firm, the *entrant*, wants to join the industry. Each incumbent owns a bottleneck input. To operate, the entrant has to buy access to the bottleneck input of one of the incumbents.

The game has two stages. In *stage* 1, the entrant negotiates an access price with the incumbents. In *stage* 2, the firms in the market choose retail prices simultaneously.

## 2.2 Consumers

There is a large number of consumers, formally a continuum, whose measure we normalize to 1. Consumers are uniformly distributed along a loop of unit length and have quadratic transportation costs, where  $y^2$  measures the cost of traveling distance  $y$ . Each consumer has a unit demand.<sup>5</sup> All consumers have the same reservation price, which is finite, and such that all consumers purchase the service.<sup>6</sup>

## 2.3 Firms

We index firms with subscript  $i = 1, 2, e$ , where firms 1 and 2 are the incumbents, and firm  $e$  is the entrant; and we index the access provider with superscript  $j = 1, 2$ .

The production of one unit of the final good involves the consumption of one unit of the bottleneck input, whose cost we normalize to 0. Denote by  $\alpha_i$  the access price of the bottleneck

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<sup>4</sup>Salop's circle is a model of localized competition. In other words, a model where each firm competes directly only with its contiguous rivals. We do not think that this is literally the case for any of our applications. The feature we are interested in, and that Salop's circle captures parsimoniously, is that, for a given firm, the products of some of its rivals are closer substitutes than the products of other rivals. Asymmetry among firms is an important feature of the model. It is challenging to introduce asymmetry in alternative models, like the spokes model of Chen and Riordan (2006), or the pyramid model of von Ungern-Sternberg (1991).

<sup>5</sup>This assumption fits well with broadband access to the Internet. Seemingly, this assumption does not fit well with mobile telephony. However, Gryzybowski and Pereira (2007a) estimated a price elasticity of demand for mobile telephone calls of  $-0.38$ , and Gryzybowski and Pereira (2007b) estimated a price elasticity of demand for the duration of mobile telephone calls of  $-0.2$ . Besides, in the EU 80% of mobile telephony subscribers have prepaid cards.

<sup>6</sup>If the market is fully covered, the entrant steals business from the incumbents.

input of firm  $i = 1, 2$ , by  $p_i$ , the retail price of firm  $i = 1, 2, e$ , denote by  $D_i$ , the demand of firm  $i = 1, 2, e$ , and by  $\pi_i^j$ , the profit of firm  $i$  when firm  $j$  is the access provider, with  $i = 1, 2, e$ , and  $j = 1, 2$ . Then,  $\pi_e^j = (p_e - \alpha_j)D_e$  and:

$$\pi_i^j = \begin{cases} p_i D_i & i \neq j \\ \underbrace{p_j D_j}_{\text{retail}} + \underbrace{\alpha_j D_e}_{\text{wholesale}} & i = j. \end{cases}$$

[Figure 1]

The incumbents are located clockwise according to their number, as illustrated in Figure 1. Firm  $e$  is located in point 0.<sup>7</sup> The distance between firms 2 and 1, moving clockwise, is  $\delta$  on  $(0, 1)$ . Parameter  $\delta$  measures the degree of asymmetry between the incumbents and the entrant. This specification encompasses several cases of interest with a single parameter: **(i)** if  $\delta = \frac{1}{2}$  there is pre-entry symmetry; **(ii)** if  $\delta = \frac{2}{3}$  there is post-entry symmetry; and **(iii)** if  $\delta \rightarrow 1$  or  $\delta \rightarrow 0$  there is pre- and post-entry product homogeneity. Brito and Pereira (2007a) discuss the optimal location of the entrant.

The bargaining game unfolds as follows. First, the incumbents simultaneously make access price offers to the entrant. Afterwards, the entrant decides which offer to accept, if any. If the entrant rejects all offers, he stays inactive and receives a payoff of 0; if he accepts one of the offers, he proceeds with the incumbents to stage 2 of the game.<sup>8</sup> A stage 1 *strategy* for firms 1 and 2 is an access pricing rule,  $\alpha_i$  that says which access prices they should offer to the entrant, given  $\delta$ . A stage 1 *strategy* for firm  $e$  is an acceptance rule, that says which offer the entrant should accept, if any, given  $\delta$ .

A stage 2 *strategy* for the firms present in the market is a retail pricing rule,  $p_i$ , that says which retail price the firms should charge, given the access price offers of the incumbents, the acceptance decision of the entrant, and  $\delta$ .

## 2.4 Equilibrium

A *subgame perfect Nash equilibrium in pure strategies* is a profile of access pricing rules, an acceptance rule, and a profile of retail pricing rules for the firms in the market, such that:

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<sup>7</sup>It might be unfeasible for the entrant to locate close to either of the incumbents. The incumbents might have proprietary technologies, or it might be hard for the entrant to mimic the brand images of the incumbents.

<sup>8</sup>In a one stage game where two players bargain over how to divide an object, i.e., in the ultimatum game, the bargaining power lies with who makes the offer. Our case is different because one of the sides of the market has two players. The number of players in each side of the market also affects the parties' bargaining power. Considering games similar to those of Binmore and Herrero (1988), Corominas-Bosch (2003), or Rubinstein and Wolinsky (1990) would lead to qualitatively similar results.

- (E1) each firm chooses a retail pricing rule to maximize profit, given the rivals' pricing rules, the entrant's acceptance decision, the incumbents' access price offers, and  $\delta$ ;
- (E2) the entrant chooses an acceptance rule to maximize profit, given the retail pricing rules, and  $\delta$ ;
- (E3) each incumbent chooses an access pricing rule to maximize profit, given the other incumbent's access price offer, the entrant's acceptance rule, the retail pricing rules, and  $\delta$ .

### 3 Equilibrium Characterization

In this section, we construct the model's equilibria by backward induction. Hence, we characterize first the equilibrium retail prices, and afterwards the equilibrium bargaining.

#### 3.1 Pricing Stage

Next, we characterize the equilibrium retail prices and profits for the cases where: (i) there is no entry, and (ii) entry occurs.

##### 3.1.1 No Entry

We use superscript "n" to denote variables or functions associated with this case.

Denote by  $l_{i,i'}$ , the location of the consumer indifferent between purchasing from firm  $i$  or firm  $i'$ , and denote by  $l_i$ , the location on the loop of firm  $i$ . For the consumer indifferent between firms  $i$  and  $i'$ :

$$p_i + (l_{i,i'} - l_i)^2 = p_{i'} + (l_{i,i'} - l_{i'})^2,$$

or, equivalently,

$$l_{i,i'} = \frac{1}{2} \left[ \frac{p_{i'} - p_i}{l_{i'} - l_i} + (l_i + l_{i'}) \right].$$

We assume that all firms have a positive demand for their products, i.e.,  $l_{i,i+1} < l_{i+1,i+2}$ , for all  $i$ . For firm  $i$ , located between firms  $i'$  and  $i''$ , demand is given by:

$$D_i(p_i, p_{i'}, p_{i''}) = l_{i,i''} - l_{i',i} = \frac{1}{2} \left[ \frac{p_{i''} - p_i}{l_{i''} - l_i} + \frac{p_{i'} - p_i}{l_i - l_{i'}} + (l_{i''} - l_{i'}) \right].$$

With this expression, one can obtain the demand for each firm in the cases where there is no entry and where there is entry.<sup>9</sup> One merely has to replace  $l_i$ ,  $l_{i'}$ , and  $l_{i''}$  by the location of the relevant firms. In the case where there is no entry, the demand of firm 1 is given by:

$$D_1(p_1, p_2) = \frac{1}{2} + \frac{(p_2 - p_1)}{2(1 - \delta)\delta}.$$

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<sup>9</sup>Only the prices of the contiguous rivals enter a firm's demand function.

The first-order condition for the retail price of firm  $i = 1, 2$  is:

$$p_i \frac{\partial D_i}{\partial p_i} + D_i = 0. \quad (1)$$

The next Lemma presents the equilibrium retail prices.

**Lemma 1:** *Without entry, in equilibrium, the incumbents charge retail prices:  $p_i^n(\delta) = \delta(1 - \delta)$ ,  $i = 1, 2$ .* ■

All consumer shares are positive without the need of any restriction on  $\delta$ . As expected, firms 1 and 2 charge the same retail price, and have the same profit.

### 3.1.2 Entry

We consider the access provider is firm  $j$  and use superscript " $j$ " to denote variables or functions associated with this case.

The first-order condition for the incumbent  $i \neq j$  is as (1). The first-order conditions for the entrant and the access provider are, respectively:

$$(p_e - \alpha_j) \frac{\partial D_e}{\partial p_e} + D_e = 0 \quad (2)$$

$$p_j \frac{\partial D_j}{\partial p_j} + D_j + \alpha_j \frac{\partial D_e}{\partial p_j} = 0 \quad (3)$$

Let  $\alpha_j^a(\delta) := \frac{\delta(4-2\delta-\delta^2)}{4}$ . The next Lemma gives the equilibrium retail prices.

**Lemma 2:** *Suppose that there is entry with firm  $j=1,2$  providing access at access price  $\alpha_j$  on  $[0, \alpha_j^a(\delta)]$ . In equilibrium, the firms charge retail prices:*

$$p_i^j(\alpha_j; \delta) = \begin{cases} \frac{[4\alpha_j(4\delta-11)-\delta(\delta-4)^2](\delta-1)}{4(\delta-4)(2\delta-3)} & i = j \\ \frac{(16\delta+20\alpha_j-8\delta^2+\delta^3)(1-\delta)}{4(\delta-4)(2\delta-3)} & i \neq j, e \\ \frac{4\alpha_j(4\delta-5)+\delta(2\delta+\delta^2-4)}{8(2\delta-3)} & i = e. \end{cases}$$

The condition that  $\alpha_j \leq \alpha_j^a(\delta)$  ensures that the consumer shares of all firm are positive.

Incumbents  $j$  and  $i \neq j$  are not symmetric. Inspection of (3) shows that by hiking its retail price, firm  $j$ , the access provider, increases the entrant's sales,  $\frac{\partial D_e}{\partial p_j} > 0$ , and hence, his own



wholesale revenues. This additional incentive leads firm  $j$  to charge a retail price no smaller than firm  $i$ . We call *wholesale effect* to this upward pressure on the retail price of the access provider, caused by the fact that by decreasing his retail price he reduces his wholesale revenues.

[Figure 2]

The access provider has a no lower retail price than the other incumbent. Otherwise, as illustrated in Figure 2, depending on the values of  $\alpha_j$  and  $\delta$ , any ranking of the retail prices is possible. Parameter  $\alpha_j$  affects the marginal cost of the entrant, and parameter  $\delta$  affects the firms' relative demands.<sup>10</sup>

The next Remark collects some auxiliary results that will be useful later.

**Remark 1:** Let  $\alpha_j$  belong to  $[0, \alpha_j^a(\delta)]$ .

(i) The profit function of firm  $i \neq j$ ,  $\pi_i^j(\cdot; \delta)$ , is increasing in the access price,  $\alpha_j$ .

(ii) The profit function of the entrant,  $\pi_e^j(\cdot; \delta)$ , is decreasing in the access price,  $\alpha_j$ , and has a root for  $\alpha_j^a(\delta)$ .

