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The Impact of Cost-Reducing R&D Spillovers on the Ergodic Distribution of Market Structures

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Abstract

We extend the literature on knowledge spillovers between firms by studying a dynamic duopoly model of R&D. Our analysis highlights the previously ignored welfare effects of spillovers through dynamic changes in industry concentration. In addition, we find that the impact of imperfect appropriability of R&D on concentration and welfare depends crucially on the manner in which spillovers are obtained. To date, the analysis of the impact of knowledge spillovers between firms has been largely restricted to static two-stage models (R&D decisions followed by product market decisions). These models generally predict suboptimal R&D expenditures and lower welfare. Such models are silent on the evolution of the market structure, and the resulting welfare implications, because they need to assume initial conditions (symmetry or asymmetry). We find that when spillovers require absorptive capacity investment in own R&D, larger spillovers lead to declines in concentration while rates of innovation increase and welfare rises. In contrast, when spillovers are costlessly obtained increases in the extent of spillovers rates of innovation fall leading to losses in welfare through both reduced consumer surplus and firm values, while the effect on concentration is ambiguous.

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1 Introduction

Imperfect appropriability of the returns to R&D generates a negative incentive for firms to innovate creating underinvestment in R&D relative to the optimum. At the same time, imperfect appropriability may mitigate socially wasteful R&D to the extent that R&D is duplicative across firms. The initial literature on the problems of imperfect appropriability and free riding in R&D analyzes the welfare implications in the presence of the trade-off between private incentives to innovate and efficient use of knowledge. Michael Spence (1984) develops a deterministic model where symmetric firms invest in cost-reducing R&D with spillovers to rivals. He finds that the absence of appropriability leads unambiguously to underinvestment in R&D. Policies that increase appropriability still leave the market inefficient because the R&D costs required to achieve a given rate of industry cost reduction are higher with full appropriability. He concludes that the best market performance is achieved in an industry with low appropriability coupled with subsidies to restore incentives to invest and improve allocative efficiency.

However, Levin and Reiss (1988) and Cohen and Levinthal (1989) show that treating external knowledge as a pure public good, acquired at zero cost by all firms in the industry, is in the genesis of the results pointing to underinvestment in the presence of R&D spillovers. They argue that spillovers are not acquired costlessly, but firms need to establish absorptive capacity to take advantage of spillovers. This point is in line with Rosenberg (1974) and Nelsons' (1982) claim that, in order to be able to take advantage of externalities, the firm has to undertake some R&D investment of its own. For example, a firm may need laboratories with research personnel that can interpret and make use of the industry pool of knowledge.

Later work by D'Aspremont and Jacquemin (1988) with further refinements by Henriques (1990), Suzumura (1992), Simpson and Vonortas (1994), and Ziss (1994) analyzes spillovers with regards to research joint ventures (which we do not consider here) and find that when joint ventures can be formed the welfare results are ambiguous.[1] These models, as those above, are typically two-stage oligopolies wherein firms first choose R&D, acquire spillovers costlessly from rivals, then compete in the product market. All of these analyses necessarily start from the assumption of a *given market structure* (usually symmetric for tractability reasons) and then proceed to evaluate the R&D incentives and welfare results. By market structure we mean the degree of asymmetry

[1] Song (2005), in an applied dynamic analysis of research joint ventures, reaches similar conclusions in his study of SEMATECH.

(or lack thereof in the symmetric case) in market shares between firms. But as Dasgupta and Stiglitz (1981) and Loury (1979) make clear, the market structure itself is endogenous to the incentives to conduct R&D with or without spillovers. Empirically, Mansfield (1984) argues that "...an industry's concentration level tends to be low if its members' products and processes can be imitated easily and cheaply... Apparently, differences among industries in the technology transfer process may be able to explain much more of the interindustry variation in concentration levels than is generally assumed." Levin and Reiss (1984) attempt to address concentration and spillovers in a two-stage, symmetric model. However, they empirically reject their own specification, which implies a positive proportional relationship between R&D and concentration, concluding that, "...it is likely that the concentration equation is particularly sensitive to our neglect of dynamics."

Thus, characterizations of the appropriability issue based on the analysis of two-stage, perfect foresight models cannot account for the effect of spillovers on concentration levels. Ultimately, it calls into question the welfare results which often hinge on the degree of appropriability and the initial conditions. That leads to the following questions that we address: 1) How do knowledge spillovers affect the incentives to conduct R&D and rates of innovation in a dynamic context? 2) In the presence of R&D with spillovers what market structures are most likely to emerge? and 3) What are the welfare consequences with spillovers when industry concentration is endogenous?

We present a dynamic duopoly model of R&D to analyze the implications of imperfect appropriability on welfare and market structure. The analysis builds on the Markov-Perfect dynamic industry model proposed by Ericson and Pakes (1995) (hereafter, "E-P"), through the introduction of a non-proprietary productivity component to R&D as part of a dynamic, stochastic process. The model features firm level heterogeneity with strategic interaction under uncertainty over an infinite horizon. This framework *jointly determines* both market structure and R&D as endogenous variables allowing us to identify changes in the likelihood of more or less concentration in the presence of R&D spillovers. Thus we are able to characterize how spillovers induce changes in the equilibrium ergodic distribution of the market structures. These changes have important welfare effects that cannot be addressed in a two-stage static model.

Our analysis also compares and contrasts the two different appropriability scenarios: 1) firms obtain R&D spillovers *costlessly*; and 2) R&D spillovers require investment in an absorptive capacity. In Spence's (1984) costless characterization, R&D falls with spillovers, but in Levin and Reiss (1988) and Cohen and Levinthal (1989) R&D can increase with spillovers due to need for absorptive capacity. The model here extends those frameworks to a dynamic stochastic framework, but also goes further by addressing the impact of spillovers on the equilibrium level of concentration providing new insights on the impact of knowledge spillovers. We find that when spillovers require absorptive capacity as a by-product of R&D investment, concentration levels decline with increases in the extent of spillovers and welfare rises through increased consumer surplus. However, when firms obtain spillovers costlessly, increases in the extent of spillovers have ambiguous effects on concentration levels while consumer surplus declines because the rate of innovation falls leading to higher prices.

The remainder of the paper proceeds as follows: Section 2 begins by laying out the two cases for how firms acquire spillovers from R&D and then specifies a fully dynamic model. Section 3 presents comparative statics on the policy functions in the costless case, while Section 4 examines the absorptive capacity case. The paper then proceeds to illustrate the effects highlighted through numerical simulation in Section 5. Section 6 concludes.

2 The Model

We model an industry where firms produce homogeneous goods and engage in Cournot competition in the spot market. We restrict our attention to a duopoly, as is commonly done in this literature, for ease of comparison. Firms can engage in R&D to lower their marginal costs in the future and thus (potentially) increase their market share *vis-a-vis* their rivals. The Cournot-Nash outcome in the spot market yields higher profits for the firm with the relatively lower marginal costs and hence drives the incentives for investing in R&D and taking advantage of R&D spillovers. The equilibrium of this class of models is Markov-Perfect in the sense of Maskin and Tirole (1988), and a rational expectations equilibrium, (See E-P, 1995).

2.1 R&D and Appropriability

Following Levin and Reiss (1988) and Cohen and Levinthal (1989), we express the total amount of innovative R&D that firm i can utilize to pursue its innovative purposes, m_i, as:

$$m_i = x_i + \gamma(x_i)bx_j \tag{1}$$

where x_i represents firm i's own R&D investment while the rival's R&D is denoted x_j. The function $\gamma(x_i)$ determines the absorptive capacity of the firm and is assumed strictly concave in x_i and satisfies $0 \leq \gamma(x_i) \leq 1 \ \forall x_i$ and $\gamma(0) = 0$. The parameter b lies in the range of $0 \leq b \leq 1$ and captures the extent of the intra-industry spillovers, i.e. the fraction of rivals' R&D that is useful and available to firm i. The extent depends on the ease of imitation, patent policies, worker mobility, the amount of knowledge embodied in the output of the innovation process, the degree of tacit knowledge required, etc. Larger b implies that a higher proportion of firms' R&D investment enters the industry pool of knowledge. If b is set to zero, then R&D is perfectly appropriable. If b equals unity, then all R&D in the industry is publicly accessible, but not necessarily used because absorptive capacity itself may be costly.[2]

We consider two cases of limited appropriability as shown in (2).[3]

$$\gamma(x_i) = \begin{cases} 1 & \text{Costless Case} \\ \frac{\gamma x_i}{1+\gamma x_i} & \text{Absorptive Capacity Case} \end{cases} . \tag{2}$$

First, following the early literature on appropriability of R&D, we consider the case that R&D spillovers are costless to obtain.[4] The absorptive capacity function becomes $\gamma(x_i) = 1, \ \forall \ x_i$ (including $x_i = 0$) and the amount of usable R&D becomes:

$$m_i = x_i + bx_j. \tag{3}$$

This specification implies that a firm need not do any research of its own in order to increase efficiency. It also entails a strong negative incentive for own R&D because innovative activity creates external benefits for rivals. That negative incentive increases with the extent of spillovers (or lack of appropriability), i.e. as b increases.

In the second case, the absorptive capacity of firm i is a monotonically increasing concave

[2] We model spillovers here based on the *flow* of R&D in the manner of the majority of the previously discussed literature such that our results can be contrasted. In our concluding section, we briefly discuss the potential effects and differences if we considered spillovers to arise from *stocks* of knowledge instead of flows of R&D. However, empirical evidence suggests that within a firm R&D is highly persistent and concentrated among large firms such that flows may be a reasonable proxy for stocks (See Malerba, Orsenigo, and Peretto, 1997).

[3] In addition, we considered a third case wherein R&D investment in own knowledge and investment in attempting to obtain spillovers were separable functions. Here, one can think of a separable function for spillovers in two ways: either the need for a basic R&D lab to keep abreast of research developments or as a mechanism for pure imitation of rivals. It turns out the analysis an results of this case are very similar to the costless case we analyze here because the separable case allows for similar substitution patterns by the firms. Since the separable case added little to the results, we chose to focus on the two main approaches found in the literature.

[4] For example, see Spence (1984) and D'Aspremont and Jacquemin (1988).

function of own R&D investment, i.e. $\gamma_x > 0$ and $\gamma_{xx} < 0$. The parameter γ in the equation above captures the productivity of this type of investment in increasing firm i's absorptive capacity and captures the ease of learning the specificities of the technological knowledge, e.g. highly sophisticated and complex in contrast to easily recognizable knowledge. The specification thus distinguishes between the extent (captured by b) and the productivity (captured by γ) of spillovers as in Levin and Reiss (1988). They note that extent reflects the degree of patent protection and secrecy, for example, but the productivity reflects the applicability of acquired R&D. Crucially, in the absorptive capacity case, R&D encompasses both the role of innovation and absorption of external knowledge. Thus, the marginal contribution to usable knowledge will exceed unity whenever an external pool of R&D activity exists, $x_j > 0$, in the presence of spillovers, $b > 0$. In the costless case, a firm can choose to eschew own R&D entirely and still innovate, but under absorptive capacity some own R&D is required for innovation. The absorptive capacity function converges to the costless case for an arbitrarily small level of $x_i > 0$ as

$$\lim_{\gamma \to \infty} \left[\frac{\gamma x_i}{1 + \gamma x_i} \right] = 1. \tag{4}$$

The outcome of firm i's innovative activity, captured by the random variable ν_i, is given by:

$$\nu_i \begin{cases} = 1 \text{ with probability } \; p(\nu_i) \\ = 0 \text{ with probability } 1 - p(\nu_i) \end{cases} \tag{5}$$

where

$$p(\nu_i) = \frac{am_i}{1 + am_i} = \frac{a(x_i + \gamma\,(x_i)\,bx_j)}{1 + a(x_i + \gamma\,(x_i)\,bx_j)}. \tag{6}$$

Thus ν_i is a binary variable indicating successful innovation when $\nu_i = 1$ and failure otherwise. a denotes the productivity of usable knowledge which represents the technological opportunity in the industry, i.e., the difficulty of innovating in the industry as it relates to the stage of development of scientific knowledge and other knowledge specific characteristics. The knowledge production function in (6) determines the probability that a firm is successful in R&D, hence the outcome of R&D is uncertain in our model.

The link from R&D to profitability is through process innovations and we track the efficiency levels of firms as the sum of innovations or number of times it was successful. The marginal costs at any point in time depend on the efficiency level of the firm at time t:

$$mc_i = mc_0 e^{-\eta w_i} \tag{7}$$

where $w_i \in \mathbb{Z}^+$ stands for the efficiency level of firm i. $\eta > 0$ captures the rate at which marginal costs decrease with a unit increase in the efficiency level and mc_0 represents the marginal cost of a firm with $w_i = 0$. Higher efficiency levels imply lower marginal costs. The industry is characterized, at time t, by the efficiency level of the firms represented by the vector $s_t = [w_{it}, w_{jt}]$ where w_{it} represents firm i's efficiency level at time t (though we drop the time subscripts throughout the remainder of the paper).

The evolution of the efficiency levels is given by:

$$w_i^{'} = w_i + \nu_i - \varepsilon \tag{8}$$

where prime indicates the next period. ν_i is the firm specific random variable that takes on the value of 1 if the firm successfully innovates and 0 otherwise as given above in (5). The second random variable, ε, is also binary taking on the value of 1 with exogneous probability $\delta > 0$ and zero otherwise. We think of an outcome of $\varepsilon = 1$ as representing an exogenous increase in the factor price index for the industry, as in E-P (1995). This shock to the efficiency level induces some degree of correlation in firms' fates, as it changes all firms' costs simultaneously. The presence of ε in the dynamic model bounds the efficiency levels from above in equilibrium yielding a finite state space.[5]

The probability of an increase in the efficiency level of firm i is then:

[5] Some comments relating our work to the previous literature is in order. Because we characterize the R&D process as one of uncertainty, our mapping from R&D effort to future marginal costs differs from most previous work (e.g. Spence, 1984, Levin and Reiss, 1988, and Cohen and Levinthal, 1989). However, consider a deterministic function mapping R&D effort directly into marginal costs where marginal costs are a decreasing, convex function of R&D effort. If one appends a bounded error term, you would obtain a similar structure to the one discussed above in expectation. The reason the error term needs to be bounded here is that our structure imposes lower and upper limits to how much marginal costs can change over a single period. Over a longer time horizon the bounds expand generating greater potential changes in the long-run through the dynamic set up of the model.

In addition, Kamien and Schwartz (1971) and Reinganum (1983) argue for the relevance of uncertainty in R&D versus determinism when modelling rivalry. Reinganum shows that uncertainty, generally assumed away in the literature, is not innocuous for the case of preemptive patenting, and its presence reverses the strength of incentives between an incumbent and potential rival in conducting R&D. Moreover, Flaherty (1980) carefully examines the stability properties of market structures in a dynamic game of perfect foresight with cost reducing R&D. She finds that the symmetric case is unstable but the asymmetric case is stable to an unanticipated shock under reasonable conditions including the linear demand model commonly used in this literature.

$$\text{Probability}(w_i^{'} = w_i + 1) = (1-\delta)\frac{a(x_i + \gamma\,(x_i)\,bx_j)}{1 + a(x_i + \gamma\,(x_i)\,bx_j)} \tag{9}$$

For illustration purposes, Figure 1 shows a parameterized example of the R&D success function in the costless case. The x-axis displays own R&D efforts and the y-axis shows the total probability of success, the probability that follows directly from own R&D (which is equivalent to the zero spillovers case) and the incremental probability that follows from spillovers. Because knowledge is perfectly and costlessly acquired, own R&D has no impact on the effect of the spillovers and the probability that the firm is successful rises merely because they are able to observe rivals' activities. The figure shows the extreme impact of this assumption, in that a firm conducting absolutely no R&D has a positive (and potentially large) probability of innovating simply because they benefit directly from their rivals' expenditures. However, the marginal contribution of external knowledge falls with greater levels of own R&D.

Figure 2 displays the absorptive capacity case. The highest curve is the total probability of success, while at low levels of own R&D the major benefits derive from absorptive capacity and not own innovative activity. These added benefits initially rise with absorptive capacity, but the additional benefit of the spillovers diminishes at higher levels of own R&D.

2.2 The Spot Market:

The demand for the industry product is assumed linear and is given by the following inverse demand function:

$$P(Q) = A - B(Q) \quad \textit{with } A, B > 0 \tag{10}$$

where $P(Q)$ is the price of the good produced, and Q the industry output. At the beginning of each period firms compete in quantities and solve the following standard profit maximization problem:

$$\max \pi_i = (P(Q) - mc_i)q_i - f \tag{11}$$

where q_i is the quantity produced by firm i and f is the fixed cost of production.

The Cournot-Nash equilibrium will determine the following optimal quantity choices:

$$q_i^* = [A + mc_j - 2mc_i]/3B \tag{12}$$

which will jointly determine the following equilibrium price:

$$P^{*}=\left(A+mc_{i}+mc_{j}\right)/3. \tag{13}$$

Given equilibrium price and quantities, equilibrium firms' profits are:

$$\pi_{i}^{*}=\max\left\{-f,\frac{\left[A+mc_{j}-2mc_{i}\right]^{2}}{9B}-f\right\} \tag{14}$$

The firm will choose whether to produce or not based on how its marginal costs compare with that of their rival as seen in the numerator of (14). If its marginal costs are too large relative to its competitor, the firm will choose not to produce and just pay fixed costs.

2.3 Optimization

In each period of time, firms choose the level of R&D expenditures that maximizes the expected present discounted value of their future stream of profits. The state of the industry is summarized by the vector of the efficiency levels of the firms. Denote $pr(w_{i}^{'},s^{'}\mid w_{i},s)$ as a firm's perceptions of the joint probability that its efficiency will evolve to $w_{i}^{'}$ in the next period, and that the market structure it faces will be $s^{'}$, conditional on the firm's current state and the current market structure. The optimal R&D choice solves the following Bellman equation subject to a non-negativity constraint on R&D:

$$V_{i}(w_{i},s)=\max_{x_{i}\geqslant 0}\{\pi(w_{i},s)-c_{x}x_{i}+\beta E_{t}V_{i}(w_{i}^{'},s^{'})\} \tag{15}$$

where $\beta=1/(1+r)$, with r standing for the interest rate, is the discount factor common to all firms, and $\pi(w_{i},s)$ denotes the profits to firm i with efficiency level ω_{i} in the spot market with market structure s. c_{x} represents the unit cost of R&D.

To understand the impact of the extent of spillovers, we need to understand the basic shape of the value function and policy functions. Figure 3 shows the solution to the value function for firm 1 using the parameters listed in Table 1 with $b=0$ and Figure 4 shows the policy function. The shapes shown here are not particular to the parameters used, but apply generally (See E-P, 1995, and Pakes and McGuire 1994). The x-axis represents firm 1's efficiency level, the y-axis is the rival firm's (firm 2) efficiency level, and the z-axis is firm 1's value in Figure 3 and R&D

effort in Figure 4. Higher efficiency levels mean lower marginal costs. Notice that holding the rival firm's efficiency level fixed, the value function takes a distinctly convex shape at low levels of efficiency, hits an inflection point then becomes concave. At the extreme ends, the value function is relatively flat. In these regions, firms enter coasting states where they cease R&D as they hit the non-negativity constraint. Firms too small to compete cease R&D because the marginal gains are too small relative to the costs given that its rival has such a large advantage. However, as the gap in efficiency levels shrinks, the value increases at an increasing rate creating larger marginal increments in value. Eventually, with higher relative efficiency levels, the firm enters the concave portion of the value function. This region represents a firm with most or all of the market that achieves little benefit from reducing costs because it captures little, if any, additional market share from its rival and it faces diminishing returns in the product market itself for additional cost reductions.

Looking at the policy function we see that as firm 1's efficiency increases R&D investment increases holding the rival firm's efficiency fixed. Eventually this R&D investment reaches a maximum (around 5 where the marginal increase in value is at its greatest) and begins declining as the marginal gains from further investment fall. When firm 1 is the more efficient firm, as the rival firm's efficiency, w_2, increases there is initially a decline in own R&D but little difference as the rival firm becomes significantly more efficient.

In the optimization problem for firm i it is convenient to let $C_S(w_i+1, s'_m)$ denote the expected value of the firm conditional on success in innovating, and let $C_F(w_i, s'_m)$ denote the expect value in the case it fails to develop an innovation. Then one can rewrite the general value function (15) above as:

$$V_i(w_i, s) = \max_{x_i, \geqslant 0} \left\{ \begin{array}{c} \pi(w_i, s) - c_x x_i + \\ \beta \left[\frac{a(x_i+\gamma(x_i)bx_j)}{1+a(x_i+\gamma(x_i)bx_j)} C_S(w_i+1, s'_m) + \frac{1}{1+a(x_i+\gamma(x_i)bx_j)} C_F(w_i, s') \right] \end{array} \right\} \tag{16}$$

Both C_S and C_F can be further broken down into components that hinge on whether the rival is successful or not. Thus (16) expands to become:

$$V_i(w_i, s) = \max_{x_i, \geqslant 0} \left\{ \begin{array}{c} \pi(w_i, s) - c_x x_i + \\ \beta \left[\begin{array}{c} \frac{a(x_i+\gamma(x_i)bx_j)}{1+a(x_i+\gamma(x_i)bx_j} \left[\left(\frac{a(x_j+\gamma(x_j)bx_i)}{1+a(x_j+\gamma(x_j)bx_i)} \right) C_{SS} + \left(\frac{1}{1+a(x_j+\gamma(x_j)bx_i)} \right) C_{SF} \right] \\ + \frac{1}{1+a(x_i+\gamma(x_i)bx_j)} \left[\left(\frac{a(x_i+\gamma(x_j)bx_i)}{1+a(x_j+\gamma(x_j)bx_i)} \right) C_{FS} + \left(\frac{1}{1+a(x_j+\gamma(x_j)bx_i)} \right) C_{FF} \right] \end{array} \right] \end{array} \right\} \tag{17}$$

and the values C_{ij} represent success or failure for the firm and the rival firm, respectively, with S indicating success for the firm and F indicating failure. Thus, C_{SS} indicates both firms innovate, C_{SF} indicates the firm is successful but the rival fails, C_{FS} is the reverse, and C_{FF} indicates both fail to innovate. Note that due to the monotonic increase in value with own efficiency and decrease with rival efficiency, the following relationships hold: $C_{SF} > C_{SS}, C_{FF} > C_{FS}$. A firm is best off when it succeeds and its rival fails, while the lowest value obtains when the firm fails but its rival succeeds in innovating.

The expansion here demonstrates where in the optimization problem the firm internalizes the effect of increasing its own R&D expenditure on its rival's probability of success. Increasing own R&D obviously increases the probability of achieving the value C_S in (16) but in the presence of spillovers raises the probability of outcome values of C_{SS} and C_{FS} where the rival also succeeds. Taking the derivative with respect to x_i and simplifying:

$$\begin{aligned} \frac{\partial V_i}{\partial x_i} &= -\frac{c_x}{\beta} + \frac{a\frac{\partial m_i}{\partial x_i}}{(1+am_i)^2} [C_S - C_F] \\ &\quad + \frac{a\frac{\partial m_j}{\partial x_i}}{(1+am_j)^2} (C_S^* - C_F^*) \end{aligned} \tag{18}$$

where C_1^* and C_2^* refer to the values if the rival succeeds or fails, respectively. $C_F^* > C_S^*$ since the firm's value is higher when its rival is a less efficient producer. Thus, the term in the second line of (18) is negative and reduces the incentives to engage in R&D while the preceding term represents the gains from innovation and provides the incentive for raising R&D. The term $-\frac{c_x}{\beta}$ represents the cost and reduces R&D.

Proposition 1 *The second-order condition for a maximization is everywhere negative for an interior solution.*

Proof. See the appendix. ■

The proof of Proposition 1 and all of the propositions that follow have been placed in the Appendix. We now turn to analyzing the costless and absorptive capacity cases.

3 Costless Spillovers

3.1 Baseline Case

The dynamic nature and complexity of the optimization precludes closed form solutions. However, in order to understand how the incentives and the distribution of market structures changes under costless and absorptive capacity functions, we will begin with a *baseline* case. The baseline case is a restricted version of the costless model wherein each firm takes the rival's probability of success as exogenous. That is, the firm does not internalize the effect of its own R&D program on its rival's pool of knowledge. The reason for using this case, as we show below, is that the degree of spillovers, b, will have essentially no impact on the rate of innovation under this restriction. With no impact on the rate of innovation, then the distribution of market structures, i.e. the degree of asymmetry between the firms, will remain largely unchanged. Then, we show below that relative to this baseline case, the effects of spillovers having diverging effects in the full models of interest, the costless and absorptive capacity cases. That is, the baseline case described here serves as a point of reference for comparing the full models under the two methods of obtaining spillovers. We compare the differences in the optimization problem and show explicitly how and why the level of R&D, rate of innovation, and market structures differ depending on whether spillovers are acquired costlessly or through absorptive capacity.

In the presence of *costless* spillovers if the firm does not internalize the effect of own R&D on the rival's probability of success, the firm treats the probabilities inside the brackets of (17) above as given, as in the E-P model.[6] Thus, the firm chooses the value maximizing level of R&D, x_i, and we obtain the first-order condition from the following value function:

$$V_i(w_i, s) = \max_{x_i, \geqslant 0} \left\{ \begin{array}{c} \pi(w_i, s) - c_x x_i + \\ \beta \left[\frac{a(x_i + \gamma(x_i) b x_j)}{1 + a(x_i + \gamma(x_i) b x_j)} C_S(w_i' + 1, s_m') + \frac{1}{1 + a(x_i + \gamma(x_i) b x_j)} C_F(w_i, s') \right] \end{array} \right\}. \quad (19)$$

[6] This baseline case still generates an equilibrium in the sense that firms' forecasts of values are the values attained and can be solved for using the Pakes-McGuire algorithm. We show this equilibrium in section 5.

The first order condition is:

$$0 = \frac{-c_x}{\beta} + \frac{a}{\left(1 + a\left(x_i + bx_j\right)\right)^2}\left(C_S - C_F\right) \tag{20}$$

which yields a solution for x_i^* and m_i^* dependent on the parameters and the value of success/failure for the firm as follows:

$$x_i^* = \frac{1}{a}\left(\frac{a\beta\left(C_S - C_F\right)}{c_x}\right)^{1/2} - \frac{1}{a} - bx_j, \text{ and} \tag{21}$$

$$m_i^* = x_i + bx_j = \frac{1}{a}\left(\frac{a\beta\left(C_S - C_F\right)}{c_x}\right)^{1/2} - \frac{1}{a}. \tag{22}$$

In (21), x_i^* is the optimal level of R&D given the productivity of R&D, a, the discount factor, β, the unit cost of R&D, c_x, the incremental value for successful R&D, $C_S - C_F$, the extent of spillovers, b, and the level of R&D undertaken by the rival firm, x_j. From this equation it is easy to see the direct negative effect of the extent of spillovers on optimal R&D levels. An increase in b lowers the optimal level of R&D since the firm directly (and costlessly) benefits from rival R&D, provided $x_j > 0$. This effect has been widely discussed in the literature (Spence, 1984) and we refer to it as the *substitution effect*. Moreover, except for the final term, bx_j, the solution is identical to the no spillovers case and the total knowledge.

When we look at the impact of spillovers on the optimal level of R&D, note that there are two endogenous variables in (21), namely the incremental value of successful R&D, $C_S - C_F$, and the level of rival R&D, x_j. For notational convenience, let $\Delta C = C_S - C_F$. Rewriting the expression with ΔC and x_j as functions of b we have:

$$x_i^* = \frac{1}{a}\left(\frac{a\beta\Delta C\left(b\right)}{c_x}\right)^{1/2} - \frac{1}{a} - bx_j\left(b\right). \tag{23}$$

Taking the first derivative with respect to the spillover parameter b yields:

$$\frac{\partial x_i^*}{\partial b} = \frac{1}{2}\left(\frac{a\beta}{c_x\Delta C\left(b\right)}\right)^{1/2}\frac{\partial\Delta C\left(b\right)}{\partial b} - x_j\left(b\right) - b\frac{\partial x_j\left(b\right)}{\partial b}. \tag{24}$$

The first term reveals that R&D could rise or fall with an increase in the extent of spillovers depending on how the incremental value of successful R&D changes. All the parameters and the term in parentheses are positive leaving the sign of the effect dependent on the sign of $\frac{\partial\Delta C(b)}{\partial b}$.

Essentially, the effect depends on how the slope of the value function changes with an increase in the extent of the spillovers. This effect can take on either positive or negative values. In fact, the term contains two effects. First, there is a positive cost effect. Because firm value rises with the level of efficiency, a firm can maintain the same position relative to its rival at a lower cost in terms of R&D expenditure. However, this R&D cost effect while increasing values, can have positive or negative effects on the marginal values. The effect will be increasing whenever R&D is increasing in efficiency and decreasing otherwise because the effect depends on the relative reduction in R&D expenditures.

Second, the marginal value may rise or fall depending upon how the transition probabilities change. Another way to think about this is to ask which firm, the less or more efficient, is willing to work harder for an improvement in relative efficiency when spillovers are greater?[7] If being successful is more likely to lead to a positive change in the level of efficiency relative to its rival, the marginal value increases and so does the incentive for R&D. On the other hand, if the spillovers lead to a more active rival, sustaining an advantage becomes more difficult and costly which flattens the value function. The slope will increase for some firms in some market structures, but decrease in others. We cannot analytically say more about $\frac{\partial \Delta C(b)}{\partial b}$, but our simulations show this effect to be ambiguous as discussed and, more importantly, quite small relative to the other effects.[8] The reason these effects are small is that the values are almost entirely driven by the profits in the product market and those profits do not depend on spillovers nor on the manner in which spillovers are obtained. Therefore, as we proceed in our analytical expostion, we treat these effects as negligible in order to focus on the crucial differences in incentives for engaging in R&D that arise due to the costless and absorptive capacity specifications.