(iii) The profit function of the access provider,  $\pi_j^j(\cdot; \delta)$ , is concave in the access price,  $\alpha_j$ , and has a point of maximum with respect to the access price at  $\alpha_j^*(\delta)$ , with  $\alpha_j^*(\delta) < \alpha_j^a(\delta)$ . ■

### 3.1.3 The Impact of Entry on Retail Prices and Profits

Next, we compare the retail prices and the profits for the cases where: (i) there is no entry, and (ii) entry occurs.

By definition of  $\alpha_j^*(\delta)$ , in Remark 1, if firm  $j$  could impose an access price to the entrant, it would choose  $\alpha_j^*(\delta)$ . For this reason, we call  $\alpha_j^*(\delta)$  the *monopoly access price* of firm  $j = 1, 2$ .<sup>11</sup>

The next Corollary describes the impact of entry on the retail prices.

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<sup>10</sup>To understand the forces at play we discuss two cases. Consider first the case where  $\delta$  belongs to  $[\frac{2}{3}, 1)$ . Equilibrium prices can be ranked as follows:  $p_e^j(\alpha_j; \delta) \geq p_j^j(\alpha_j; \delta) \geq p_i^j(\alpha_j; \delta)$ . The entrant sets the highest price because he has the highest marginal cost and because he faces a larger demand: any of the incumbents has closer rivals. The incumbent that does not provide access sets the lowest price because he has lower costs and because he is not affected by the wholesale effect. Consider now the case where  $\delta$  belongs to  $(0, \frac{2}{3})$ . Firm  $e$  has a smaller demand and larger costs than its rivals. The level of demand and the level of own costs push prices in opposite directions and, thus, any price ranking may occur.

<sup>11</sup>It is easy to check that:  $\alpha_j^*(\delta) = \frac{(\delta-4)^2(11\delta^2-32\delta+22)\delta}{4(118-16\delta^3+91\delta^2-175\delta)}$ . This is concave in  $\delta$ . A small  $\delta$  represents low wholesale demand and hence a low monopoly access price. A large  $\delta$  represents strong retail price competition between the incumbents and, hence, as the access provider's retail price is increasing in  $\alpha$  the access provider will set a lower access price.

**Corollary 1:** *Let  $\alpha_j$  on  $[0, \alpha_j^*(\delta)]$  be given.<sup>12</sup> Suppose that there is entry with access provided by firm  $j$ . The retail price of firm  $i \neq j$  is lower than when there is no entry. The retail price of firm  $j$  may be lower or higher than when there is no entry. The retail price of firm  $e$  may be lower or higher than the retail prices of the incumbents when there is no entry. ■*

[Figure 3]

With entry, the retail price of firm  $i \neq j$  decreases compared with the case where there is no entry because it has a closer rival. As illustrated in Figure 3, the retail price of firm  $j$ , the access provider, may increase or decrease with entry, due to the wholesale effect. The retail price of the entrant may be higher or lower than the retail prices of the incumbents when there is no entry, depending on the values of the access price,  $\alpha_j$ , and the asymmetry parameter,  $\delta$ .

As Figure 3 illustrates, if the access price is high, entry increases the retail prices of the access provider. If the access price or the asymmetry parameter are high, the retail price of the entrant may be larger than the retail prices of the incumbents when there is no entry. In particular, if  $\delta > 0.848$ , the retail price of the entrant may be larger than the retail prices of the incumbents when there is no entry, even when  $\alpha_j = 0$ .<sup>13</sup>

The next Corollary describes the impact of entry on profits.

**Corollary 2:** *Let  $\alpha_j$  on  $[0, \alpha_j^*(\delta)]$  be given. Suppose that there is entry with access provided by firm  $j$ . The profit of both incumbents may be lower or higher with entry than without entry. ■*

[Figure 4]

As Figure 4 illustrates, for the incumbent that does not provide access, the decrease in the retail prices, stated in Corollary 1, may be compensated by an eventual increase in consumer shares that may result from one of its competitors setting higher retail prices, especially if the access price is near the monopoly level. For the access provider, it may or may not be profitable to concede access. The larger the access price and the asymmetry parameter are, the

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<sup>12</sup>Clearly, no equilibrium can occur in which the access provider sets the access price above its monopoly level. Decreasing the access price increases profits without changing the outcome of the bargaining stage.

<sup>13</sup>For such a large  $\delta$ , the competitors of the entrant are located at a large distance, which gives the entrant local market power. Nevertheless, consumers are better off with entry due to the reduction in transportation costs. They could always switch to firms 1 and 2, which for low values of  $\alpha_j$  set lower prices than before entry.

more likely it is profitable for an incumbent to concede access.<sup>14</sup> For  $\delta$  on  $(0.720, 1)$ , it may be profitable to sell access if the access price is high enough.

### 3.2 Bargaining Stage

Next, we characterize the equilibria of the bargaining stage. We call access price offers on  $[\alpha_j^a(\delta), +\infty)$  *unprofitable*, and access price offers on  $[0, \alpha_j^a(\delta))$  *profitable*.<sup>15</sup>

The next Proposition characterizes the equilibrium of the bargaining stage.

**Proposition 1:** *The bargaining stage has three equilibria.*

- (i) *If  $\delta$  belongs to  $(0, 0.720)$ , then the incumbents make unprofitable offers, and the entrant rejects them.*
- (ii) *If  $\delta$  belongs to  $(0.720, 0.748)$ , then one of incumbents offers the monopoly access price,  $\alpha_j^*(\delta)$ , the other makes an unprofitable offer, and the entrant accepts the former offer.*
- (iii) *If  $\delta$  belongs to  $(0.748, 1)$ , then both incumbents offer 0, and the entrant accepts one of the offers at random.* ■

If  $\delta$  is small, i.e., if  $\delta$  belongs to  $(0, 0.720)$ , providing access is not profitable for any of the incumbents, as explained in section 4.1.3. Hence, if one of the incumbents makes an unprofitable offer, the other incumbent should also make one.

If  $\delta$  takes intermediate values, i.e., if  $\delta$  belongs to  $(0.720, 0.748)$ , one of the incumbents offers the monopoly access price,  $\alpha_j^*(\delta)$ . The other incumbent prefers not to undercut this offer, both because the entrant is a distant competitor, and because the entrant has high marginal costs.

If  $\delta$  is large, i.e., if  $\delta$  belongs to  $(0.748, 1)$ , both incumbents offer access at marginal cost. When the entrant sells a product that has no close competitor it has a large demand. The prospect of high wholesale profits leads to strong competition to be the access provider. This competition between the incumbents lowers the access price to marginal cost.

For  $\delta$  on  $(0, 0.748)$ , it is also an equilibrium for both incumbents to offer  $\alpha_i = 0$ , and for the entrant to choose an offer at random. However, we rule out this equilibrium because playing

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<sup>14</sup>A high  $\alpha_j$  represents both a higher wholesale mark-up, and larger equilibrium retail prices. Either of these two effects is likely to compensate any business stealing effect caused by entry. A high  $\delta$  also means that the entrant sells a product that has no close substitute. This implies both that the entrant faces a large demand, and that the post-entry price competition is less intense. Either of these two factors benefits the access provider.

<sup>15</sup>From Remark 1, the profit function of the entrant,  $\pi_e^j(\cdot; \delta)$ , is strictly decreasing in  $\alpha_j$ . Thus, access prices larger than or equal to  $\alpha_j^a(\delta)$  imply a zero consumer share and zero profits for the entrant.

$\alpha_i = 0$  is weakly dominated by playing an unprofitable offer.<sup>16</sup>

We conclude this section by discussing the case of a monopolist incumbent and the prisoners' dilemma, which help to put our results in perspective.

**Monopolist** Suppose that there is only one incumbent, firm 1. The entrant, firm  $e$ , locates on 0, and firm 1 locates on  $l$  on  $(0, 0.5)$ . Denote by  $v$ , the consumers' reservation price, and by  $\alpha$ , the access price. If there is no entry, firm 1 charges  $p_1^m = v - \frac{1}{4}$ , and earns  $\pi_1^m = v - \frac{1}{4}$ . If there is entry, firm 1 and firm  $e$ , respectively, charge  $p_1 = p_e = \alpha + l(1 - l)$ , and earn,  $\pi_1 = \alpha + \frac{l(1-l)}{2}$  and  $\pi_e = \frac{l(1-l)}{2}$ . For  $\alpha > \alpha^p := v - \frac{1}{4} - l(1 - l)$ , the retail price of firm 1 increases with entry. For  $\alpha > \alpha^\pi := v - \frac{1}{4} - \frac{l(1-l)}{2} > \alpha^p$ , the profits of firm 1 increase with entry. Three comments are in order. First, if  $l \approx 0$ , then  $\pi_e \approx 0$ . Thus, if there are fixed costs entry does not occur. Second, even if  $l > 0$  and there are no fixed costs, firm 1 only concedes access voluntarily if  $\alpha > \alpha^\pi > \alpha^p$ . This implies that the retail prices increase with entry. Third, if an open access policy mandates access at the long-run incremental cost, i.e.,  $\alpha = 0$ , firm 1 has incentives to engage in exclusionary practices, like a price squeeze or non-price discrimination. ■

**Prisoners' Dilemma** If  $\delta$  belongs to  $(0.748, 1)$ , the incumbents face a prisoners' dilemma with respect to giving access to their bottleneck inputs. To clarify this point, consider the following simplification of the stage 1 bargaining game. Suppose that each of the incumbents has only two possible strategies: (i) deny selling access, or (ii) sell access at the optimal access price, undercutting the rival if necessary.<sup>17</sup> It turns out that the second strategy is dominant for the incumbent. Suppose that firm 2 denies access. Firm 1 best responds by selling access at the monopoly access price,  $\alpha_1^*(\delta)$ . Suppose now that at firm 2 sells access at any access price no higher than its monopoly price. Firm 1 best responds by undercutting the rival's offer, independently of its value, as shown in the proof of Proposition 1. However, the incumbents have higher profits if there is no entry than if entry occurs. In other words, although selling access to firm  $e$  is a dominant strategy, the incumbents would like to commit not to do so. The occurrence of the prisoners' dilemma does not depend on the access prices falling to zero. As

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<sup>16</sup>If incumbent  $i'$  makes an unprofitable offer, incumbent  $i$  is better off by also making an unprofitable offer instead of  $\alpha_i = 0$ , because  $\pi_i^n > \pi_i^i(0, \delta)$ , for all  $\delta$ . If incumbent  $i'$  makes any positive profitable offer, incumbent  $i$  is better off by also making an unprofitable offer instead of  $\alpha_i = 0$ , because  $\pi_i^{i'}(\alpha_{i'}, \delta) > \pi_i^i(0, \delta)$ , for all  $\delta$  and all  $\alpha_{i'} > 0$ . Finally, if  $\alpha_{i'} = 0$ , incumbent  $i$  is indifferent between making an unprofitable offer or offering  $\alpha_i = 0$ , because  $\pi_i^{i'}(0, \delta) = \pi_i^i(0, \delta)$ .

<sup>17</sup>The first strategy corresponds to making an unprofitable offer. The second strategy is a simplification of our game because it does not specify the access price, and implies that firms do not set access prices above their monopoly level.

illustrated in Figure 4, the profits of the access provider are higher when there is no entry than when entry occurs, if the access price is not too high. ■

## 4 Extensions

In this section, we present three variations of the model of section 3. The first variation introduces a third incumbent. The second variation has the entrant, rather than the incumbents, making the access price offers. The third variation has the incumbents colluding.