The substitution terms in (24),$-x_j(b) - b\frac{\partial x_j(b)}{\partial b}$, display the direct effect of the change in spillovers on the amount of available knowledge. The secondary effect, captured by $-b\frac{\partial x_j(b)}{\partial b}$, shows that when spillovers increase a rival's R&D expenditures, it leads to a decrease in own R&D and essentially the firm is more willing to free ride on its rival's activity. In contrast, a decrease in rival R&D, leads to an increase in own R&D and less dependence on spillovers. The

[7] The question is similar to the approach taken in Budd, Harris, and Vickers (1993) who look at whether asymmetry between two firms tends to increase or decrease in a dynamic model of duopoly without spillovers.

[8] A technical appendix, available upon request, provides details on the impact and magnitude of changes in ΔC in equilibrium. Overall, these effects are small relative to the other effects within the first-order condition that we analyze here. The changes in ΔC do have an impact, but are very rarely so large as to alter any of our results. Moreover, when the changes are substantial to affect the results, these only occur in a small fraction of the ergodic distribution that occurs in equilibrium.

degree of substitutability in strategies is directly proportional to the extent parameter.

While the overall incentives driven by the substitution effect reduce R&D expenditure, the linear costless specification for knowledge spillovers generates an extreme result when we look at the rate of innovation. Recall that firms combine the optimal amount of own and rival R&D to form usable knowledge which then enters the R&D production function given in (6). Thus, changes in these probabilities show how the rate of innovation in the industry varies with spillovers. When there are no spillovers, the rate of innovation is:

$$\Pr\left(v\left(x_i^*\right)\right) = \frac{ax_i}{1+ax_i} = 1 - \left(\frac{a\beta\Delta C}{c_x}\right)^{-1/2}. \tag{25}$$

Under complete appropriability, the rate of innovation is negatively related to the cost of R&D, c_x, and positively related to the productivity of R&D, a, the discount factor, β, and the marginal gain from success, ΔC. With $b > 0$, we find that the rate of innovation is:

$$\begin{aligned}
\Pr\left(v\left(x_i^*\right)\right) &= \frac{am_i}{1+am_i} \\
&= \frac{a\left[\left(\frac{1}{a}\left(\frac{a\beta\Delta C(b)}{c_x}\right)^{1/2} - \frac{1}{a} - bx_j\right) + bx_j\right]}{1 + a\left[\left(\frac{1}{a}\left(\frac{a\beta\Delta C(b)}{c_x}\right)^{1/2} - \frac{1}{a} - bx_j\right) + bx_j\right]} \\
&= 1 - \left(\frac{a\beta\Delta C\left(b\right)}{c_x}\right)^{-1/2}
\end{aligned} \tag{26}$$

The result above is identical to the case when $b = 0$ (the standard result in E-P), which leads to the following proposition.

Proposition 2 *In the baseline case when knowledge spillovers are costlessly obtained, neither the direct substitution effect nor the indirect substitution effect alter the optimal rate of innovation. Only alterations in the marginal values of innovation will change the rate of innovation and hence alter the degree of concentration. I.e., holding the values constant,* $\frac{\partial \Pr(v(x_i^*))}{\partial b} = 0$.

Proof. See the appendix. ■

The rate of innovation in the baseline case is the same as the no spillovers case, $b = 0$ (and the no spillovers results do not depend on how spillovers are obtained, obviously). Proposition 2 will be instrumental in understanding how the costless and absorptive capacity cases differ under the full models of interest. By examining the rates of innovation and levels of knowledge we can show whether rates of innovation rise or fall with spillovers compared to this baseline case where they are unaffected.

The key aspect of Proposition 2 resides in the fact that the substitution effect neutralizes the impact of spillovers on own R&D, leaving the rate of innovation identical to the rate of innovation without spillovers. The reason is the linear specification, standard in the literature (e.g. Spence, 1984, D'Aspremont and Jacquemin, 1988), which directly adds outside knowledge to own R&D. The optimal choice of own R&D directly accounts for the spillovers, and without any change in the value function, would lead to the identical rate of innovation without spillovers. As a result, the only remaining effect on the probability of success or rate of innovation, for any given market structure, resides in how the spillovers alter the shape of the value function. If there are no effects on the shape of the value function, then spillovers are irrelevant.

Furthermore, without any changes in the rates of innovation, the degree of concentration will be unaffected. Thus, the static, costless spillover two-period models' welfare predictions will be unaffected in the case where firms do not account for how their own R&D benefits rivals. As indicated, when we account for the full effects (below) the rates of innovation change and the market structure changes which, in turn, alters the welfare results.

Given that the rates of innovation are unchanged (and do not depend on the elasticity of R&D w.r.t b) then total R&D efforts must be declining as the extent of the spillovers rises. This result is intuitively straightword. Firms utilize the outside knowledge as a substitute for their own efforts and reduce costs which leads to lower R&D efforts, but the use of outside knowledge allows the rates of innovation to remain the same.

3.2 The Full Model, Costless Case

Now we return to the costless case when firms internalize the effect of their own R&D on rivals' chances of success. From the value function given in (17) we obtain the following first order condition:

$$FOC \quad : \quad 0 = \frac{-c_x}{\beta} + \frac{a}{(1+am_i)^2}(C_S - C_F) + \frac{ab}{(1+am_j)^2}[C_S^* - C_F^*]. \tag{27}$$

There are two differences between this first order condition and that of the baseline case. First, the crucial difference between the baseline case and the full model lies with the final term. This term is unambiguously negative, but not present in the baseline case. The term reflects the negative incentive to engage in R&D because it raises the rival's ability to innovate. Therefore, the optimal level of x_i falls with this term. Intuitively, firms choose lower levels of R&D when they account for the fact that the rival becomes more likely to lower costs through the spillovers. That means the rate of innovation *falls* with spillovers unlike the baseline case where they remained constant. That statement is conditional on the second difference. The difference in values for success and failure, represented by $C_S - C_F$, or ΔC from before, undoubtedly changes. As previously noted, analytically we cannot say much about them without solving a parameterized version of the model. Our simulations show these effects to be small relative to the other effects.[9]

Solving implicitly for m_i we have:

$$m_i^* = \frac{1}{a}\left[\left(\frac{a\beta\Delta C}{c_x}\right) + \frac{ab\beta}{c_x}\left(\frac{1+am_i}{1+am_j}\right)^2 [C_S^* - C_F^*]\right]^{1/2} - \frac{1}{a} \tag{28}$$

which differs from the baseline case in the appearance of the entire second term in brackets, $\frac{ab\beta}{c_x}\left(\frac{1+am_i}{1+am_j}\right)^2 [C_S^* - C_F^*]$. That term does not appear in the baseline case and it is unambiguously negative because $C_S^* < C_F^*$. Thus, the level of available knowledge is lower than the baseline case (and the $b = 0$ no spillovers case) holding the difference in values constant. Moreover, the term is increasing in magnitude with the degree of spillovers, b.

Proposition 3 *When spillovers are costless, holding changes in values constant, the level of available knowledge, the rate of innovation, and the overall level of R&D investment in the full model is less than in the baseline case.*

Proof. See the appendix. ■

Proposition 3 establishes that, when spillovers are costless, the R&D investment and the rate of innovation falls with increased spillovers. This results in two opposing effects on concentration. First, note the term in parantheses multiplying the negative incentive portion of (28), $\left(\frac{1+am_i}{1+am_j}\right)^2$. The term is increasing in own R&D and decreasing in rival R&D. Since the entire effect is

[9] Even though the first order condition in the Bellman equation changes, the profits in the product market remain the same. In addition, in the vast majority of the state space in our simulations, over 91%, the change in the slope of the value function reinforces the results derived here. Only in particular elements of the state space does the change in the slope reverse the result and these cases only occur in a small fraction of the equilibrium erogodic distribution.

negative, because $C_S^* < C_F^*$, it implies that the firm with the larger level of knowledge reduces R&D more than the firm with the lower level of knowledge. This effect generates a push towards more symmetry and, hence, lower industry concentration. Basically, with costless spillovers, the smaller firm benefits more than the larger firm leading to more similar rates of innovation as the extent of spillovers increases.

On the other hand, since R&D levels unambiguously decline with spillovers, the smaller firm becomes more likely to hit the non-negativity constraint where own R&D goes to zero. For that firm, its rate of innovation can still be positive because of the costless spillovers, but its rate of innovation will always be lower than the rivals. This result, again, follows from the linear specification for costless spillovers. With $m_i = x_i + bx_j$, when a firm hits the non-negativity constraint its level of knowledge will be bx_j. The rival's level of knowledge will by x_j which is always larger provided spillovers are not complete, i.e. $b < 1$. Thus, the market will tend towards higher concentration with increased spillovers as the smaller firm becomes more likely to entirely eschew own R&D and free ride on spillovers.

The welfare effects here are clearly negative. First, there is the effect of reduced level of R&D as highlighted in the static two-period model of Spence (1984). However, our analysis indicates a sustained dynamic lowering of the rate of innovation which implies higher long run prices. In addition, concentration may rise or fall which will further affect consumer welfare.

4 Absorptive Capacity

4.1 Baseline Case

We now consider the case when firms acquire spillovers by establishing absorptive capacity through their own R&D program. Starting with the original value function, under absorptive capacity, but assuming, for the ease of comparison to the baseline case, that firms do not account for the impact of own R&D on rivals, the first-order condition is:

$$0 = \frac{-c_x}{\beta} + \frac{a\left(1 + \frac{\gamma}{(1+\gamma x_i)^2} bx_j\right)}{(1 + am_i)^2} (C_S - C_F).$$

Solving for the optimal level of own R&D:

$$x_i^{*,AC} = \left(\frac{\beta\Delta C(b)}{ac_x}\right)^{1/2}\left(\frac{\partial m_i^*(b)}{\partial x_i^*}\right)^{1/2} - \frac{1}{a} - \frac{\gamma x_i^*}{(1+\gamma x_i^*)} b x_j(b). \tag{29}$$

The expression in (29), while not an explicit solution for x_i^*, is readily compared with the baseline costless case in (21). There are two differences in these solutions. First, in (29) the second term in parentheses, expressed as $\frac{\partial m_i}{\partial x_i}$, does not appear in the costless case. The term is:

$$\frac{\partial m_i}{\partial x_i} = 1 + \frac{b x_j}{(1+\gamma x_i)^2} \geq 1$$

which must be greater than or equal to unity. In the costless case, external knowledge is added linearly, $m_i = x_i + b x_j$, and thus $\frac{\partial m_i}{\partial x_i} = 1$. Under absorptive capacity, the marginal increase in knowledge from own R&D exceeds unity whenever b and x_j are non-zero, therefore generating a greater incentive to engage in R&D and raising the rate of innovation.

The second difference lies with the direct negative spillover effect, the last term in (29), which is now modified by the absorptive capacity expression. Because the effect of absorptive capacity, $\frac{\gamma x_i^*}{(1+\gamma x_i^*)}$, lies between 0 and 1, the direct negative spillover effect plays less of a role when absorptive capacity is required to utilize external knowledge. Thus, we see a shifting of incentives towards increased R&D under absorptive capacity.

Proposition 4 *In the baseline case, holding changes in values constant, when R&D spillovers are obtained through investing in absorptive capacity, R&D, the level of available knowledge and the rate of innovation is greater than when spillovers are obtained costlessly whenever* $b > 0$.

Proof. See the appendix. ■

In effect, the presence of absorptive capacity changes the margins on which the R&D decision is made. With increased productivity of own R&D through spillovers, absent in the costless case, optimal R&D expenditures rise. Solving for the rate of innovation we have:

$$\Pr(\upsilon(x_i^*)) = 1 - \left(\frac{a\beta\Delta C}{c_x}\right)^{-1/2}\left(\frac{(1+\gamma x_i^*)^2}{(1+\gamma x_i^*)^2 + \gamma b x_j}\right)^{1/2}. \tag{30}$$

The solution is the same as the baseline costless case except that it includes the final term in parentheses, which can be written as $\left(\frac{\partial m_i^*}{\partial x_i^*}\right)^{-1/2}$. The term, $\frac{\partial m_i^*}{\partial x_i^*}$, is the marginal gain in knowledge from R&D, which was alway 1 in the costless case. In the absorptive capacity case, in contrast, $\frac{\partial m_i}{\partial x_i} \geq 1$, and thus $\left(\frac{\partial m_i^*}{\partial x_i^*}\right)^{-1/2}$ must lie between 0 and 1 . Therefore, for an interior solution, from (30) we can see that the rate of innovation is greater than in the costless baseline case. Since the marginal increment in knowledge from own R&D rises due to absorptive capacity for the same parameters, the rate of innovation is higher than when costlessly obtained. In the costless baseline case, the rate of innovation did not change with spillovers. Here, the increase in the extent of spillovers raises the incentive to engage in R&D and acquire spillovers leading to higher rates of innovation, a complete contrast with the full costless model.

The rate of innovation rises and the higher rates of innovation will promote greater product market competition reducing the level of concentration. Once absorptive capacity is introduced, the firm with the smaller R&D program also enjoys the larger marginal gain from the spillovers in terms of its probability of successful innovation. We cannot say, however, whether R&D for an individual firm rises or falls relative to the *no spillovers* case because of the competing effects. On the one hand, R&D will fall for the same reasons we observed previously wherein the greater availability of outside knowledge provides a substitute for expenditures on R&D. However, in order to make greater use of external R&D, the firm will want to engage in more of its own R&D to enhance its absorptive capacity. That leads to smaller firms having large incentives to engage in at least some R&D. As a result, they are less likely to optimally choose zero own R&D making it more difficult for a larger firm to sustain any cost advantage over time. Taken together, under absorptive capacity, the effects push towards lower concentration and greater rates of innovation.

4.2 The Full Model, Absorptive Capacity Case

Now, to complete the comparison, we study the first-order condition in the full model of the absorptive capacity case. The optimality condition is:

$$\begin{aligned} 0 &= \frac{-c_x}{\beta} + \frac{a\left(1 + \frac{\gamma}{(1+\gamma x_i)^2} b x_j\right)}{(1 + a m_i)^2} (C_S - C_F) \\ &\quad + \frac{ab \frac{\gamma}{(1+\gamma x_i)^2} x_j}{(1 + a m_j)^2} (C_S^* - C_F^*) \end{aligned} \tag{31}$$

which is similar to the first-order condition of the full model version of the costless case repeated here:

$$FOC = 0 = \frac{-c_x}{\beta} + \frac{a}{(1+am_i)^2}(C_S - C_F) + \frac{ab}{(1+am_j)^2}[C_S^* - C_F^*].$$

Note the effect of absorptive capacity which enters the final two terms in the numerators. The first of these two terms reflects the positive incentives to engage in R&D and contains $\left(1 + \frac{\gamma}{(1+\gamma x_i)^2} bx_j\right)$. This component reflects the same increase in the margin of success of the firm due to taking advantage of spillovers. This component takes a value of 1 if the rival does no R&D and increases with higher levels of x_j. Thus, the magnitude of the second term increases with spillovers generating higher levels of R&D at the optimum. The second, and negative term, shows the marginal increase in the probability of rival success, as in the costless case, but is reduced by the derivative of the absorptive capacity function which is less than one. Overall then, as a result of the absorptive capacity role of R&D, the positive incentives for R&D rise and the negative incentives decline in magnitude relative to the costless baseline case (where innovation rates were unchanged).