### 4.1 The Tetrapoly

Next, we analyze the case where there are three incumbents. The purpose of the variation is twofold. First, to analyze the impact of the number of incumbents on our results. Second, to analyze who has more incentives to give access, a direct or a distant rival of the entrant, and who the entrant prefers to get access from. The results of this section generalize to the case where there are other incumbents, in addition to firms 1 and 2.<sup>18</sup>

Consider the model of section 3, except that there is a third incumbent, firm 3. Firm 3 is located equidistantly between firms 1 and 2, opposite to firm  $e$ , as depicted in Figure 1.

#### 4.1.1 Pricing Stage

Next, we characterize the equilibrium retail prices and profits for the case where: **(i)** there is no entry, **(ii)** there is entry with access provided by the non-contiguous rival, firm 3, and **(iii)** there is entry with access provided by a contiguous rival, either firm 1 or firm 2.

When entry is provided by a non-contiguous competitor, firm 3, we use superscript "3" to denote variables or functions associated with this case. When entry is provided by a contiguous competitor, firm  $k = 1, 2$ , we use superscript " $k$ " to denote variables or functions associated with this case.

When there is no entry, the first-order conditions are like those of section 4.1.1. When there is entry with access provided by a contiguous rival, the first-order conditions are like those of section 4.1.2. When there is entry with access provided by firm 3, the first-order conditions are also like those of section 4.1.2, except for the access provider, whose first-order condition is (1).

Let  $\alpha_3^a(\delta) := \frac{3\delta}{8+4\delta}$ , and  $\alpha_k^a(\delta) := \frac{3\delta}{4+8\delta}$ . The next Lemma gives the equilibrium retail prices.

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<sup>18</sup>As long as the additional incumbents locate equidistantly between firms 1 and 2, moving clockwise.

**Lemma 3:** *In equilibrium:*

(i) *Without entry, the firms charge retail prices:*

$$p_i^n(\delta) = \begin{cases} \frac{(1-\delta)(\delta+3)\delta}{4(2\delta+1)} & i = 1, 2 \\ \frac{(1-\delta)(1+4\delta-\delta^2)}{8(2\delta+1)} & i = 3. \end{cases}$$

(ii) *If there is entry with firm 3 providing access at access price  $\alpha_3$  on  $[0, \alpha_3^a(\delta)]$ , the firms charge retail prices:*

$$p_i^3(\alpha_3; \delta) = \begin{cases} \frac{(1-\delta)(3\delta+4\alpha_3)}{12} & i = 1, 2 \\ \frac{(1-\delta)(3+4\alpha_3)}{24} & i = 3 \\ \frac{4(4-\delta)\alpha_3+3\delta}{24} & i = e. \end{cases}$$

(iii) *If there is entry with firm  $k = 1, 2$  providing access at access price  $\alpha_k$  on  $[0, \alpha_k^a(\delta)]$ , the firms charge retail prices:*

$$p_i^k(\alpha_k; \delta) = \begin{cases} \frac{(1-\delta)(3\delta+11\alpha_k)}{12} & i = k \\ \frac{(1-\delta)(3+8\alpha_k)}{24} & i = 3 \\ \frac{(1-\delta)(3\delta+5\alpha_k)}{12} & i \neq k, 3, e \\ \frac{3\delta+4(5-2\delta)\alpha_k}{24} & i = e. \end{cases}$$

■

Inspection of the first-order conditions shows that the wholesale effect is present only when the access provider is a contiguous rival of the entrant.

When there is no entry, or when there is entry with access provided by firm 3, firms 1 and 2 charge the same retail price. Otherwise, as in section 4.1, any ranking of the retail prices is possible.

With the obvious change of notation, the results of Remark 1 also apply for the three cases of the tetrapoly.

Versions of Corollaries 1 and 2 hold with the following differences.

If there is entry with access provided by firm 3, the retail prices of all incumbents are lower than when there is no entry. For the same access price, the retail prices are higher for all firms when the access provider is firm 1 or firm 2, than when it is firm 3. In both cases the reason is that when firm 3 is the access provider there is no wholesale effect.

If there is entry, the profits of the incumbents that do not provide access are always lower than when there is no entry. The decrease in the retail prices more than compensates any eventual increase in consumer shares that may occur.

If all incumbents charge the same access price, firms 1, 2, and  $e$  always have higher profits when firm 1 is the access provider, than when firm 3 is the access provider; industry profits are also higher. This occurs because retail prices are higher in the former case due to the wholesale effect, and also because, by assumption, the market is always fully covered.

#### 4.1.2 Bargaining Stage

Next, we characterize the equilibria of the bargaining stage. We assume that when indifferent between firms 1, 2, or 3, the entrant chooses one firm randomly. Let  $\beta(\delta) := \frac{2\delta+1}{\delta+2}$ .

When the three incumbents make profitable offers, the entrant should accept the offer of the non-contiguous rival, firm 3, if and only if  $\alpha_3 < \beta(\delta) \min\{\alpha_1, \alpha_2\}$ . As  $\beta(\delta) < 1$ , the entrant prefers, under equal circumstances, to buy access from a contiguous rival, rather than from the non-contiguous rival. Due to the wholesale effect, the contiguous rivals set higher retail prices when they provide access than the non-contiguous rival. An increase in  $\delta$ , increases the set of values of the access prices for which the entrant prefers to buy access from a contiguous rival, than from the non-contiguous rival.

Even if the other incumbents make no profitable offers, a contiguous rival of the entrant may prefer not to offer its monopoly access price. As explained in section 4.1.3, the proximity to the entrant determines whether it is better to provide access at the monopoly price, or to have no entry. If the non-contiguous rival of the entrant made the best profitable alternative offer, in the entrant's perspective, a contiguous rival should always undercut this offer. However, if the other contiguous rival of the entrant made the best profitable alternative offer, in the entrant's perspective, the contiguous rival may prefer not to undercut it, if it is sufficiently high. This happens because, when the access provider is a contiguous rival, retail prices may increase with entry, which may be better for a contiguous rival than providing access.<sup>19</sup>

Even if the other incumbents make no profitable offers, the non-contiguous rival does not necessarily prefer to offer access at the monopoly price. Besides, the non-contiguous rival may prefer not to undercut an offer by another incumbent, if it is sufficiently high.

The next Proposition characterizes the equilibrium of the bargaining stage.

**Proposition 2:** *The bargaining stage has two equilibria.*

(i) *If  $\delta$  belongs to  $(0, 0.426)$ , then the incumbents make unprofitable offers and the entrant rejects them.*

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<sup>19</sup>When the rival's offers are too high, it is even possible to undercut them while setting the monopoly price. For firm  $j$ , for instance, this happens when  $\alpha_3 \geq \beta(\delta)\alpha_j^*(\delta)$ , or  $\alpha_{j'} \geq \alpha_j^*(\delta)$ ,  $j, j' = 1, 2$  and  $j' \neq j$ .

(ii) If  $\delta$  belongs to  $(0.426, 1)$ , then all the incumbents offer zero,  $\alpha_k = 0$ , and the entrant chooses an offer at random. ■

Comparison of Propositions 1 and 2 shows that if there are three incumbents, rather than two, the equilibrium where access is sold at the monopoly price disappears. Otherwise, the equilibrium of the bargaining stage is qualitatively identical in the two cases.<sup>20</sup>

To conclude this section, we discuss the case where one of the incumbents, firm 2, does not negotiate access to its input, perhaps because it is capacity constrained.

**Remark 2:** *If only firms 1 and 3 negotiate, and if  $\delta$  belongs to  $(0.426, 0.474)$ , in addition to the equilibria described in Proposition 2, there is another equilibrium in which: firm 1 offers the monopoly access price,  $\alpha_1^*(\delta)$ , firm 3 makes an unprofitable offer, and the entrant accepts the offer of firm 1.* ■

If firm 2 does not negotiate access to its bottleneck input, and if  $\delta$  takes intermediate values, it is profitable for firm 1 to sell access at the monopoly price because its wholesale revenues are sufficiently large, and its retail price increases. However, the demand of the entrant is relatively small. Thus, it is better for firm 3 to benefit from the increase in the retail price of firm 1, than to undercut firm 1 and become the access provider of a small entrant.<sup>21</sup>

We conclude this section by revisiting the prisoners' dilemma.

**Prisoners' Dilemma** If  $\delta$  belongs to  $(0.526, 1)$ , the incumbents face a prisoners' dilemma with respect to giving access to their bottleneck inputs. As in the case where there are two incumbents, the occurrence of the prisoners' dilemma does not depend on the access prices falling to zero. Interestingly, for  $\delta$  on  $(0.426, 0.526)$ , the unique equilibrium is also for firms to sell access at a zero access price. However, there is no prisoner's dilemma. If firms 1 and 2 deny selling access, firm 3 best responds by also denying to sell access. Hence, providing access is not a dominant strategy for all firms. ■

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<sup>20</sup>As in the case of the triopoly, for all  $\delta$ , there is also weakly dominated equilibrium where all incumbents offer  $\alpha_i = 0$ , and the entrant chooses an offer at random.

<sup>21</sup>If only firms 1 and 2 negotiated, the equilibria would be the same as in Proposition 1.



## 4.2 The Entrant Makes the Access Price Offers

The ability to make take-it-or-leave-it offers gives the incumbents market power. To analyze the impact of the parties' bargaining power on the incumbents' incentives to concede access to their bottleneck inputs, we develop a version of the bargaining stage where the entrant makes the access price offers.<sup>22</sup>

Consider the model of section 3, except that the bargaining stage unfolds as follows. The entrant makes a public offer to each of the incumbents, and the incumbents decide if they accept or reject their offers. If both incumbents accept their offers, the offer yielding the highest profit for the entrant prevails; in case of a draw, the entrant selects an offer at random.

The equilibrium of the retail price competition stage is as described in section 4.1.

Denote by  $\alpha_k^\mu(\delta)$  the access price for which the incumbent  $k$  is indifferent between providing access and there being no-entry, i.e.,  $\pi_k^k(\alpha_k^\mu(\delta), \delta) \equiv \pi_k^n$ , with  $\alpha_k^\mu(\delta) \leq \alpha_k^*(\delta)$ .<sup>23</sup> We say that access price offers on  $[\alpha_k^\mu(\delta), +\infty)$  are *individually rational*, and that access price offers on  $[0, \alpha_k^\mu(\delta))$  are *non-individually rational*.

The next Proposition characterizes the equilibrium of the bargaining stage.

**Proposition 3:** *The bargaining game has two equilibria.*

(i) *If  $\delta$  belongs to  $(0, 0.720)$ , the entrant makes two non-individually rational offers, and the incumbents reject them.*

(ii) *If  $\delta$  belongs to  $(0.720, 1)$ , the entrant offers  $\tilde{\alpha}_j(\delta) + \varepsilon$  and  $\alpha_k^\mu(\delta)$ , both offers are accepted, and the entrant chooses firm  $j$  as its access provider.* ■

Entry occurs for the same values of  $\delta$ , regardless of who makes the access price offers.

It is unclear whether the entrant prefers to make the offers. Consider initially that  $\delta$  belongs to  $(0.720, 0.748)$ . In this case, it is as if only one of the incumbents is allowed to make offers, as the other one prefers not to undercut offers equal to or above the monopoly price. In this bilateral bargaining situation, the entrant clearly prefers to make the offers and pay a lower access price. If  $\delta$  belongs to  $(0.748, 1)$ , the incumbents undercut each other's offers, and end

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<sup>22</sup>One of the parties might have more bargaining power than the other because: it is less impatient, perhaps because it has a lower discount rate, or it is less risk averse, perhaps because the outcome of this venture has a smaller impact on its wealth. See Binmore and Harbord (2005) and Binmore et al. (1986) for a more thorough discussion of these issues.