Solving implicitly for m_i^*:

$$m_i^* = \frac{1}{a}\left[\left(\frac{a\beta}{c_x}\right)(1+\gamma'(x_i)bx_j)\Delta C + \frac{ab\beta x_j}{c_x}\left(\frac{1+am_i}{1+am_j}\right)^2 \gamma'(x_i)[C_S^* - C_F^*]\right]^{1/2} - \frac{1}{a} \qquad (32)$$

The negative component (second term inside the brackets) enters as we saw in the full costless case. Thus, overall knowledge is reduced, as expected, by the negative incentive stemming from spillovers helping the rival. However, when compared with the full costless case, we see that knowledge, innovation, and R&D are higher. The first derivative of the absorptive capacity function, $\gamma'(x_i)$, enters both the positive and negative terms. Since $0 \leq \gamma'(x_i) < 1$, the positive term increases in absolute value while the negative term falls. Thus, knowledge is higher under the fall absorptive capacity case than under the full costless case and that difference increases with spillovers. Proposition 5 summarizes the result.

Proposition 5 *In the full model, holding changes in values constant, and knowledge spillovers are obtained through absorptive capacity, the level of knowledge and the rate of innovation is higher than when spillovers are obtained costlessly.*

Proof. See the appendix. ■

Intuitively, these results make sense. When R&D investment is required to take advantage of spillovers, it increases the incentives for firms to engage in R&D because it raises marginal benefits from external knowledge. In addition, the marginal external benefit to rivals is mitigated because their ability to succeed depends on their own engagement in R&D.

With the opposing effects, we cannot make a general statement about whether knowledge is higher or lower than the costless baseline case where the rate of innovation was unaffected by b. However, in the case of symmetry we can show that knowledge (and therefore the rate of innovation) is higher.

Proposition 6 *In the full model, holding changes in values constant, when knowledge spillovers are obtained through absorptive capacity, and the firms are* symmetric*, the level of knowledge, and the rate of innovation, is higher than when spillovers are obtained costlessly in the baseline case. Therefore the rate of innovation rises with the extent spillovers under absorptive capacity.*

Proof. See the appendix. ■

With the increase in knowledge over the costless baseline case, the rate of innovation rises with spillovers provided the firms are symmetric. We can say more though when the firms are not symmetric. (32) reveals a push towards symmetry. The effect is easiest to observe from solving for the rate of innovation:

$$\Pr\left(\upsilon\left(x_i^*\right)\right) = 1 - \left[\left(\frac{a\beta\Delta C}{c_x}\right) + \left(\frac{a\beta}{c_x}\right)\gamma'\left(x_i\right)bx_j\left(\Delta C + \left(\frac{1+am_i}{1+am_j}\right)^2\Delta C^*\right)\right]^{-1/2}. \quad (33)$$

If there are no spillovers, $b = 0$, we obtain exactly the same solution as in (26), because the entire second term in brackets vanishes. With spillovers, the terms in front, $\left(\frac{a\beta}{c_x}\right)\gamma'\left(x_i\right)bx_j$, are all positive leaving the sign dependent on the relative magnitudes of ΔC and $\left(\frac{1+am_i}{1+am_j}\right)^2\Delta C^*$, where ΔC is the marginal gain in value for success and ΔC^* is the marginal loss in value from the rival's success. Under asymmetry, whether the rate of innovation rises depends on $\left(\frac{1+am_i}{1+am_j}\right)^2$ and here we observe a push towards symmetry. When firm i conducts relatively more R&D than its rival j, the ratio increases (exceeds 1) meaning the firm places more weight on the negative consequence of R&D, namely creating spillovers for the rival. On the other hand, whenever firm i conducts relatively less R&D the ratio is less than one and the rate of innovation rises. As shown in Proposition 6, when the firms are symmetric, $\Delta C > \Delta C^*$, and thus the entire term is overall positive leading to higher rates of innovation. Therefore, the firm with the lower R&D program will have greater incentive to increase R&D and its probability of innovation will rise.

Note how these results contrast with the baseline costless case where we began. There, spillovers did not affect the rate of innovation. Now, spillovers will raise the rate of innovation and when firms are asymmetric, the incentives shift towards more R&D and higher rates of innovation for the smaller firm. Contrasted with the costless case, where the rates of innovation unambiguously fell, we see a very different effect on the market structure. Moreover, the result differs with the analysis found in Levin and Reiss (1988) which predicts a decrease in the rate of (process) innovation and concentration with an increase in the extent of spillovers. That result follows from the symmetric assumption which only allows concentration to adjust through the number of firms. Here, the rate of innovation increases when we allow for asymmetry between firms because the incentives to engage in R&D shift towards the smaller firms. That effect, in turn, reduces concentration, but generates a positive (as opposed to negative in Levin and Reiss) feedback effect on innovation.

To summarize the analysis, Table 2 shows our results for comparative purposes and our expectations for the simulations that follow. Compared to the no spillovers situation, $b = 0$, R&D unambiguously declines in the costless spillovers cases, both in the baseline and full model, though it falls more in the full model where the rate of innovation declines. In the absorptive capacity cases, however, the level of knowledge and the rate of innovation increase. R&D may increase or decrease, but if R&D declines, it will decline less than in the costless case. Whether the rate of innovation rises more in the baseline case of absorptive capacity or the full model is unclear because of the endogeneity of the market structure. On the one hand, firms internalize the negative effect on their own values of increasing the knowledge available to the rival firm. That effect clearly lowers the rate of innovation. However, the additional push towards symmetry that this effect entails would raise the overall rate of innovation because competition and the marginal gains from success are largest when the firms are symmetric.

For welfare, we anticipate the following. With no change in the rate of innovation, we expect little change in consumer surplus in the costless baseline case. However, with lower expenditures on R&D but similar rates of innovation, firm values should rise (where we have been assuming no change above). In the full model costless case, the fall in the rate of innovation will raise prices and lower welfare, although this may be offset to some extent by lower concentration. On the other hand, in both absorptive capacity cases, we expect that consumer surplus will increase through both higher rates of innovation which lead to lower costs and the general push towards more symmetry which increases competition and further lowers prices.

5 Simulation

5.1 Methodology

The preceding analysis captures the impact of spillovers on optimal R&D decisions, and hence the rates of innovation. We use this section to illustrate the effects discussed above and to show how the market structure changes with the extent of spillovers by simulating the model.

The numerical algorithm solves for the Markov-perfect Nash equilibrium policy functions for investment and values associated with each possible market structure. The algorithm delivers the optimal strategies $\{x_i^*(w_i, s)\}$ for all $w_i \in W$, and all $s \in S$. The simulation program then uses these equilibrium policies to stochastically generate the evolution of the market structure of the industry, which is an ergodic process. We simulated the model $100,000$ times under each scenario to obtain the numerical results for the ergodic distributions of market structures and the expected discounted value of the welfare measures. We also vary the extent of spillovers, b, from zero to unity, i.e. from fully appropriable R&D to complete spillovers, with jumps of 10% in the extent of knowledge spillovers. The parameter values used for the simulations of the model are given in Table 1. In a few cases, firms began to exceed the state space when spillovers reached $b = 0.9$ and 1.0. Thus, we omit those results here since the range from 0 to 0.8 demonstrates the results of the preceding section.

5.2 Costless Spillovers

We begin with the representation of spillovers as they have been primarily used in the literature preceding the work of Levin and Reiss (1988), but continuing in the analysis of the welfare implications of research joint ventures (e.g. D'Aspremont and Jacquemin, 1988, Suzumura, 1992). The graphs in Figure 5 show how R&D, the rate of innovation, firm values, and concentration change as the extent of spillovers, b, increases. In all graphs we distinguish between the leader and the follower. The lead firm is the firm with the highest efficiency level (largest market share).[10] In doing so it allows us to see the changes in incentives for the large and small firms as spillovers increase.

For purposes of comparison and relating the findings of the previous section to the simulations, we also simulated the baseline models. As shown in Proposition 4, the rate of innovation will only change if there is variation in the difference in values, ΔC, but these changes are quite small. The probability of success hardly varies even though R&D expenditures decline substantially and in percentage terms more so for the follower firm. There is also a small increase in values as the firms spend less on R&D. Since the market consists of only two firms, the market share of the largest firm, the C1 index, is a sufficient statistic for concentration. While some variation exists, the changes show that there is less than a one percentage point shift in the market shares on average. In the other cases below we see more substantial changes.

Figure 6 shows the ergodic distribution for the market structures when $b = 0.0$, $b = 0.2$, $b = 0.5$, and $b = 0.8$ The x-axes show the efficiency level for the leader firm and the y-axes show the efficiency level for the follower firm. Thus, the x-y plane represents all possible market structures which is the cross-product of the two efficiency levels. The z-axes shows the distribution of the market structures that occur in equilibrium. The upper left diagram shows the distribution of market structures without spillovers, $b = 0.0$. Two modes emerge where the taller peak represents an asymmetric, concentrated market structure. Here, the lead firm possesses a significant efficiency advantage over its rival and thus captures much of the market share. The second mode, in contrast, lies along the diagonal where firms' hold equal, or nearly equal, efficiency levels. Thus, the parameterization chosen provides a good baseline for comparison as we will be able to detect changes in the mass under these two modes that represent quite different market structures.

As b increases, the distribution does change but not dramatically so as we would expect when the rates of innovation remain largely the same. The changes that do occur follow from small adjustments in the slope of the value function.[11] Compare these results with Figures 7 and 8 which show the same set of graphs for the full model, i.e. accounting for the impact of their R&D

[10] When the firms have identical efficiency levels we assign one to being the leader and the other the follower. Since the firms are facing identical problems, the identities do not matter or alter the results in any way.

[11] Using the simluations, we calculate that the changes to the value function, assumed away for analytical tractibility reasons in the preceding section, account for only about 17% of the change in R&D. Moreover, the direction of the effects varies, sometimes positive, sometimes negative. In contrast, the component we do analyze, the substitution effect, is consistently negative (as expected) and accounts for the remainder of the effects and dominates.

on the rival. The difference is striking. As in Proposition 6, R&D falls further, while the rate of innovation for the lead firm declines and rises slightly for the follower. There is a small decrease in concentration for the middle levels of the extent of spillovers, but this change still only represents a two percentage point change, at most, in the C1 concentration ratio.

To understand the non-monotonic movement in the concentration ratio, the distribution graphs in Figure 8 show that as both firms reduce R&D expenditures, the symmetric mode, where both firms are relatively efficient, disappears with high levels of spillovers. The mass of the distribution moves towards the asymmetric mode. With falling levels of R&D, the zero R&D outcome becomes more likely which leaves the larger firm with the higher rate of innovation. At the same time the frequency of symmetric, or nearly symmetric, low efficiency levels for both firms increases which reduces concentration . This effect follows from the incentives for the larger firm to cut R&D more than the smaller firm due to spillovers. Thus, we see offsetting effects on concentration as the two firms compete less fiercely in R&D and in initial decline followed by an increase in concentration with higher levels of b . The end result is lower consumer surplus due to the low rate of innovation and higher prices.

5.3 Absorptive Capacity

Under the baseline absorptive capacity specification, Figure 9 displays the effects of increasing spillovers on the mean equilibrium levels of R&D, probability of success, firm values and C1. For both the leader and the follower, R&D investment falls on average due to the negative incentive associated with less appropriability, but also with spillovers less R&D is required to achieve any given level of success. The change in total R&D efforts is smaller than under the costless case above expected from Proposition 7. Firm values also rise, but with a more pronounced increase for the smaller firm. The concentration in industry also declines.

The key difference shown in Figure 9 is the reversal of efforts by the leader and the follower. Under no spillovers, or costless spillovers, the lead firm maintained higher R&D efforts and, even in the presence of spillovers, a higher rate of innovation as in Proposition 8. However, with costly absorptive capacity required to obtain external knowledge, as the extent of the spillovers increases, we see the smaller, follower firm engaging in more R&D and reaching a higher rate of innovation. These changes, in turn do affect the market structure in a fairly dramatic, but intuitive way.

Figure 10 shows the substantial effects of the change in incentives. As the extent of the spillovers increases, the market become more symmetric. Both of the same modal market structures are present. However, the probability mass shifts from the highly unequal market structure to the more symmetric market structure as the extent of spillovers rises Given the shift towards symmetry, not surprisingly the C1 concentration ratio falls with spillovers. The market share shifts approximately seven percentage points, which moves the mean market share distribution to about 55-45 from 62.5-37.5.

We show the results of the full model under absorptive capacity in Figures 11 and 12. The outcome is very similar to the previous case, but the additional push towards symmetry in (32) shows that the follower's average R&D expenditure exceeds the leader's for lower levels of spillovers. The level of knowledge and rates of innovation are clearly higher than in the costless case (Proposition 9). The result of the increased R&D competition can be seen in the lower right panel of Figure 11 which shows an even larger decrease in the degree of concentration. This stronger push towards symmetry appears in Figure 12 where at $b = 0.8$ the symmetric mode is clearly dominant indicating higher R&D competition between the firms.

Overall, the simulations reveal how the costless and absorptive capacity cases differ. When investment in R&D is required to take advantage of spillovers, the marginal benefits of R&D increase. That leads to more intense competition and greater benefits from R&D fall on the follower firm. In turn that leads to a less concentrated market structure with higher rates of innovation. The higher rates of innovation raise consumer surplus because competition intensifies. In contrast, costlessly obtained spillovers reduce the rate of innovation as firms substitute rival R&D for their own. The impact on the market structure is non-monotonic with a tendency towards decreased concentration as overall rates of innovation decline, but towards increased concentration as the incentives of the less efficient firm lead it to reduce R&D even more than the leader for high levels of spillovers.

Finally, Table 3 compares firm values and consumer surplus under high spillovers ($b = 0.8$) versus the no spillovers case ($b = 0$). We continue to distinguish between the lead and follower firms because the impact differs. In the costless baseline case, both firms benefit but the majority of the benefits fall on the smaller of the two firms because spillovers allow the firm to free ride on its rival and reduce R&D expenditures. There is a small positive change in consumer surplus, but it is an order of magnitude smaller than changes in the other cases. Under the full costless model, the follower firm continues to benefit, but the lead firm's value declines while consumer surplus falls with higher prices due to lower rates of innovation.

In contrast, under absorptive capacity, consumer surplus increases with the higher rates of innovation and the less concentrated market structure. The effect is more pronounced in the full model where the market pushes towards greater symmetry. The increased symmetry is also reflected in the mean changes in the values of the lead and follower firms. The follower firm's value rises in both cases, but by nearly 50% in the full model. The lead firm's value rises slightly when firms ignore the externality but falls when the market becomes more competitive.

While the preceding illustrates the results of the analytical section, we also conducted a sensitivity analysis of our parameterization. Specifically, we varied the key parameters governing R&D productivity, a, the rate of cost reduction, η, the level of fixed costs f, the rate of factor price increases, δ, and the productivity of absorptive capacity, γ. We found no change in the qualitative results presented here.[12]

6 Conclusions

Our model of an infinite horizon duopoly engaged in R&D competition and Cournot-Nash competition in the spot market shows strong effects on the ergodic distribution of the market structures and welfare previously ignored in the literature. In light of Mansfield's (1984) stylized fact stating that industries exhibit greater degrees of competition when imitation is easier, we find that result in the model under absorptive capacity, but not in the costless case. However, whether spillovers are welfare enhancing or not depends on whether they are obtained costlessly or through absorptive capacity. Costless spillovers reduce the level of R&D, the rate of innovation, and lower welfare while generating offsetting effects on concentration. The welfare lossses follow from the higher prices in equilibrium because firms are innovating less. In contrast, when firms need to establish absorptive capacity to acquire external knowledge the rate of innovation rises for two reasons. First, the positive impact on the incentives to increase R&D raise the marginal productivity of R&D from the firm's point of view. Second, the effect is strongest for the firm that would, in the absence of spillovers conduct the least amount of R&D. The result is a push towards a more symmetric market structure where the industry rate of innovation is highest. Higher rates of innovation lead to lower long-run prices for consumers which raises welfare.

[12] These results are available on request.

There are two issues possibly worth addressing in further research. The first is entry and exit. Since the previous literature worked with static two-period models, introducing entry and exit was not feasible in that context. Our model can handle entry and exit, but we doubt it would change the results in any dramatic fashion. It would basically add an incentive wherein an incumbent may seek to capture the whole market by driving its rival out of the market completely and conduct R&D to deter entry. Spillovers, however, would lessen that incentive as a cost effective strategy. In addition, entry may offset the welfare gains as the appropriability problems becomes more severe for any one firm when multiple rivals exist.

Secondly, we have followed, for the most part, the specifications of imperfect appropriability used for the past 25 years in the literature on R&D, spillovers, and competition. However, these papers often discuss, but rarely model or measure explicitly, external knowledge as a stock, rather than a flow (which is how we, and many of the papers cited, treat it in the model). Consider a large dominant firm that owes its position to having a vastly superior production process to all others, but which, for whatever reason, ceases to engage in further R&D. Under the specifications discussed here that firm can no longer be a source of spillovers. Yet it seems more than plausible that other firms will still try to emulate that most efficient firm and seek information on that production process. Thus, we believe a more interesting avenue for future work is to treat the efficiency levels as a stock of innovations which enter the knowledge production function rather than the flow of R&D. This alteration may well change incentives and the resulting equilibrium market structures, and thus we feel it is a worthy avenue for future work.

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Appendix I: Proofs of Propositions

Proposition 1 *The second-order condition for a maxmization is everywhere negative for an interior solution.*

Proof. Starting from:

$$
\begin{aligned}
\frac{\partial V_i}{\partial x_i} &= -\frac{c_x}{\beta}+\frac{a\frac{\partial m_i}{\partial x_i}}{(1+am_i)^2}\left[\left(\frac{am_j}{1+am_j}\right)C_{SS}+\left(\frac{1}{1+am_j}\right)C_{SF}\right] \\
&\quad -\frac{a\frac{\partial m_i}{\partial x_i}}{(1+am_i)^2}\left[\left(\frac{am_j}{1+am_j}\right)C_{FS}+\left(\frac{1}{1+am_j}\right)C_{FF}\right] \\
&\quad \left(\frac{am_i}{1+am_i}\right)\frac{a\frac{\partial m_j}{\partial x_i}}{(1+am_j)^2}\left(C_{SS}-C_{SF}\right)+\left(\frac{1}{1+am_i}\right)\frac{a\frac{\partial m_j}{\partial x_i}}{(1+am_j)^2}\left(C_{FS}-C_{FF}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial V_i^2}{\partial x_i^2} &= -\frac{2a^2}{(1+am_i)^3}\left(\frac{\partial m_i}{\partial x_i}\right)^2[C_S-C_F]+\frac{a}{(1+am_i)^2}\left(\frac{\partial^2 m_i}{\partial x_i^2}\right)[C_S-C_F] \\
&\quad +\frac{a^2}{(1+am_i)^2(1+am_j)^2}\left(\frac{\partial m_i}{\partial x_i}\right)\left(\frac{\partial m_j}{\partial x_i}\right)[C_{SS}+C_{FF}-C_{SF}-C_{FS}] \\
&\quad -\frac{2a^2}{(1+am_j)^3}\left(\frac{\partial m_j}{\partial x_i}\right)^2[C_S^*-C_F^*]+\frac{a}{(1+am_j)^2}\left(\frac{\partial^2 m_j}{\partial x_i^2}\right)[C_S^*-C_F^*] \\
&\quad +\frac{a^2}{(1+am_i)^2(1+am_j)^2}\left(\frac{\partial m_i}{\partial x_i}\right)\left(\frac{\partial m_j}{\partial x_i}\right)[C_{SS}+C_{FF}-C_{SF}-C_{FS}]
\end{aligned}
$$

From the definitions of m_i and m_j:

$$
\begin{aligned}
\frac{\partial m_i}{\partial x_i} &= 1+\gamma'(x_i)\,bx_j \geq 1,\ \frac{\partial^2 m_i}{\partial x_i^2}=\gamma''(x_i)\,bx_j<0 \\
0 &< \frac{\partial m_j}{\partial x_i}=\gamma(x_j)\,b\leq 1,\ \frac{\partial^2 m_j}{\partial x_i^2}=0
\end{aligned}
$$

The second-order condition can be slightly written by combining the 2nd and 4th lines and since the last term goes to zero:

$$
\begin{aligned}
\frac{\partial V_i^2}{\partial x_i^2} &= -\frac{2a^2}{(1+am_i)^3}\left(\frac{\partial m_i}{\partial x_i}\right)^2[C_S-C_F]+\frac{a}{(1+am_i)^2}\left(\frac{\partial^2 m_i}{\partial x_i^2}\right)[C_S-C_F] \\
&\quad +\frac{2a^2}{(1+am_i)^2(1+am_j)^2}\left(\frac{\partial m_i}{\partial x_i}\right)\left(\frac{\partial m_j}{\partial x_i}\right)[C_{SS}+C_{FF}-C_{SF}-C_{FS}] \\
&\quad -\frac{2a^2}{(1+am_j)^3}\left(\frac{\partial m_j}{\partial x_i}\right)^2[C_S^*-C_F^*]
\end{aligned}
$$

The terms in the first line above are unambiguously negative. The sign of the second two lines needs to be established. Focusing on the final, unambiguously positive term first where $C_S^* - C_F^* < 0$, we can derive an equivalent expression from the FOC. The first-order condition is:

$$-\frac{c_x}{\beta} + \frac{a}{(1+am_i)^2}\left(\frac{\partial m_i}{\partial x_i}\right)[C_S - C_F] + \frac{a}{(1+am_j)^2}\left(\frac{\partial m_j}{\partial x_i}\right)[C_S^* - C_F^*] = 0.$$

After some algebra we can show that:

$$-\frac{2a^2}{(1+am_j)^3}\left(\frac{\partial m_j}{\partial x_i}\right)^2[C_S^* - C_F^*] = \frac{2a^2(1+am_j)}{(1+am_i)^2(1+am_j)^2}\left(\frac{\partial m_i}{\partial x_i}\right)\left(\frac{\partial m_j}{\partial x_i}\right)[C_S - C_F] - \frac{2ac_x}{\beta(1+am_j)}\left(\frac{\partial m_j}{\partial x_i}\right)$$

The final equality shows the last term in the second-order condition equals the right-hand side, where the first term is positive and the second-term, owing to the minus sign in front, is negative. Thus the positive term in the second-order condition is something smaller in absolute value than

$$\frac{2a^2(1+am_j)}{(1+am_i)^2(1+am_j)^2}\left(\frac{\partial m_i}{\partial x_i}\right)\left(\frac{\partial m_j}{\partial x_i}\right)[C_S - C_F].$$

We can compare this term with the second line of the second order condition above:

$$\frac{2a^2}{(1+am_i)^2(1+am_j)^2}\left(\frac{\partial m_i}{\partial x_i}\right)\left(\frac{\partial m_j}{\partial x_i}\right)[C_{SS} + C_{FF} - C_{SF} - C_{FS}].$$

The second-order condition will hold if the combined value is negative. Adding the terms and factoring we have:

$$\left[\frac{2a^2}{(1+am_i)^2(1+am_j)^2}\left(\frac{\partial m_i}{\partial x_i}\right)\left(\frac{\partial m_j}{\partial x_i}\right)\right]\left(\begin{array}{c} C_{SS} + C_{FF} - C_{SF} - C_{FS} + am_jC_{SS} \\ +C_{FS} - am_jC_{SF} - C_{FF} \end{array}\right).$$

The term in brackets is positive, while the term in parentheses reduces to:

$$(1+am_j)(C_{SS} - C_{SF}) < 0$$

which is negative since $C_{SF} > C_{SS}$. Thus, the second-order condition holds. *Q.E.D.* ■

Proposition 2 *In the baseline case when knowledge spillovers are costlessly obtained, neither the direct substitution effect nor the indirect substitution effect alter the optimal rate of innovation. Only alterations in the marginal values of innovation will change the rate of innovation and hence alter the degree of concentration. I.e., holding the values constant,* $\frac{\partial \Pr(v(x_i^*))}{\partial b} = 0$.

Proof. Substituting the optimal R&D level (21) into the knowledge production function:

$$\begin{aligned}
\Pr\left(v\left(x_i^*\right)\right) &= \frac{am_i}{1+am_i} \\
&= \frac{a\left(x_i^*+bx_j\right)}{1+a\left(x_i^*+bx_j\right)} \\
&= \frac{a\left[\left(\frac{1}{a}\left(\frac{a\beta\Delta C(b)}{c_x}\right)^{1/2}-\frac{1}{a}-bx_j\right)+bx_j\right]}{1+a\left[\left(\frac{1}{a}\left(\frac{a\beta\Delta C(b)}{c_x}\right)^{1/2}-\frac{1}{a}-bx_j\right)+bx_j\right]} \\
&= 1-\left(\frac{a\beta\Delta C\left(b\right)}{c_x}\right)^{-1/2}
\end{aligned}$$

■

Q.E.D.

Proposition 3 *When spillovers are costless, holding changes in values constant, the level of available knowledge, the rate of innovation, and the overall level of R&D investment in the full model is less than in the baseline case.*

Proof. In the baseline case:

$$m_i^{*BASELINE}=\left[\left(\frac{a\beta\Delta C}{c_x}\right)\right]^{1/2}$$

whereas in the fully model m_i^{*FS} is given by

$$m_i^{*FULL}=\frac{1}{a}\left[\left(\frac{a\beta\Delta C}{c_x}\right)+\frac{ab\beta}{c_x}\left(\frac{1+am_i}{1+am_j}\right)^2\left[C_S^*-C_F^*\right]\right]^{1/2}-\frac{1}{a}.$$

Since $C_S^*<C_F^*$, then $m_i^{*FULL}<m_i^{*COSTLESS}$. Since the rate of innovation is determined by the probability of successful which is a strictly monotonically increasing function of m, the rate of innovation declines. Finally a lower level of m, immediately implies that the total levels of R&D must decline. *Q.E.D.* ■

Proposition 4 *In the baseline case, holding changes in values constant, when R&D spillovers are obtained through investing in absorptive capacity, R&D, the level of available knowledge and the rate of innovation are higher than when spillovers are obtained costlessly whenever* $b>0$.

Proof. Compare the R&D solutions for the baseline and full costless cases. Two terms appear in the latter but not in the former. First, we have

$$\frac{\partial m_i}{\partial x_i}=1+\frac{bx_j}{\left(1+\gamma x_i\right)^2}\geq 1$$

which raises the optimal R&D level. Second, the absorptive capacity function modifies the direct spillover effect

$$-\frac{\gamma x_i^*}{(1+\gamma x_i^*)} b x_j\,(b)$$

but $\frac{\gamma x_i^*}{(1+\gamma x_i^*)}$ is bounded between 0 and 1, while in the costless case it takes on the value of 1. Therefore the overall negative effect is reduced in absolute value leading to higher levels of x_i^*.