<sup>23</sup>There are two roots for  $\pi_j^j(\alpha_j^\mu(\delta), \delta) \equiv \pi_j^n$ . Condition  $\alpha_j^\mu(\delta) \leq \alpha_j^*(\delta)$  singles out the lowest of the two. Function  $\alpha_j^\mu(\delta)$  is increasing with  $\delta$  and only exists for  $\delta \geq 0.720$ .

up offering access at marginal cost. This is the best possible outcome for the entrant, and it cannot be replicated when the entrant makes the offers as these offers would lead to double rejection.

### 4.3 The Cartel

Next, we discuss the impact of the incumbents operating as a cartel on the incentives to concede access to their bottleneck inputs.

Consider the model of section 3, except that the incumbents set the access price and the retail prices in order to maximize their joint profits, i.e., the incumbents behave as a cartel. The entrant competes with the cartel on the retail market. Given that incumbents maximize joint profits, it is irrelevant who provides access. Assume that the consumers' valuation,  $v$ , is sufficiently high to ensure that the market is fully covered.

We use superscript "c" to denote variables or functions associated with the case of the cartel without entry, and use superscript "ce" for the case of the cartel with entry. Denote the highest access price for which the market is fully covered when the incumbents behave as a cartel by:

$$\bar{\alpha}^{ce} := \begin{cases} v - \frac{(\delta+2)(13\delta+2)}{144} & \text{if } \delta \geq \frac{8}{11}, \\ v - \frac{2\delta^2-2\delta+3}{12} & \text{if } \delta < \frac{8}{11}. \end{cases}$$

The next Lemma presents the equilibrium retail prices.

**Lemma 4:** *In equilibrium:*

(i) *Without entry, the cartel charges retail prices:*

$$p_i^c(\delta) = \begin{cases} v - \left(\frac{1-\delta}{2}\right)^2 & \text{if } \delta \leq \frac{1}{2} \\ v - \left(\frac{\delta}{2}\right)^2 & \text{if } \delta > \frac{1}{2}. \end{cases}$$

(ii) *Suppose there is entry with the access price  $\alpha$  on  $[0, \bar{\alpha}^{ce}]$ . The firms charge retail prices:*

$$p_i^{ce}(\alpha; \delta) = \begin{cases} \alpha + \frac{\delta(4-\delta)}{12} & i = 1, 2 \\ \alpha + \frac{\delta(\delta+2)}{12} & i = e \end{cases}$$

■

All retail prices increase by the same amount with  $\alpha$ , so the retail price differences across firms, and therefore consumer shares, do not change with  $\alpha$ . As a consequence, cartel profits increase in  $\alpha$  and value  $\bar{\alpha}^{ce}$  is the profit maximizing access price.

As expected, for the same levels of the access price, with entry the retail prices of the cartel are higher than when the incumbents behave independently. Besides, the retail prices of the incumbents increase with entry if  $\delta$  is sufficiently large, i.e., if  $\delta$  belongs to  $(0.5, 1)$ .

The next Remark compares the cartel profits with and without entry.

**Remark 3:** *The cartel profits are higher with entry than without entry, if and only if  $\delta$  belongs to  $[0.664, 1)$ .* ■

From Proposition 1, if the incumbents behave independently, conceding access is the unique equilibrium, only if  $\delta$  belongs to  $[0.720, 1)$ . Comparing this interval with the one presented in Remark 3, one can conclude that a cartel allows entry for a larger set of values of  $\delta$  than an industry where the incumbents do not collude. The intuition is simple. If  $\delta$  is large, some consumers are located at a large distance from firms 1 and 2. This means that the retail prices of the cartel have to be low to induce these consumers to purchase the service. An entrant located between firms 1 and 2 reduces the transportation costs of these consumers substantially, and can therefore charge a higher retail price. Hence, despite the increase in competition, conceding access generates two positive effects for the cartel. First, the cartel can charge higher retail prices to the consumers that it keeps after entry. Second, the cartel obtains a large wholesale profit from the consumers lost to the entrant.

## 5 Conclusion

This article sheds light into the question of whether vertically integrated oligopolists have incentives to concede access to their bottleneck production factors to downstream entrants. We showed that vertically integrated oligopolists have different incentives to provide access than a vertically integrated monopolist. In particular, for some levels of asymmetry, incumbents face a prisoners' dilemma with respect to conceding access. However, the model has other types of equilibria. We also showed that entry does not lead to an unqualified decrease in retail prices. Our main results are robust with respect to: there being three incumbents, the entrant making the access price offers, and the existence of collusion among the incumbents.

We do not address in this article, and leave for further research, the important question of evaluating the impact of the entry on the incentives of the incumbents to invest in their technologies.

## References

- [1] Armstrong, M., 1998, "Network Interconnection in Telecommunications", *The Economic Journal*, **108**, 545-564.
- [2] Armstrong, M., S. Cowan and J. Vickers, 1996, "The Access Pricing Problem: a Synthesis", *Journal of Industrial Economics*, **44**, 131-150.
- [3] Armstrong, M. and J. Vickers, 1998, "The Access Pricing Problem with Deregulation", *Journal of Industrial Economics*, **46**, 115-121.
- [4] Baumol, W., 1983, "Some Subtle Issues in Railroad Deregulation", *International Journal of Transportation Economics*, **10**, 341-55
- [5] Baumol, W. and J. Sidak, 1994, The Pricing of Inputs Sold to Competitors, *Yale Journal on Regulation*, **11(1)**, 170 – 202
- [6] Biglaiser, G. and DeGarba, P., 2001, "Downstream Integration by a Bottleneck Input Supplier whose Regulated Wholesale Prices are above Costs", *Rand Journal of Economics*, **31(2)**, 13750
- [7] Binmore, K. and Herrero, M., 1988, "Matching and Bargaining on Dynamic Markets", *Review of Economic Studies*, **55(1)**, 17-31
- [8] Bork, R., 1954, "Vertical Integration and the Sherman Act: The Legal History of an Economic Misconception", *University of Chicago Law Review*, 22, 157-201
- [9] Brito, D. and Pereira, P., 2007a, "Product Differentiation when Competing with Suppliers of Bottleneck Inputs", *Autoridade da Concorrência*.
- [10] Brito, D. and Pereira, P., 2007b, "Mobile Virtual Network Operators: Beyond the Hyperbolae", *Competition Policy International*, **3(1)**, 271-280.
- [11] Chen, Y. and M. Riordan, 2006, "Price and Variety in the Spokes Model", *The Economic Journal*, forthcoming.
- [12] Commission for Communications Regulation, 2004, "Market Analysis - Wholesale Mobile Access and Call Origination", Document No: 04/118 and 04/118a
- [13] Corominas-Bosch, M., 2003, Bargaining in a network of buyers and sellers, *Journal of Economic Theory*

- [14] Economides, N., 1998, "The Incentive for non-Price Discrimination by an Input Monopolist", *International Journal of Industrial Organization*, **16**, 271-84
- [15] European Commission, 2004, European Electronic Communications Regulation and Markets 2004 (10th report), *European Commission*
- [16] Foros, Ø. and B. Hansen. 2001. "Competition and Compatibility among ISPs", *Information Economics and Policy*, **13(4)**, 411-425.
- [17] Grzybowski, L. and P. Pereira, 2007a, "Complementarity between Calls and Messages in Mobile Telephony", *University of Alicante*
- [18] Grzybowski, L. and P. Pereira, 2007b, "The Consumer Loss of the Minimum Duration for Mobile Telephone Calls", *University of Alicante*
- [19] Hart, O. and J. Tirole, "Vertical Integration and Market Foreclosure", *Brookings Papers on Economic Activity*, Summer 1990, (Special Issue), 205-76.
- [20] Krattenmaker, T. and Salop, S., 1986, "Anticompetitive Exclusion: Raising Rivals' Costs to Achieve Power of Price", *Yale Law Journal*, **96**, 209-93.
- [21] Laffont, J.J., P. Rey and J. Tirole, 1998, "Network Competition: I. Overview and Nondiscriminatory Pricing", *The RAND Journal of Economics*, **29(1)**, 1-37.
- [22] Laffont, J.J. and J. Tirole, 1993, "A Theory of Incentives in Procurement and Regulation", Cambridge, MA: MIT Press.
- [23] Laffont, J.J. and J. Tirole, 1994, "Access Pricing and Competition", *European Economic Review*, 38, 1673-1710.
- [24] Lewis, T. and D. Sappington, 1999, "Access Pricing with Unregulated Downstream Competition", *Information Economics and Policy*, **11**, 73-100.
- [25] Masmoudi, H. and F. Prothais, 1994, "Access Charges: An Example of Application of the Fully Efficient Rule - Mobile Access to the Fixed Network," mimeo, Ecole Polytechnique and France Télécom.
- [26] Office of the Director of Telecommunications Regulation, 2000, "The Regulatory Framework for Access in the Mobile Market; Report on the Consultation", Document ODTR 00/53

- [27] Oftel, 1999, "Oftel Statement on Mobile Virtual Network Operators", *Oftel*
- [28] Ordover, J., Saloner, G. and Salop, S., 1990, "Equilibrium Vertical Foreclosure", *American Economic Review*, **80(1)**, 127-42
- [29] Ordover, J. and G. Shaffer, 2006, "Wholesale Access in Multi-Firm Markets: when is it profitable to supply a competitor?", mimeo, New York University, 1–50.
- [30] Peitz, M., 2005, "Asymmetric access price regulation in telecommunications markets", *European Economic Review*, **49**, 341–358.
- [31] Rey, P. and J. Tirole, 2006, "Primer on Foreclosure," forthcoming, Handbook of Industrial Organization III.
- [32] Rubinstein, A. and Asher Wolinsky, 1990, Decentralized Trading, Strategic Behavior and the Walrasian Outcome, *Review of Economic Studies*, **57**, 63–78
- [33] Salop, S., 1979, Monopolistic competition with outside goods, *Bell Journal of Economics*, **10**, 141–156.
- [34] Salop, S. and Scheffman, D., 1987, "Cost-Raising Strategies"; *Journal of Industrial Economics*, **36(1)**, 19-34
- [35] Sibley, D. and Weisman, D., 1998, "Raising Rival's Costs: The Entry of an Upstream Monopolist into Downstream Markets", *Information Economics and Policy*, **10**, 451-70
- [36] Vogelsang, I., 2003, "Regulation of Access to Telecom Networks", *Journal of Economic Literature*, **41** (3), 830-862.
- [37] Vickers, J., 1995, "Competition and Regulation in Vertically Related Markets", *Review of Economic Studies*, **62(1)**, 1-17
- [38] Von Ungern-Sternberg, T., 1991. "Monopolistic competition in the pyramid". *The Journal of Industrial Economics* **XXXIX** (4), 355–368.
- [39] Willig, R., 1979, "The Theory of Network Access Pricing," in H.M. Trebing (ed.) Issues in Public Policy Regulation, Michigan State University papers.

# Appendix

In the appendix we prove the results in the main text. The proofs of Lemmas 1, 2, and 3, Corollary 1 and 2, and Remark 1 are obvious, and therefore are omitted.