Substituting x_i^* into the R&D production function we have:

$$\Pr(\upsilon(x_i^*)) = 1 - \left(\frac{c_x}{a\beta\Delta C\,(b)}\right)^{1/2}\left(\frac{\partial m_i^*\,(b)}{\partial x_i^*}\right)^{-1/2}$$

and noting that:

$$\frac{\partial m_i}{\partial x_i} = 1 + \frac{b x_j}{(1+\gamma x_i)^2} \geq 1,$$

immediately implies that the term $\frac{\partial m_i^*(b)}{\partial x_i^*}$ is larger than in the costless case where $\frac{\partial m_i}{\partial x_i} = 1$. Then

$$\frac{d\Pr(v_i)}{db} > 0.$$

Thus,

$$\frac{d\Pr(v_i)}{db} = \frac{a}{(1+am_i)^2}\frac{dm_i}{db} > 0.$$

which holds since $\frac{dm_i}{db} > 0$. *Q.E.D.* ■

Proposition 5 *In the full model, holding changes in values constant, and knowledge spillovers are obtained through absorptive capacity, the level of knowledge is higher than when spillovers are obtained costlessly.*

Proof. Comparing the optimal level of knowledge in the costless and absorptive capacity cases respectively we have:

$$m_i^{*COSTLESS} = \frac{1}{a}\left[\left(\frac{a\beta\Delta C}{c_x}\right) + \frac{ab\beta}{c_x}\left(\frac{1+am_i}{1+am_j}\right)^2 [C_S^* - C_F^*]\right]^{1/2} - \frac{1}{a}$$

$$m_i^{*AC} = \frac{1}{a}\left[\left(\frac{a\beta\Delta C}{c_x}\right)(1+\gamma'(x_i)\,bx_j) + \frac{ab\beta}{c_x}\left(\frac{1+am_i}{1+am_j}\right)^2 \gamma'(x_i)\,x_j\,[C_S^* - C_F^*]\right]^{1/2} - \frac{1}{a}$$

In both solutions, inside the brackets the first term is positive since $\Delta C > 0$ while the second term is negative since $C_S^* - C_F^* < 0$. The first term is overall larger since $\gamma'(x_i)\,bx_j \geq 0$ while the negative term is reduced by $0 \leq \gamma'(x_i) \leq 1$, which make the absorptive capacity level of m_i^* larger than in the costless case. *Q.E.D.* ■

Proposition 6 *In the full model, holding changes in values constant, when knowledge spillovers are obtained through absorptive capacity, and the firms are* symmetric*, the level of knowledge, and the rate of innovation, is higher than when spillovers are obtained costlessly in the baseline case. Therefore the rate of innovation rises with the extent spillovers under absorptive capacity.*

Proof. Using (28) and (32), the difference in available knowledge between the two cases is given by the following terms:

$$\begin{aligned} & \frac{ab\beta x_j}{c_x}\gamma'(x_i)\,\Delta C + \frac{ab\beta}{c_x}\left(\frac{1+am_i}{1+am_j}\right)^2 \gamma'(x_i)\,x_j\,[C_S^* - C_F^*] \\ = \; & \frac{ab\beta x_j}{c_x}\gamma'(x_i)\left[(C_S - C_F) + \left(\frac{1+am_i}{1+am_j}\right)^2 (C_S^* - C_F^*)\right] \end{aligned}$$

Under symmetry then the expression becomes:

$$\frac{ab\beta x_j}{c_x}\gamma'(x_i)\,[C_S - C_F + C_S^* - C_F^*]$$

Since $C_S > C_F^* > C_S^* > C_F$, then

$$\begin{aligned} C_S - C_F^* &> C_S^* - C_F^*, \text{ and using } C_F^* > C_F, \text{ then} \\ C_S - C_F &> C_S - C_F^* > C_S^* - C_F^* \end{aligned}$$

Thus, the entire term is positive indicating an increase in knowledge relative to the baseline, costless case under symmetry. *Q.E.D.* ■

Table 1: Parameter Values		
Linear Demand Intercept	A	8
Linear Demand Slope	B	1
Discount Rate facing firms	$1/(1+r)$	1/1.08
Rate of increase of the factor price index	δ	0.7
Productivity of R&D Investment	a	3
Productivity of Absorptive Capacity	γ	4
Unit cost of Innovative R&D Spending	c_x	2
Fixed Costs	f	0.1
Rate of Decrease in Marginal Costs	η	0.3

Table 2: Summary of Analytical Results for Increases in b

Case/Variable	R&D	Rate of Innovation	Concentration	Welfare
Costless Baseline	↓	No change	No change	↑ slightly
Abs Cap. Baseline	↑	↑	↓	↑
Costless Full Model	↓	↓	?	**?**/↓
Abs Cap. Full Model	↑	↑	↓	↑

Table 3: Percentage Changes in Welfare with Spillovers
(Percentage Represents Change in Surplus from No Spillovers,
$b = 0.0$, to High Spillovers, $b = 0.8$)

Case	Lead Firm Surplus	Follower Firm Surplus	Consumer Surplus	Total
Costless Baseline	7.7%	26.7%	0.8%	6.5%
Abs Cap. Baseline	2.3%	35.0%	6.8%	8.8%
Costless Full Model	**-15.2%**	**8.9%**	**-16.8%**	**-13.2%**
Abs Cap. Full Model	**-5.7%**	**46.4%**	**7.9%**	**8.1%**

Figure 1: Costless Case
(b=0.5, a=0.5, Xj=4.0)

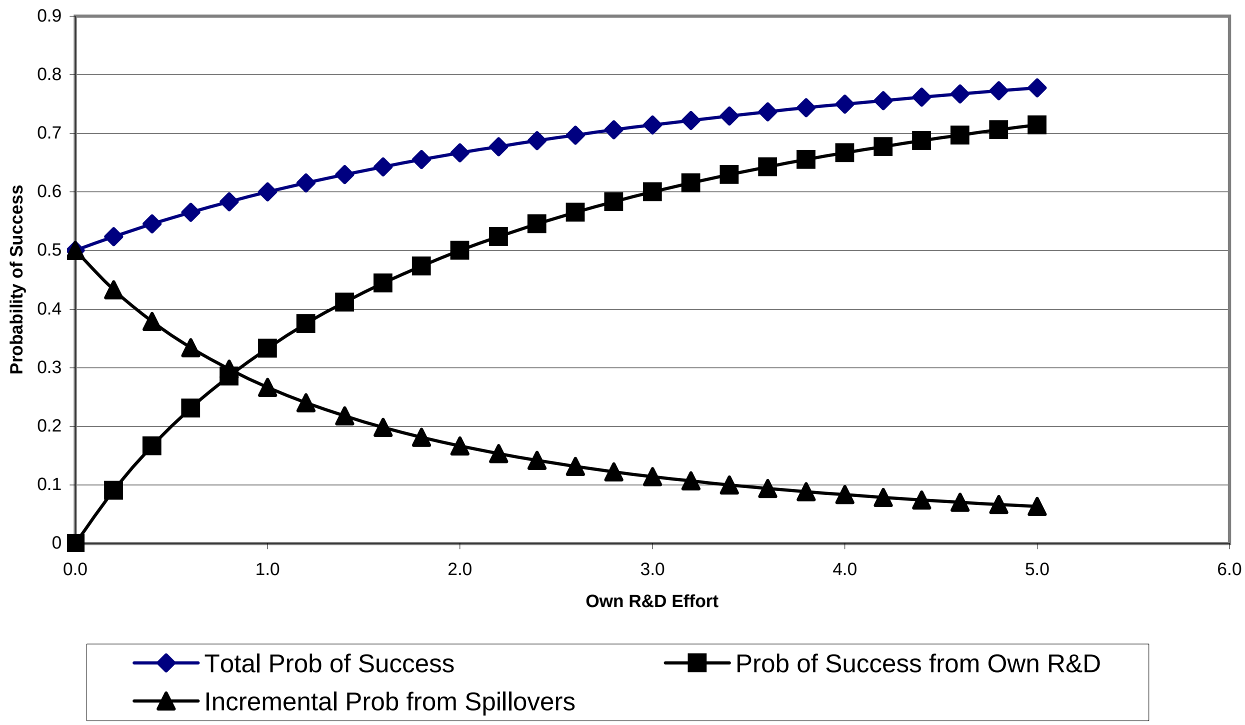

Figure 2: Absorptive Capacity Case
(b=0.5, gamma=5, a=0.5, Xj=4.0)

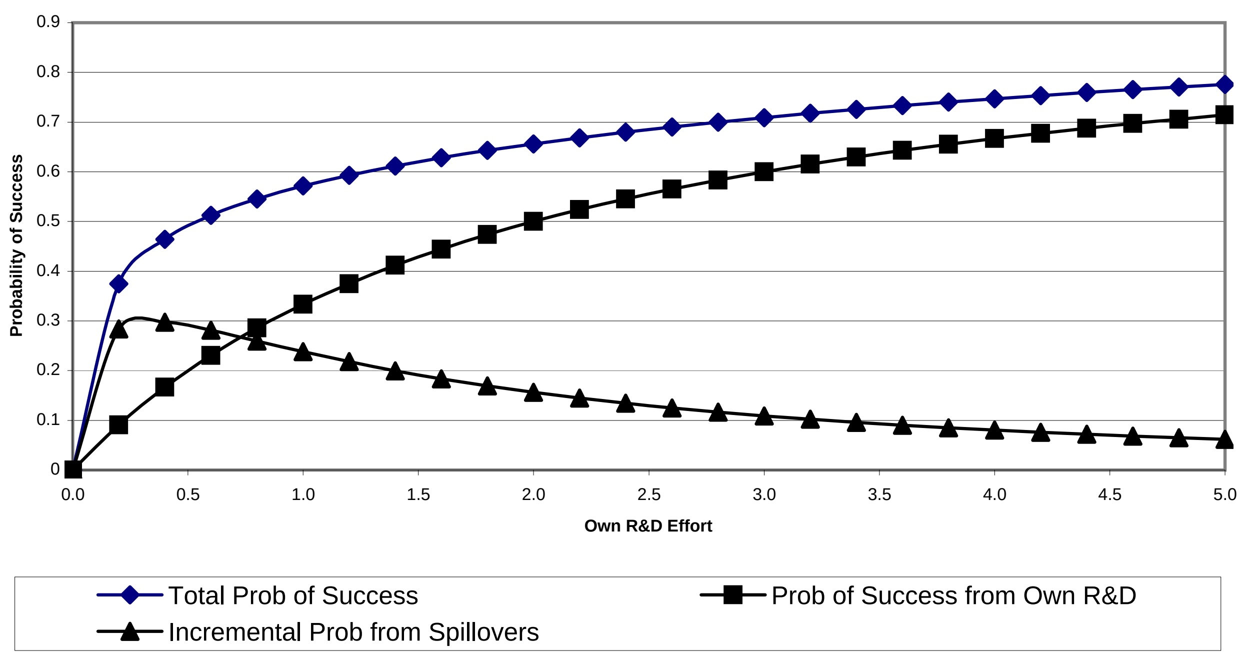

Figure 3: Value Function in Duopoly

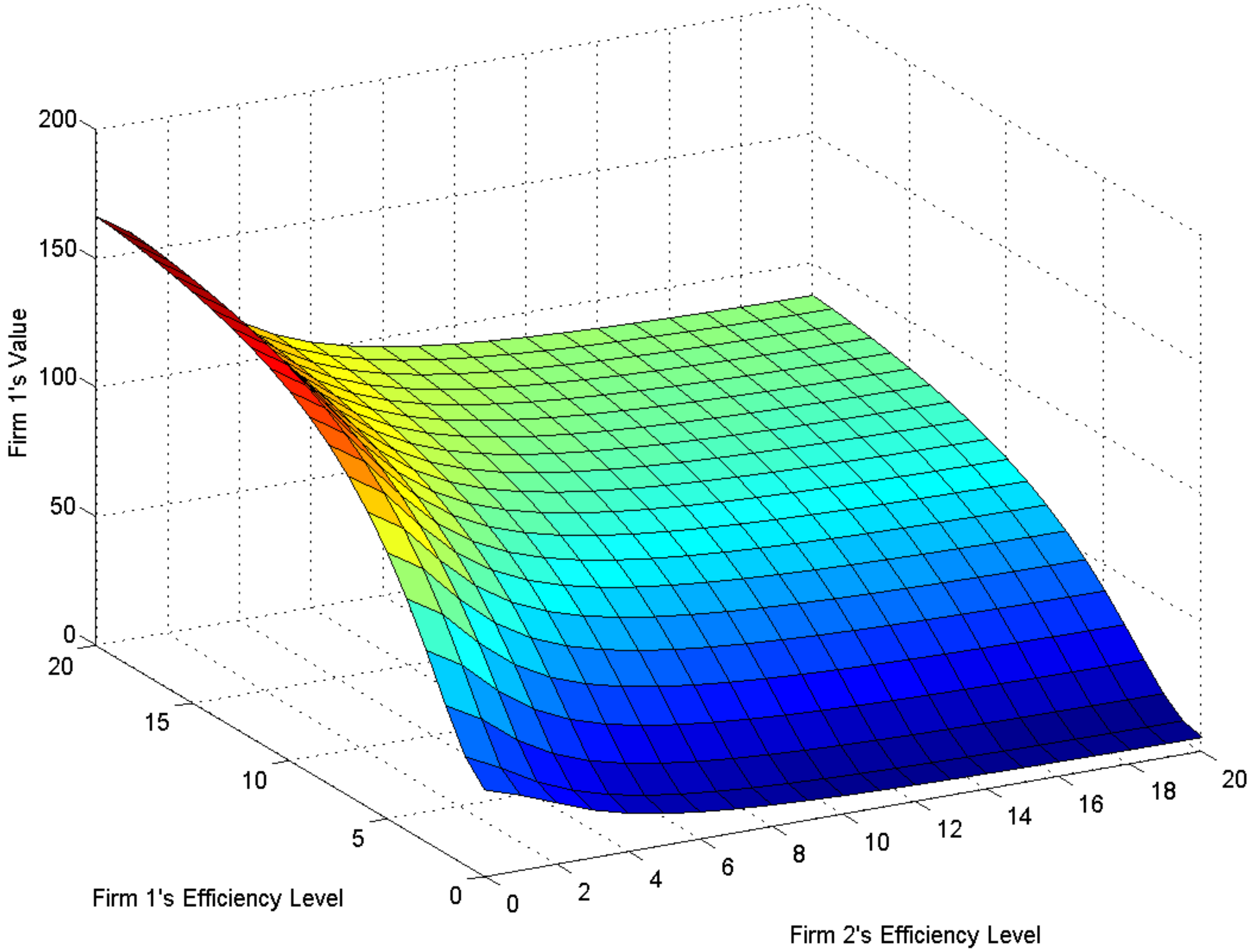

Figure 4: Firm 1's R&D Investment

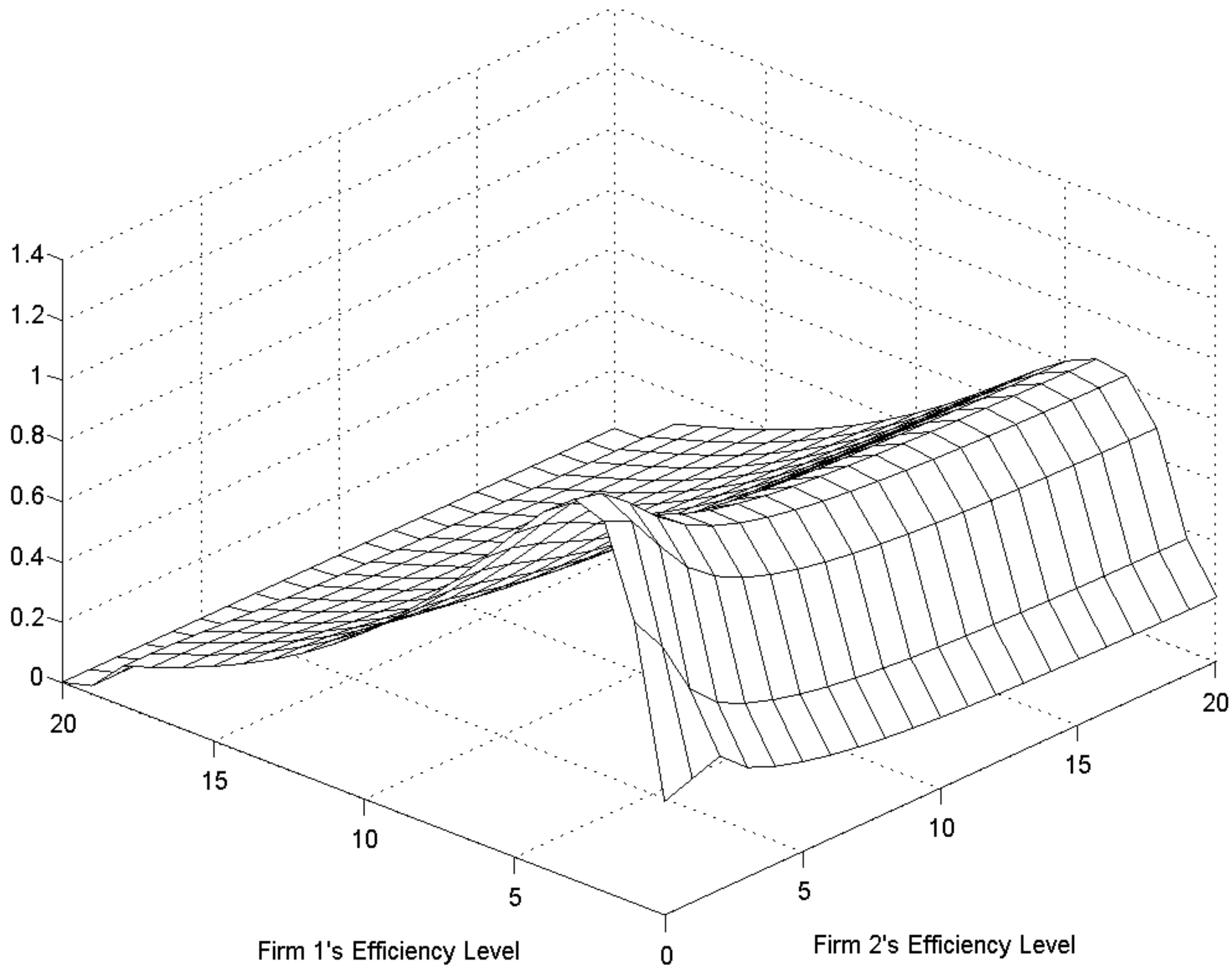

Figure 5: Baseline Costless Case

Changes in Mean Values of R&D, Rate of Innovation, Firm Values, and C1 Concentration Ratio