We start by presenting the expressions for equilibrium profits and monopoly access price levels for the tetraopoly case. When there is no entry, incumbents profits are:

$$\begin{aligned}\pi_1^n(\delta) &= \pi_2^n(\delta) = \frac{(1-\delta)(\delta+1)(\delta+3)^2\delta}{32(2\delta+1)^2} \\ \pi_3^n(\delta) &= \frac{(1-\delta)(\delta^2-4\delta-1)^2}{32(2\delta+1)^2}\end{aligned}$$

In the case of entry with access provided by firm 1 (or firm 2) we have, for  $\alpha_1$  in  $[0, \alpha_1^a(\delta)]$ :

$$\begin{aligned}\pi_1^1(\alpha_1; \delta) &= \frac{(1-\delta)(3\delta+11\alpha_1)(3\delta-\alpha_1)}{144\delta} + \frac{3\delta-4\alpha_1(1+2\delta)}{12\delta}\alpha_1 \\ \pi_2^1(\alpha_1; \delta) &= \frac{(1-\delta)(3\delta+5\alpha_1)^2}{144\delta} \\ \pi_3^1(\alpha_1; \delta) &= \frac{(1-\delta)(3+8\alpha_1)^2}{288} \\ \pi_e^1(\alpha_1; \delta) &= \frac{(4\alpha_1-3\delta+8\delta\alpha_1)^2}{288\delta}\end{aligned}$$

and  $\alpha_1^*(\delta) = \frac{3\delta(11-5\delta)}{85\delta+59}$ .

When entry occurs with access provided by firm 3 and  $\alpha_3$  in  $[0, \alpha_3^a(\delta)]$ .

$$\begin{aligned}\pi_1^3(\alpha_3; \delta) &= \pi_2^3(\alpha_3; \delta) = \frac{(1-\delta)(3\delta+4\alpha_3)^2}{144\delta} \\ \pi_3^3(\alpha_3; \delta) &= \frac{(1-\delta)(4\alpha_3+3)^2}{288} + \frac{3\delta-8\alpha_3-4\delta\alpha_3}{12\delta}\alpha_3 \\ \pi_e^3(\alpha_3; \delta) &= \frac{(8\alpha_3-3\delta+4\delta\alpha_3)^2}{288\delta}\end{aligned}$$

and  $\alpha_3^*(\delta) = \frac{3(4-\delta)\delta}{4(5\delta+\delta^2+12)}$ .

**Proposition 1:** From Remark 1, the profit function of the access provider,  $\pi_j^j(\cdot; \delta)$ , is concave with respect to the access price. Denote by  $\bar{\alpha}_j(\delta)$ , the strictly positive level of the access price of firm  $j$  that generates the same profit for the provider as a 0 access price, i.e.,  $\pi_j^j(\bar{\alpha}_j(\delta); \delta) \equiv \pi_j^j(0; \delta)$ . Also, from Remark 1, the profit function of the entrant,  $\pi_e^j(\cdot; \delta)$ , is strictly decreasing in  $\alpha_j$ . Thus, access prices larger than  $\alpha_j^a(\delta)$  imply a null consumer share and no profits for the entrant and will not be accepted. As  $\alpha_j^a(\delta) < \bar{\alpha}_j(\delta)$ , we restrict, without loss of generality, the stage 1 strategy space of firm  $j$  to be  $\alpha_j = [0, \alpha_j^a(\delta)]$ . As incumbents are symmetric, the entrant will accept the lowest offer provided that it is lower than  $\alpha_j^a(\delta)$ .

Let  $\alpha_i \leq \alpha_i^*(\delta)$ . Denote by  $\check{\alpha}_{ij}(\delta)$  the highest offer of firm  $i$  such that firm  $j$  prefers to undercut it, i.e.,  $\pi_j^j(\check{\alpha}_{ij}(\delta) - \varepsilon, \delta) > \pi_j^i(\alpha_{ij}(\delta), \delta)$ . We have that  $\check{\alpha}_{ij}(\delta) := \frac{16\delta - 16\delta^2 + 3\delta^3}{4(7-4\delta)}$  and it can be shown that  $\check{\alpha}_{ij}(\delta) < \alpha_i^*(\delta)$ , if and only if  $\delta < 0.74783$ , and that  $\check{\alpha}_{ij}(\delta) < \alpha_i^a(\delta)$ , for all  $\delta$ .

Let  $\alpha_i > \alpha_i^*(\delta)$ . Denote by  $\alpha_{ij}(\delta)$  the highest offer of firm  $i$  such that firm  $j$  prefers to undercut it with its monopoly access price, i.e.,  $\pi_j^j(\alpha_j^*(\delta), \delta) > \pi_j^i(\alpha_{ij}(\delta), \delta)$ . We have that  $\alpha_{ij}(\delta) := \frac{20\delta(1-\delta)(\delta-2)(175\delta-91\delta^2+16\delta^3-118)(\delta-4)^2-20\sqrt{\delta^2(\delta-1)(\delta-2)(128\delta-53\delta^2+4\delta^3-80)(175\delta-91\delta^2+16\delta^3-118)(2\delta-3)^2(\delta-4)^4}}{400(\delta-1)(\delta-2)(175\delta-91\delta^2+16\delta^3-118)}$ . It can be shown that  $\alpha_{ij}(\delta) < \alpha_i^*(\delta)$  if and only if  $\delta < 0.74783$ .

Let  $\delta < 0.72033$  : Then, if  $\alpha_j \in [\check{\alpha}_{ij}(\delta), \alpha_j^a(\delta)]$  firm  $i$  should set  $\alpha_i = \alpha_i^a(\delta)$ . For  $\alpha_j \in (0, \check{\alpha}_{ij}(\delta)]$  firm  $i$  should set  $\alpha_i = \alpha_j - \varepsilon$ .

Let  $0.72033 < \delta < 0.74783$ : Then, if  $\alpha_j = \alpha_j^a(\delta)$  firm  $i$  should set  $\alpha_i = \alpha_i^*(\delta)$ . If  $\alpha_j \in (\check{\alpha}_{ij}(\delta), \alpha_j^a(\delta))$ , firm  $i$  should set  $\alpha_i = \alpha_i^a(\delta)$ . For  $\alpha_j \in (0, \check{\alpha}_{ij}(\delta)]$  firm  $i$  should set  $\alpha_i = \alpha_j - \varepsilon$ .

Let  $\delta > 0.74783$  : Then, if  $\alpha_j = \alpha_j^a(\delta)$  firm  $i$  should set  $\alpha_i = \alpha_i^*(\delta)$ . If  $\alpha_j \in (\alpha_{ij}(\delta), \alpha_j^a(\delta))$  firm  $i$  should set  $\alpha_i = \alpha_i^a(\delta)$ . If  $\alpha_j \in (\alpha_j^*(\delta), \alpha_{ij}(\delta)]$  firm  $i$  should set  $\alpha_i = \alpha_i^*(\delta)$ . If  $\alpha_j \in [0, \alpha_j^*(\delta)]$ , firm  $i$  should set  $\alpha_i = \alpha_j - \varepsilon$ .

Thus, a given firm's best response functions is

$$\alpha_i(\alpha_j) = \begin{cases} \alpha_i^a(\delta) & \alpha_j \geq \check{\alpha}_{ij}(\delta) \text{ and } \delta < 0.72033 \text{ or} \\ & \alpha_j \in (\check{\alpha}_{ij}(\delta), \alpha_j^a(\delta)) \text{ and } 0.72033 < \delta < 0.74783 \text{ or} \\ & \alpha_j \in (\alpha_{ij}(\delta), \alpha_j^a(\delta)) \text{ and } \delta > 0.74783 \\ \alpha_i^*(\delta) & \text{if } \alpha_j = \alpha_j^a(\delta) \text{ and } 0.72033 < \delta < 0.74783 \text{ or} \\ & \alpha_j \in (\alpha_j^*(\delta), \alpha_{ij}(\delta)] \cup \{\alpha_j^a(\delta)\} \text{ and } \delta > 0.74783 \\ \alpha_j - \varepsilon & \text{else} \end{cases}$$

Given symmetry, it is straightforward to obtain the equilibria. ■

**Proposition 2:** We start by showing that the optimal acceptance rule of the entrant is:<sup>24</sup>

$$AC(\alpha_1, \alpha_2, \alpha_3; \delta) = \begin{cases} \text{none} & \text{if } \alpha_i^a(\delta) \leq \alpha_i, i = 1, 2, 3 \\ \text{one} & \text{if } \alpha_1 < \alpha_1^a(\delta), \beta(\delta)\alpha_1 > \alpha_3, \alpha_1 > \alpha_2 \\ \text{two} & \text{if } \alpha_2 < \alpha_2^a(\delta), \alpha_2 < \frac{1}{\beta(\delta)}\alpha_3, \alpha_2 < \alpha_1 \\ \text{three} & \text{if } \alpha_3 < \alpha_3^a(\delta), \alpha_3 < \beta(\delta)\alpha_1, \alpha_3 < \beta(\delta)\alpha_2. \end{cases}$$

When both firms 1 and 2 make profitable offers, the entrant will prefer the lowest offer, due to the incumbent's symmetric location. When faced with an additional offer by firm 3, the

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<sup>24</sup>In case of a draw, the entrant selects an offer randomly. For simplicity we omit these cases in the entrant's optimal acceptance rule. Additionally, we assume that when indifferent between entering or not, the entrant will stay out of the market.



entrant prefers purchasing access from firm 1 (or firm 2) rather than from firm 3, if and only if,  $\pi_e^1(\alpha_1; \delta) > \pi_e^3(\alpha_3; \delta)$ , or equivalently,  $\alpha_3 > \beta(\delta)\alpha_1$  with  $\beta(\delta) := \frac{2\delta+1}{\delta+2}$ .<sup>25</sup>

We now show that the access price best response correspondence of firm  $j = 1, 2$  is:

(i) If there is no profitable alternative for the entrant:

$$\zeta_j(\alpha_{-j}; \delta) = \begin{cases} \alpha_j^a(\delta) & \text{if } \delta \leq 0.42619 \\ \alpha_j^*(\delta) & \delta > 0.42619. \end{cases}$$

(ii) If the best profitable alternative for the entrant is  $\alpha_3$ , i. e. if  $\alpha_3 < \alpha_3^a(\delta)$  and  $\alpha_3 < \beta(\delta)\alpha_{j'}, j' = 1, 2$  and  $j' \neq j$ :

$$\zeta_j(\alpha_{-j}; \delta) = \begin{cases} \alpha_j^*(\delta) & \text{if } \beta(\delta)\alpha_j^*(\delta) < \alpha_3 \\ \frac{\alpha_3}{\beta(\delta)} - \varepsilon & \text{if } 0 \leq \alpha_3 \leq \beta(\delta)\alpha_j^*(\delta). \end{cases}$$

(iii) If the best profitable alternative for the entrant is  $\alpha_{j'}, j' = 1, 2$  and  $j' \neq j$ , i. e.,  $\alpha_{j'} < \alpha_{j'}^a(\delta)$  and  $\alpha_3 > \beta(\delta)\alpha_{j'}$ :

$$\zeta_j(\alpha_{-j}; \delta) = \begin{cases} \alpha_j^*(\delta) & \text{if } \alpha_j^*(\delta) < \alpha_{j'} \leq \alpha_{j'j}(\delta) \\ \alpha_{j'} - \varepsilon & \text{if } \alpha_{j'} \leq \min \{ \alpha_j^*(\delta), \check{\alpha}_{j'j}(\delta) \} \\ \alpha_j^a(\delta) & \text{else} \end{cases}$$

(iv) If the entrant is indifferent between firm 3 and firm  $j'$ , i.e., if  $\alpha_3 = \beta(\delta)\alpha_{j'}$  and  $\alpha_{j'} < \alpha_{j'}^a(\delta)$ :

$$\zeta_j(\alpha_{-j}; \delta) = \begin{cases} \alpha_j^*(\delta) & \text{if } \alpha_j^*(\delta) < \alpha_{j'} \\ \alpha_{j'} - \varepsilon & \text{if } \alpha_{j'} \leq \min \{ \alpha_j^*(\delta), \check{\alpha}_{j'j}(\delta) \} \\ \alpha_j^a(\delta) & \text{else} \end{cases}$$

where  $\alpha_{j'j}(\delta)$ ,  $\check{\alpha}_{j'j}(\delta)$  and  $\check{\alpha}_{j'j}(\delta)$  are defined below, and  $\alpha_{-j}$  denotes the vector of offers by the incumbents other than  $j$ .

Without loss of generality, we will obtain firm 1's best response correspondence.

If firm 2 and firm 3 have both made unprofitable offers to the entrant, i.e.,  $\alpha_i \geq \alpha_i^a(\delta)$ ,  $i = 2, 3$ , firm 1 has two options: **(i)** set its monopoly access price,  $\alpha_1^*(\delta)$ , or **(ii)** make an unprofitable offer,  $\alpha_1 = \alpha_1^a(\delta)$ , which will result in no entry taking place. The first option is better than the second if and only if:<sup>26</sup>

$$\pi_1^1(\alpha_1^*(\delta); \delta) > \pi_1^n(\delta) \Leftrightarrow \delta > 0.42619$$

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<sup>25</sup>Note that  $\beta(\delta)\alpha_1^a = \alpha_3^a$ . Thus, if  $\alpha_3 > \alpha_3^a(\delta)$  and  $\alpha_1 < \alpha_1^a(\delta)$  we have  $\beta(\delta)\alpha_1 < \alpha_3$ . Also,  $\alpha_1 < \alpha_1^a(\delta)$  and  $\alpha_2 > \alpha_2^a(\delta)$  implies  $\alpha_1 < \alpha_2$ . This means that if only one offer is profitable for the entrant, he will accept it.

<sup>26</sup>We assume that when indifferent between providing access and not, the incumbents prefer not to.

Assume now that at least one of the other incumbents' offers is profitable and such that firm 3's offer is the best one, i.e.,  $\alpha_3 < \min \{\alpha_3^a(\delta), \beta(\delta)\alpha_2\}$ .<sup>27</sup> Firm 1 has two options: **(i)** undercut firm 3, or **(ii)** make an unprofitable offer. If the offer of firm 3 is high, i.e., if  $\alpha_3 > \beta(\delta)\alpha_1^*(\delta)$ , firm 1 can undercut firm 3 with its monopoly access price, whereas if the offer of firm 3 is low, i.e., if  $\alpha_3 \leq \beta(\delta)\alpha_1^*(\delta)$ , firm 1 has to offer an access price lower than its monopoly access price,  $\alpha_1 = \frac{\alpha_3}{\beta(\delta)} - \varepsilon < \alpha_1^*(\delta)$ , to be chosen by the entrant as its provider.

Undercutting with monopoly price, i.e., when  $\alpha_3 > \beta(\delta)\alpha_1^*(\delta)$ , is more profitable than not undercutting if and only if

$$\begin{aligned} \pi_1^1(\alpha_1^*(\delta); \delta) &> \pi_1^3(\alpha_3; \delta) \Leftrightarrow \frac{3(15 - 7\delta - 5\delta^2)\delta}{4(85\delta + 59)} > \frac{(1 - \delta)(3\delta + 4\alpha_3)^2}{144\delta} \Leftrightarrow \\ &\Leftrightarrow \alpha_3 < \alpha_{31}(\delta) := \frac{3(85\delta + 59)\delta(1 - \delta) - 6\delta\sqrt{3(\delta - 1)(85\delta + 59)(7\delta + 5\delta^2 - 15)}}{-4(85\delta + 59)(1 - \delta)} \end{aligned}$$

As  $\alpha_3^a(\delta) < \alpha_{31}(\delta)$  for all  $\delta$ , this is always true for  $\alpha_3 < \alpha_3^a(\delta)$ .

Undercutting below monopoly price is more profitable than not undercutting if and only if

$$\lim_{\varepsilon \rightarrow 0^+} \pi_1^1\left(\frac{\alpha_2}{\beta(\delta)} - \varepsilon; \delta\right) > \pi_1^3(\alpha_3; \delta) \Leftrightarrow \alpha_3 < \check{\alpha}_{31}(\delta) := \frac{6\delta(2\delta + 1)(\delta^2 - \delta + 6)}{208\delta + 133\delta^2 + 7\delta^3 + 84}$$

which is always true for  $\alpha_3 \leq \beta(\delta)\alpha_1^*(\delta)$  because  $\beta(\delta)\alpha_1^*(\delta) < \check{\alpha}_{31}(\delta)$  for all  $\delta$ . Hence, undercutting firm 3 is always the best-response for firm 1.

Assume now that firm 2's offer is the best one, i.e.,  $\alpha_2 < \min \{\alpha_2^a(\delta), \alpha_3/\beta(\delta)\}$ .<sup>28</sup> Firm 1 has two options **(i)** undercut firm 2 with  $\alpha_1 = \alpha_1^*(\delta)$  if  $\alpha_2 > \alpha_1^*(\delta)$ , or with  $\alpha_1 = \alpha_2 - \varepsilon$  if  $\alpha_2 \leq \alpha_1^*(\delta)$ , or **(ii)** make an unprofitable offer. For  $\alpha_2 > \alpha_1^*(\delta)$ , firm 1 prefers to undercut if and only if

$$\begin{aligned} \pi_1^1(\alpha_1^*(\delta); \delta) &> \pi_1^2(\alpha_2; \delta) \Leftrightarrow \\ &\Leftrightarrow \alpha_2 < \alpha_{21}(\delta) := \frac{15\delta(85\delta + 59)(1 - \delta) - 30\delta\sqrt{3(\delta - 1)(85\delta + 59)(7\delta + 5\delta^2 - 15)}}{25(85\delta + 59)(\delta - 1)} \end{aligned}$$

Note that  $\alpha_2^*(\delta) < \alpha_{21}(\delta)$  if and only if  $\delta > 0.25242$ . For  $\alpha_2 \leq \alpha_1^*(\delta)$ , note that  $\lim_{\varepsilon \rightarrow 0^+} \pi_1^1(\alpha_2 - \varepsilon; \delta) > \pi_1^2(\alpha_2; \delta)$ , if and only if,  $\alpha_2 < \check{\alpha}_{21} := \frac{3\delta}{5\delta + 7}$ , with  $\alpha_2^*(\delta) < \check{\alpha}_{21}$  if and only if  $\delta > 0.25242$ . Thus, for  $\delta > 0.25242$  firm 1 will always undercut firm 2's offer.

Finally, assume that the entrant is indifferent between accepting the offer of firm 2 and the offer of firm 3. Again, firm 1 has two options **(i)** undercut with  $\alpha_1 = \alpha_1^*(\delta)$  if  $\alpha_2 > \alpha_1^*(\delta)$ , or with  $\alpha_1 = \alpha_2 - \varepsilon$  if  $\alpha_2 \leq \alpha_1^*(\delta)$ , or **(ii)** make an unprofitable offer. For  $\alpha_2 > \alpha_1^*(\delta)$ , firm 1 prefers to undercut if and only if

<sup>27</sup>This is also the case when firm 3 is the only one making a profitable offer, i.e.,  $\alpha_3 < \alpha_3^a(\delta)$  and  $\alpha_2 \geq \alpha_2^a(\delta)$ .

<sup>28</sup>This is also the case when firm 2 is the only one making a profitable offer, i.e.,  $\alpha_2 < \alpha_2^a(\delta)$  and  $\alpha_3 \geq \alpha_3^a(\delta)$ .

$$\pi_1^1(\alpha_1^*(\delta); \delta) > \pi_1^2(\alpha_2; \delta)/2 + \pi_1^3(\beta(\delta)\alpha_2; \delta)/2$$

which is always true for  $\alpha_2 < \alpha_2^a(\delta)$ . For  $\alpha_2 \leq \alpha_1^*(\delta)$ , note that  $\lim_{\varepsilon \rightarrow 0^+} \pi_1^1(\alpha_2 - \varepsilon; \delta) > \pi_1^2(\alpha_2; \delta)/2 + \pi_1^3(\beta(\delta)\alpha_2; \delta)/2$ , if and only if,  $\alpha_2 < \check{\alpha}_{21}(\delta) := \frac{6(\delta + \delta^2 + 10)(\delta + 2)\delta}{(400\delta + 241\delta^2 + 27\delta^3 + 196)}$  which is larger than  $\alpha_1^*(\delta)$ . Hence, in case of a draw, firm 1 will always undercut its rivals.

We now turn to the access price best response correspondence of firm 3, which is:

(i) If there is no profitable alternative for the entrant:

$$\zeta_3(\alpha_{-3}; \delta) = \begin{cases} \alpha_3^a(\delta) & \text{if } \delta \leq 0.526 \\ \alpha_3^*(\delta) & \text{if } \delta > 0.526. \end{cases}$$

(ii) If the best profitable alternative for the entrant is  $\alpha_i$  or if there are two profitable equal offers:

$$\zeta_3(\alpha_{-3}; \delta) = \begin{cases} \alpha_3^*(\delta) & \text{if } \frac{\alpha_3^*(\delta)}{\beta(\delta)} < \alpha_i < \alpha_{i3}(\delta) \\ \beta(\delta)\alpha_i - \varepsilon & \text{if } 0 \leq \alpha_i \leq \min \left\{ \frac{\alpha_2^*(\delta)}{\beta(\delta)}, \check{\alpha}_{i3}(\delta) \right\} \\ \alpha_3^a(\delta) & \text{else} \end{cases}$$

where  $\check{\alpha}_{i3}(\delta)$  and  $\alpha_{i3}(\delta)$  are defined below.

Start by assuming first that both firm 1 and firm 2 have made an unprofitable offers to the entrant,  $\alpha_i \geq \alpha_i^a(\delta)$ ,  $i = 1, 2$ . Firm 3 has two options: **(i)** set its monopoly access price,  $\alpha_3^*(\delta)$ , or **(ii)** make an unprofitable offer,  $\alpha_3 > \alpha_3^a(\delta)$ . The first option is more profitable if and only if

$$\pi_3^3(\alpha_3^*(\delta); \delta) > \pi_3^n(\delta) \Leftrightarrow \delta > 0.526$$

Consider now the case in which firm  $j = 1, 2$  has made the best profitable offer to the entrant, i.e.,  $\alpha_j < \alpha_j^a(\delta)$  and  $\alpha_j \leq \alpha_{j'}$ . Again, firm 3 has two options: **(i)** undercut firm  $j$ , or **(ii)** make an unprofitable offer.