(Solid Line is Leader Firm and Dashed Line is Follower Firm)

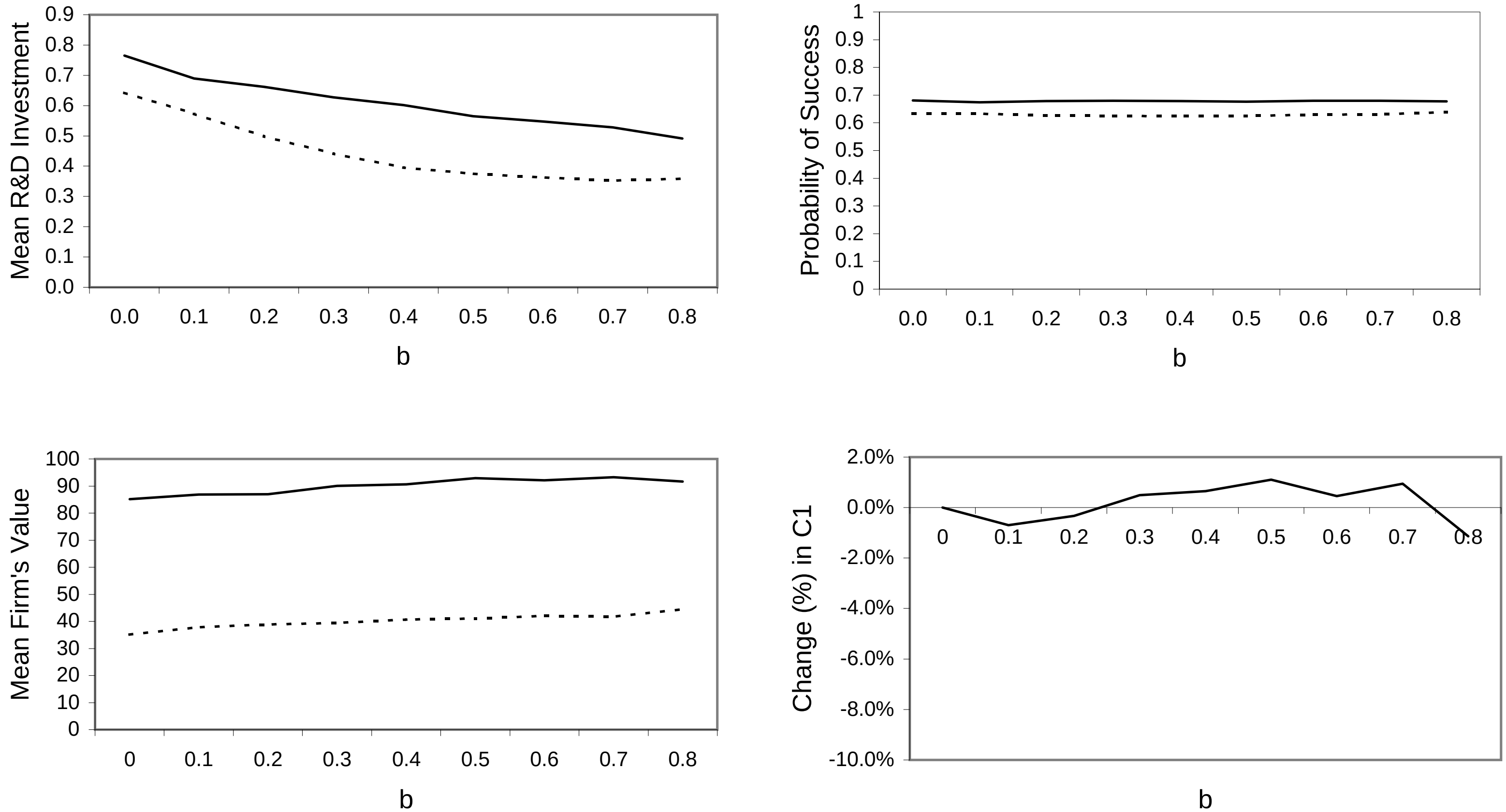

Figure 6: Costless, Baseline Case
Impact of Spillovers on Distribution of the Market Structure

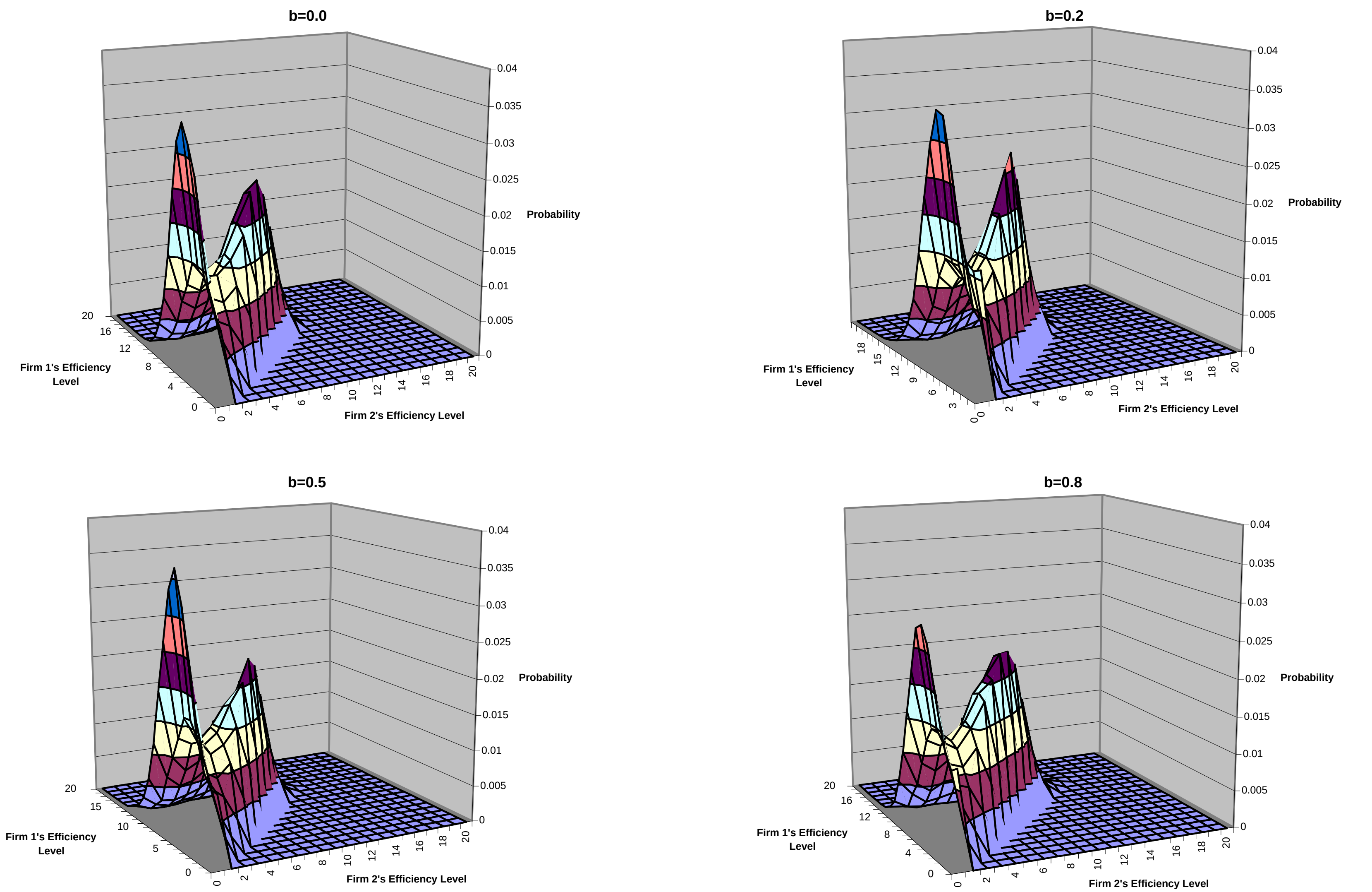

Figure 7: Costless Case, Full Model

Changes in Mean Values of R&D, Rate of Innovation, Firm Values, and C1 Concentration Ratio

(Solid Line is Leader Firm and Dashed Line is Follower Firm)

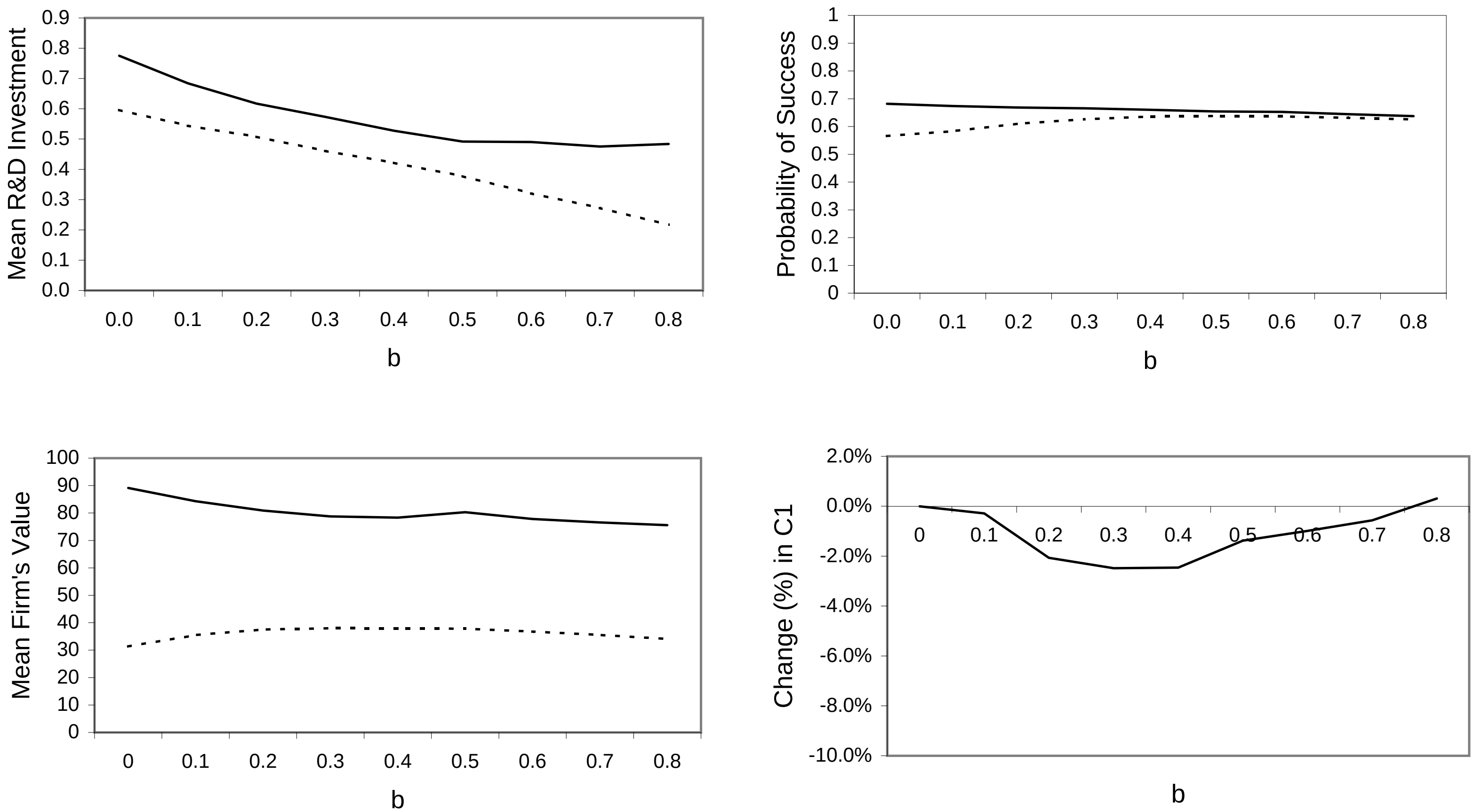

Figure 8: Costless, Full Model
Impact of Spillovers on Distribution of the Market Structure