If the offer of firm  $j$  is very high, i.e., if  $\alpha_j > \frac{\alpha_3^*}{\beta(\delta)}$ , firm 3 may undercut with its monopoly price. This is more profitable than not undercutting if

$$\pi_3^3(\alpha_3^*(\delta); \delta) > \pi_3^j(\alpha_j; \delta) \Leftrightarrow \alpha_j < \alpha_{j3}(\delta) := \frac{3}{8} \left( \sqrt{1 - \frac{\delta(\delta - 4)^2}{(5\delta + \delta^2 + 12)(\delta - 1)}} - 1 \right)$$

Hence, if  $\frac{\alpha_3^*}{\beta(\delta)} < \alpha_j < \alpha_{j3}(\delta)$  firm 3 should set its monopoly price. Otherwise it should make an unprofitable offer.

If the offer of firm  $j$  is lower, i.e., if  $\alpha_j \leq \frac{\alpha_3^*}{\beta(\delta)}$ , firm 3 may undercut it by setting  $\alpha_3 = \beta(\delta)\alpha_j - \varepsilon$ . This is more profitable than not undercutting if

$$\lim_{\varepsilon \rightarrow 0^+} \pi_3^3(\beta(\delta)\alpha_j - \varepsilon; \delta) > \pi_3^j(\alpha_j; \delta) \Leftrightarrow \alpha_j < \check{\alpha}_{j3}(\delta) := \frac{9\delta^2(\delta + 2)}{2(\delta + 1)(19\delta + 4\delta^2 + 4)}$$

Finally, we will show that any other possibility than those presented in Proposition 2 does not constitute an equilibrium of this game. Start by noting that the acceptance of an offer larger than the monopoly access price,  $\alpha_j^*(\delta)$ , cannot occur in equilibrium as the access provider would benefit from reducing its offer to  $\alpha_j^*(\delta)$ .

(i) A positive profitable offer by firm 3 is accepted by the entrant. This cannot occur in equilibrium because any such offer should be undercut either by firm 1 or by firm 2, for any  $\delta$ .

(ii) A positive profitable offer by firm 2 is accepted by the entrant and  $\delta > 0.25242$ . This cannot occur in equilibrium because with  $\delta > 0.25242$  we have  $\alpha_{21}(\delta) > \alpha_2^*(\delta)$  and  $\check{\alpha}_{21}(\delta) > \alpha_2^*(\delta)$ . This means that any offer equal to or below  $\alpha_2^*(\delta)$  will be undercut by firm 1.

(iii) A positive profitable offer by firm 2 is accepted by the entrant and  $\delta \leq 0.25242$ . If  $\alpha_2 < \check{\alpha}_{21}(\delta)$  firm 1 will undercut. If  $\alpha_2 \geq \check{\alpha}_{21}(\delta)$  firm 1 and firm 3 will best respond by making unprofitable offers. But in such case, firm 2 best responds by also making an unprofitable offer.

(iv) A positive profitable offer by firm 1 is accepted by the entrant. Equal to (ii) and (iii) above

(v) A pair of positive profitable offers by firm 2 and firm 3 is accepted by the entrant. This cannot occur in equilibrium because any such offers should be undercut by firm 1.

(vi) The equilibrium cannot have the three firms making unprofitable offers for  $\delta > 0.426$ . Either firm 1 or firm 2 would gain by setting the monopoly access price. ■

**Remark 2:** Start by noting that  $\alpha_1^*(\delta) > \frac{\alpha_3^*}{\beta(\delta)}$ , that  $\alpha_{13}(\delta) > \alpha_1^*(\delta)$  for  $\delta > 0.474$ . This means that firm 3 may undercut firm 1's monopoly price with its monopoly price, but is willing to do so, if and only if  $\delta > 0.474$ .

We will now check that the following alternatives do not occur in equilibrium:

A positive profitable offer by firm 3 accepted by the entrant: any such offer should be undercut by firm 1, for any  $\delta$ .

A positive profitable offer by firm 1 with  $\alpha_1 > \alpha_1^*(\delta)$  accepted by the entrant: decreasing such offer would increase firm 1's profits and still be profitable.

A positive profitable offer by firm 1 with  $\alpha_1 = \alpha_1^*(\delta)$  accepted by the entrant for  $\delta > .474$ : firm 3 would undercut it.

A positive profitable offer by firm 1 with  $\alpha_1 < \alpha_1^*(\delta)$ . Two things might happen: either firm 3 is willing to undercut it or not. In the first case it is trivially not an equilibrium. In the second case, firm 1 would benefit from increasing its access price until the monopoly level  $\alpha_1^*(\delta)$ . If firm 3 was not willing to undercut the initial lower level, it would also choose not to undercut the higher price. ■

**Proposition 3:** The strategy space for incumbent  $i = 1, 2$  is  $\{accept, reject\}$  where *accept* means "accept offer  $\alpha_i$ " and *reject* stands for "reject offer  $\alpha_i$ ". Let us assume, without loss of generality, that the offers made are such that  $\pi_e^1(\alpha_1; \delta) \geq \pi_e^2(\alpha_2; \delta)$  or, in other words,  $\alpha_1 \leq \alpha_2$ . Then, the game incumbents play is represented in its normal form by:

		Firm 2:	
		<i>accept</i>	<i>reject</i>
Firm 1:	<i>accept</i>	$\pi_1^1(\alpha_1; \delta), \pi_2^1(\alpha_1; \delta)$	$\pi_1^1(\alpha_1; \delta), \pi_2^1(\alpha_1; \delta)$
	<i>reject</i>	$\pi_1^2(\alpha_2; \delta), \pi_2^2(\alpha_2; \delta)$	$\pi_1^n(\delta), \pi_2^n(\delta)$

Start by noting that:

- (i)  $(accept, accept)$  is an equilibrium if and only if  $\pi_1^1(\alpha_1; \delta) \geq \pi_1^2(\alpha_2; \delta)$ .
  - (ii)  $(accept, reject)$  is an equilibrium if and only if  $\pi_1^1(\alpha_1; \delta) \geq \pi_1^n(\delta)$ .
  - (iii)  $(reject, accept)$  is an equilibrium if and only if  $\pi_1^1(\alpha_1; \delta) \leq \pi_1^2(\alpha_2; \delta)$  and  $\pi_2^2(\alpha_2; \delta) \geq \pi_2^n(\delta)$ .
  - (iv)  $(reject, reject)$  is an equilibrium if and only if  $\pi_1^1(\alpha_1; \delta) \leq \pi_1^n(\delta)$  and  $\pi_2^2(\alpha_2; \delta) \leq \pi_2^n(\delta)$ .
- Let  $\pi_j^j(\alpha_j^\mu(\delta); \delta) \equiv \pi_j^n(\delta)$ . Then,

$$\alpha_j^\mu(\delta) : = \frac{\delta (32\delta - 11\delta^2 - 22) (\delta - 4)^2}{4 (175\delta - 91\delta^2 + 16\delta^3 - 118)} + \frac{\sqrt{\delta^2 (2000\delta + 1096\delta^3 - 2304\delta^2 - 171\delta^4 - 4\delta^5 - 608) (2\delta - 3)^2 (\delta - 4)^2}}{4 (175\delta - 91\delta^2 + 16\delta^3 - 118)}$$

Recall that we say that access price offers on  $[\alpha_j^\mu(\delta), +\infty)$  are *individually rational (IR)*, and that access price offers on  $[0, \alpha_j^\mu(\delta))$  are *non-individually rational*. There are three cases of interest:

a) No *IR* offer is made. In this case,  $\pi_1^1(\alpha_1; \delta) < \pi_1^n$  and  $\pi_2^2(\alpha_2; \delta) < \pi_2^n$ . There are two candidates for equilibrium:  $(reject, reject)$  and  $(accept, accept)$ . The second one is based on firm 2 playing a weakly dominated strategy and is therefore excluded. If there is no individually rational offer, the entrant stays out of the market. Note that there are no *IR* offers if  $\delta < 0.720$ .

b) Two *IR* offers are made. Then,  $\alpha_2 \geq \alpha_1 \geq \alpha_1^\mu(\delta)$ . The candidates for equilibrium are:

- (i)  $(accept, accept)$  is an equilibrium if and only if  $\pi_1^1(\alpha_1; \delta) \geq \pi_1^2(\alpha_2; \delta)$ .
- (ii)  $(accept, reject)$  is an equilibrium.
- (iii)  $(reject, accept)$  is an equilibrium if and only if  $\pi_1^1(\alpha_1; \delta) \leq \pi_1^2(\alpha_2; \delta)$ .

If (i) or (ii) are the outcome of the game, the entrant's payoff is not a function of  $\alpha_2$ . If (iii) is the outcome of the game, decreasing  $\alpha_2$  until it equals  $\alpha_1$  increases the entrant's profits. Additionally, making a lower offer to firm 2 increases the range of offers to firm 1

such that  $(accept, accept)$  is the outcome instead of  $(reject, accept)$  and the entrant clearly prefers the former. Hence, when making two  $IR$  offers, these will be equal and (at least) one of them will be accepted. Hence, the entrant should offer the lowest  $IR$  offers possible, i.e.,  $\alpha_2 = \alpha_1 = \alpha_1^\mu(\delta) = \alpha_2^\mu(\delta)$ .

c) Only one  $IR$  offer is made. The  $IR$  offer must be the highest offer, i.e., the offer made to firm 2, and it must be that  $\alpha_2 \geq \alpha_2^\mu(\delta) = \alpha_1^\mu(\delta) > \alpha_1$ . There are two possible candidates for equilibrium:  $(accept, accept)$  and  $(reject, accept)$  depending on whether  $\pi_1^1(\alpha_1; \delta) \geq \pi_1^2(\alpha_2; \delta)$  or  $\pi_1^1(\alpha_1; \delta) \leq \pi_1^2(\alpha_2; \delta)$ . In both cases, by lowering the offer to firm 2, the entrant does not become strictly worse-off:

If  $\alpha_1$  is accepted, i.e, if  $\pi_1^1(\alpha_1; \delta) \geq \pi_1^2(\alpha_2; \delta)$ , the entrant gets the same profit.

If  $\alpha_2$  is accepted, i.e., if  $\pi_1^1(\alpha_1; \delta) \leq \pi_1^2(\alpha_2; \delta)$ , the entrant becomes better off than when making a higher offer.

Note again that decreasing  $\alpha_2$  increases the set of values for  $\alpha_1$  such that  $\pi_1^1(\alpha_1; \delta) \geq \pi_1^2(\alpha_2; \delta)$ , meaning that lower  $\alpha_1$  are accepted. Hence,  $\alpha_2 = \alpha_2^\mu(\delta)$ .

Let  $\tilde{\alpha}_1(\delta)$  in  $(0, \alpha_1^\mu(\delta))$  be such that  $\pi_1^1(\tilde{\alpha}_1(\delta); \delta) = \pi_1^2(\alpha_2^\mu(\delta); \delta)$ . It can be shown that  $\pi_1^1(\alpha_1^\mu(\delta); \delta) = \pi_1^n(\delta) > \pi_1^2(\alpha_2^\mu(\delta); \delta) > \pi_1^1(0; \delta)$  for all  $\delta$ . Hence, continuity of  $\pi_1^1(\alpha_1; \delta)$  and the fact that  $\pi_1^1(\alpha_1; \delta)$  is increasing in  $\alpha_1$  for any  $\alpha_1 \leq \alpha_1^\mu(\delta)$  ensures that there is an  $\tilde{\alpha}_1(\delta)$  in  $(0, \alpha_1^\mu(\delta))$  such that  $\pi_1^1(\tilde{\alpha}_1(\delta); \delta) = \pi_1^2(\alpha_2^\mu(\delta); \delta)$ . Regarding the entrant's choice of  $\alpha_1$ , there are two possibilities: (i)  $\alpha_1 \in [0, \tilde{\alpha}_1(\delta)]$  and (ii)  $\alpha_1 \in (\tilde{\alpha}_1(\delta), \alpha_1^\mu(\delta))$

With  $\alpha_1 \in [0, \tilde{\alpha}_1(\delta)]$  and  $\alpha_2 = \alpha_2^\mu(\delta)$  the equilibrium is  $(reject, accept)$  and the entrant gets a payoff of  $\pi_e^2(\alpha_2^\mu(\delta); \delta)$ . With  $\alpha_1 \in (\tilde{\alpha}_1(\delta), \alpha_1^\mu(\delta))$  and  $\alpha_2 = \alpha_2^\mu(\delta)$ , the equilibrium is  $(accept, accept)$  and the entrant gets a payoff of  $\pi_e^2(\alpha_1, \delta)$ . As  $\alpha_1 < \alpha_2^\mu(\delta)$  the second case yields a higher payoff for the entrant.

From alternatives a), b) and c), the one that maximizes the entrant's payoff is c): setting  $\alpha_2 = \alpha_2^\mu(\delta)$  and  $\alpha_1 = \tilde{\alpha}_1(\delta) + \varepsilon$  with  $\varepsilon \rightarrow 0^+$ . ■

**Lemma 4: (i)** The highest price that firms 1 and 2 can set that will ensure that all consumers located between them still purchase the service is  $p_1 = p_2 = V - (\frac{\delta}{2})^2$  if  $\delta \geq \frac{1}{2}$  or  $p_1 = p_2 = V - (\frac{1-\delta}{2})^2$  if  $\delta < \frac{1}{2}$ . The cartel sells to all consumers and has a profit given by  $\pi_{1+2}^c = V - (\frac{\delta}{2})^2$  if  $\delta \geq \frac{1}{2}$  or  $\pi_{1+2}^c = V - (\frac{1-\delta}{2})^2$  if  $\delta < \frac{1}{2}$ .

**(ii)** The derivation of the equilibrium prices is straightforward. Next, we show that the expression of  $\bar{\alpha}^{ce}$  conforms with the definition. Consumers with the lowest surplus are either the one indifferent between firms 1 and  $e$  or the one indifferent between firms 1 and 2. The

total surplus, i.e., valuation minus price minus transportation cost, is, respectively,

$$V - \left[ \alpha + \frac{1}{12} \delta (4 - \delta) \right] - \left[ \frac{1}{12} (5\delta - 2) \right]^2$$

$$V - \left[ \alpha + \frac{1}{12} \delta (4 - \delta) \right] - \left[ \frac{1}{2} (1 - \delta) \right]^2$$

Both must be positive so that the market is covered. Hence, as the cartel's profit is increasing in the access price, this will be set at:

$$\bar{\alpha}^{ce} = \min \left\{ V - \frac{1}{144} (\delta + 2) (13\delta + 2), V - \frac{1}{12} (2\delta^2 - 2\delta + 3) \right\}$$

That is,

$$\bar{\alpha}^{ce} = \begin{cases} V - \frac{1}{144} (\delta + 2) (13\delta + 2) & \delta \geq 8/11 \\ V - \frac{1}{12} (2\delta^2 - 2\delta + 3) & \delta < 8/11 \end{cases}$$

■

**Remark 3:** From Lemma 4, without entry, cartel profits are:

$$\pi_{1+2}^c = \begin{cases} V - (\frac{1-\delta}{2})^2 & \text{if } \delta < 1/2 \\ V - (\frac{\delta}{2})^2 & \text{if } \delta \geq 1/2 \end{cases}$$

and with entry, cartel profits are:

$$\pi_{1+2}^{ce} = \begin{cases} V + \frac{1}{72} (28\delta - 20\delta^2 + \delta^3 - 18) & \text{if } \delta < 8/11 \\ V + \frac{1}{144} (4\delta - 29\delta^2 + 2\delta^3 - 4) & \text{if } \delta \geq 8/11 \end{cases}$$

If  $\delta < 1/2$ ,  $\pi_{1+2}^c > \pi_{1+2}^{ce}$ . If  $1/2 < \delta < 8/11$  we have that  $\pi_{1+2}^c < \pi_{1+2}^{ce}$  if and only if  $\delta > 0.66389$ . Finally, if  $\delta > 8/11$  we have  $\pi_{1+2}^{ce} > \pi_{1+2}^c$ . ■

# Figures

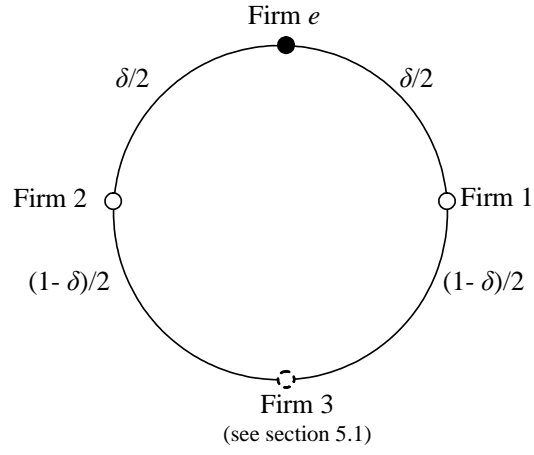


Figure 1: Firms' locations on the loop.

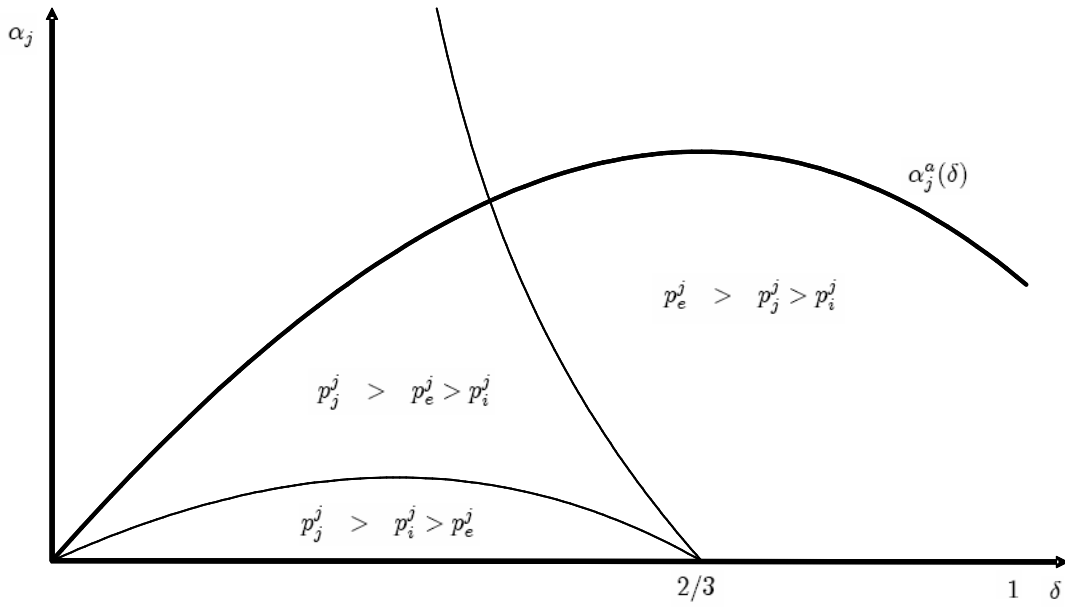


Figure 2: Price rankings when firm  $j$  is the access provider.



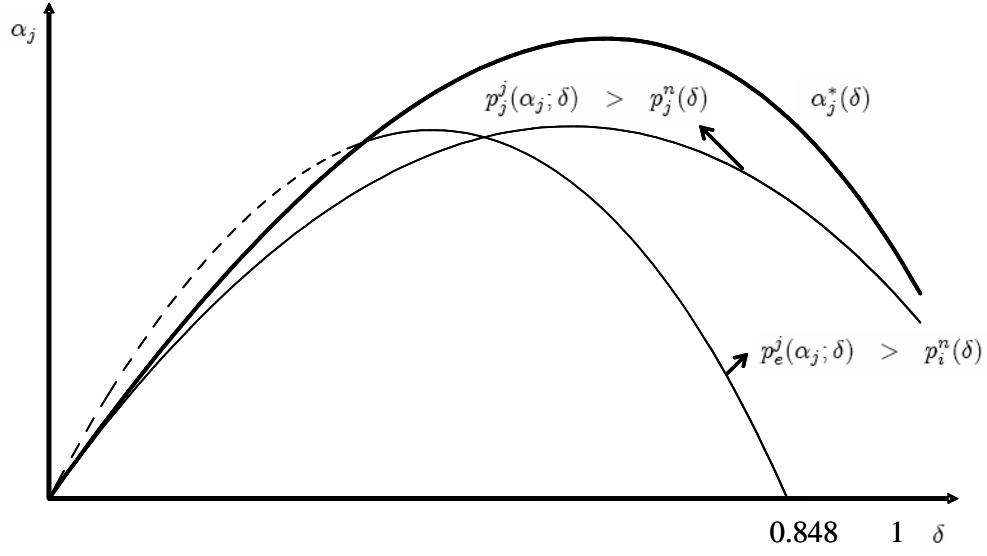


Figure 3: Change in the retail prices of the access provider and of the entrant.

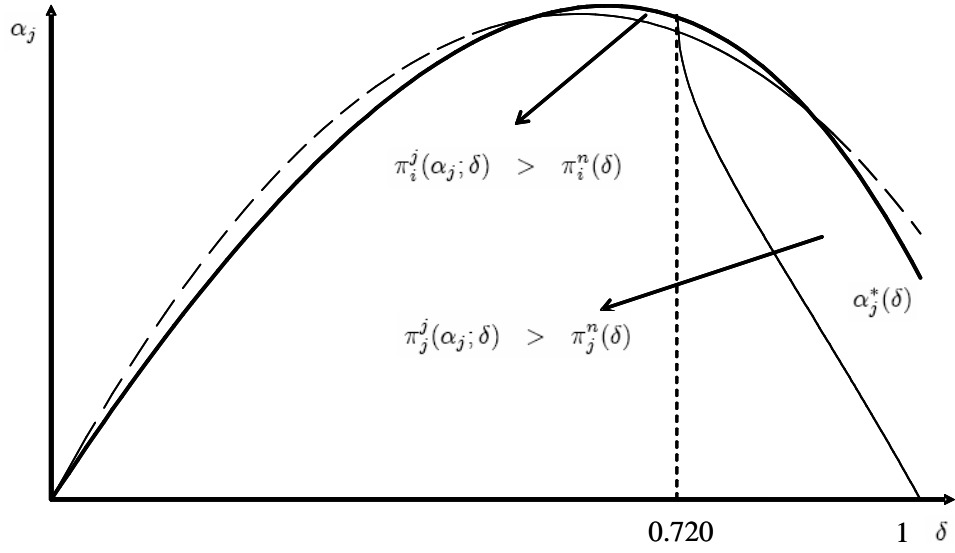


Figure 4: Changes in the incumbents' profits when firm  $j$  is the access provider.