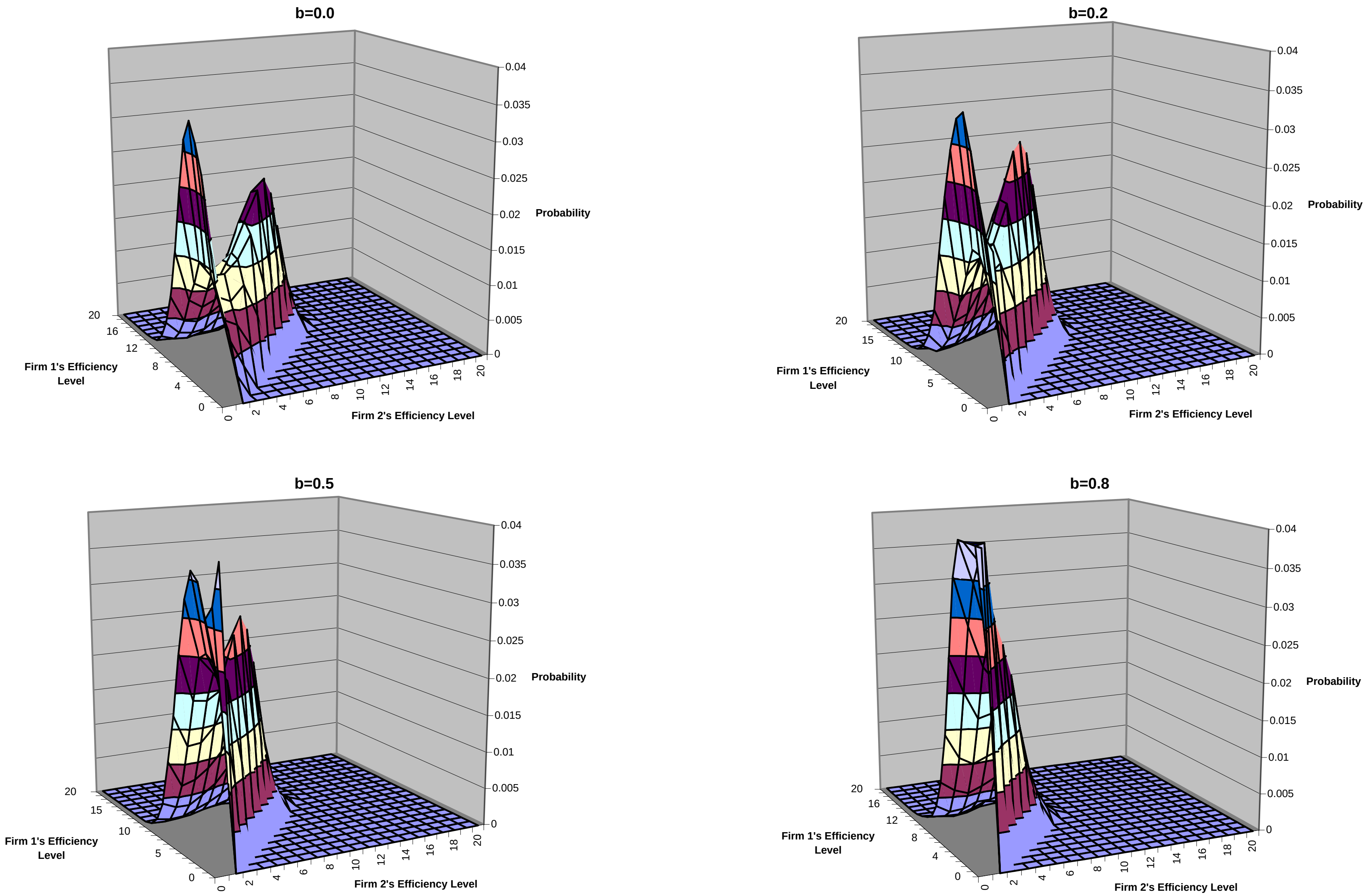

Figure 9: Baseline Absorptive Capacity Case

Changes in Mean Values of R&D, Rate of Innovation, Firm Values, and C1 Concentration Ratio

(Solid Line is Leader Firm and Dashed Line is Follower Firm)

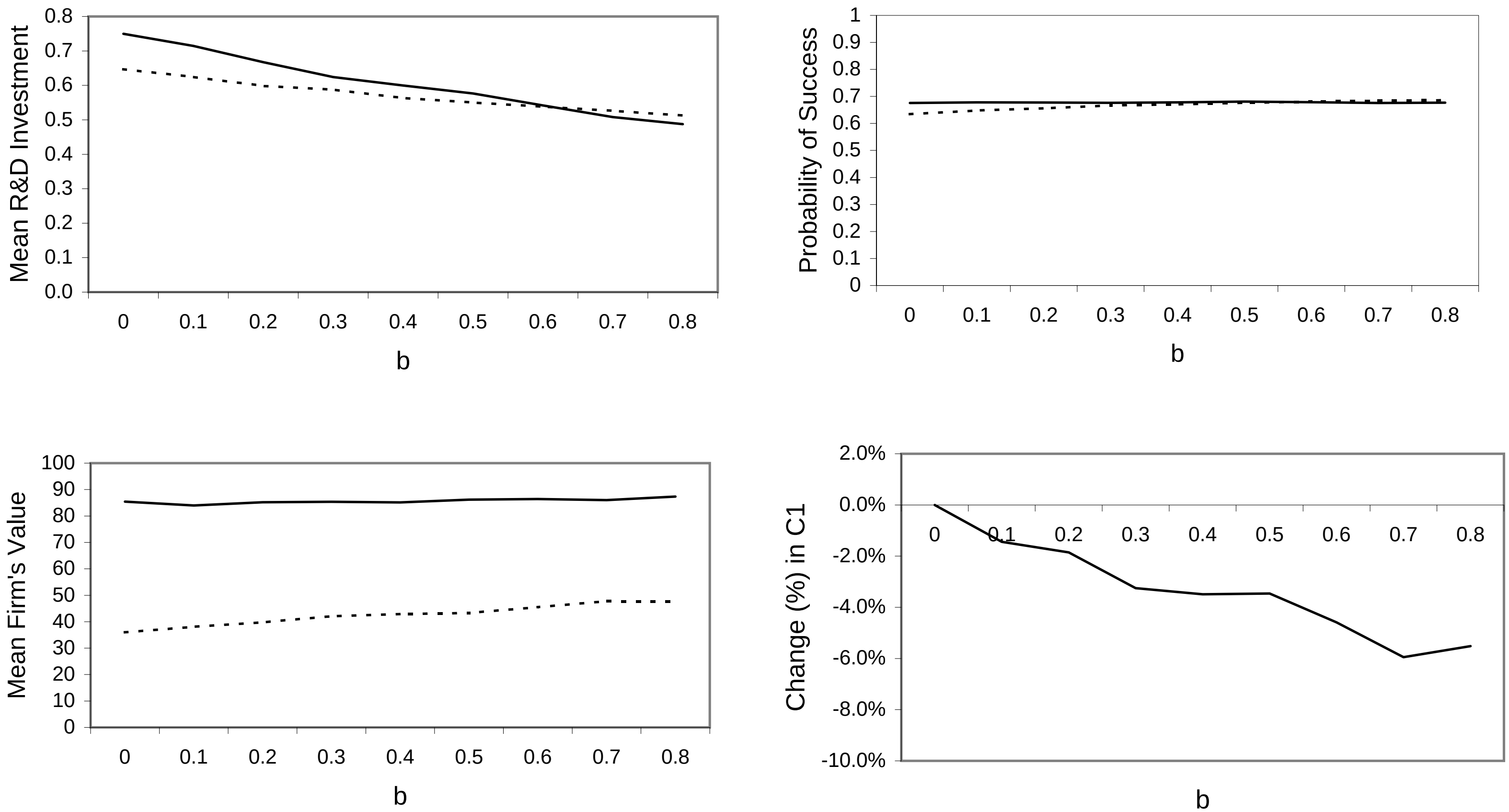

Figure 10: Absorptive Capacity, Baseline Case
Impact of Spillovers on Distribution of the Market Structure

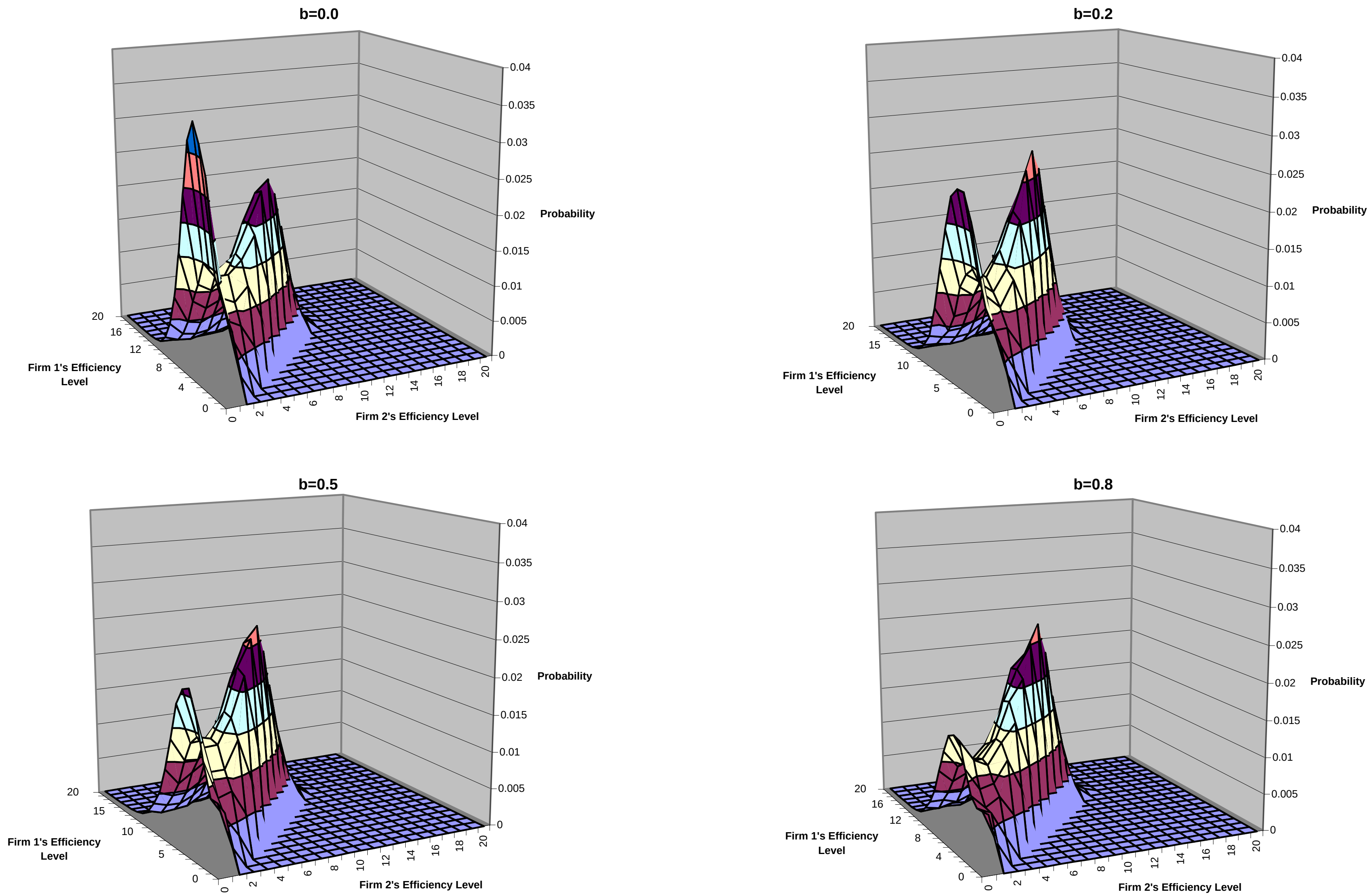

Figure 11: Absorptive Capacity Case, Full Model

Changes in Mean Values of R&D, Rate of Innovation, Firm Values, and C1 Concentration Ratio

(Solid Line is Leader Firm and Dashed Line is Follower Firm)

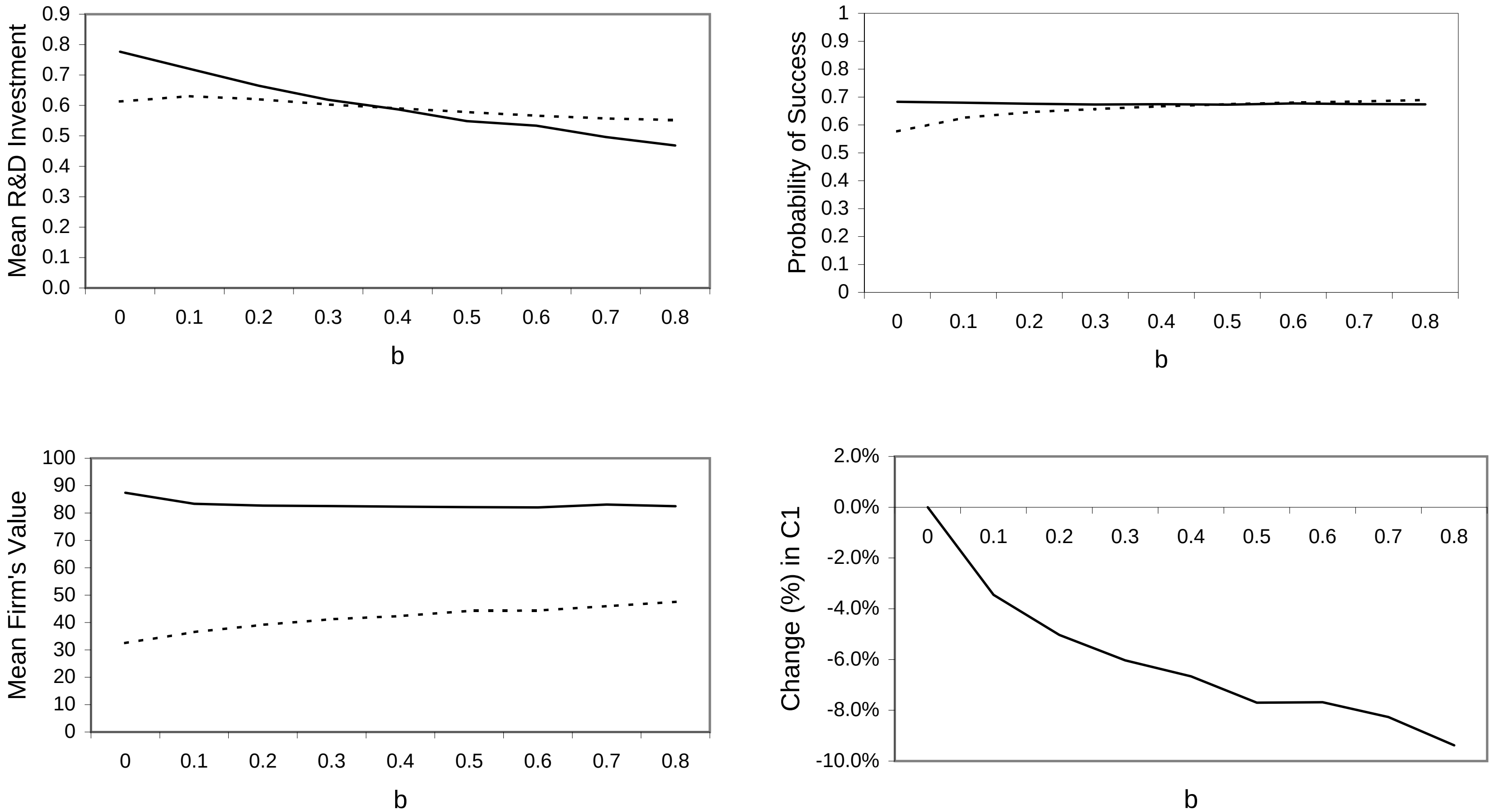

Figure 12: Absorptive Capacity, Full Model
Impact of Spillovers on Distribution of the Market Structure