

# Dynamic price competition with capacity constraints and strategic buyers

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## Abstract

We analyze a dynamic durable good oligopoly model where sellers are capacity constrained over the length of the game. Buyers act strategically in the market, knowing that their purchases may affect future prices. The model is examined when there is one and multiple buyers. Sellers choose their capacities at the start of the game. We find that there are only mixed strategy equilibria. Buyers may split orders, preferring to buy a unit from both high and low price sellers to buying all units from the low price seller. Sellers enjoy a rent above the amount needed to satisfy the market demand that the other seller cannot meet. Buyers would like to commit not to buy in the future or hire agents with instructions to always buy from the lowest priced seller. Further, sellers' market shares tend to be asymmetric with high probability, even though they are *ex ante* identical.

*JEL numbers:* D4, L1

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# 1 Introduction

In many durable goods markets, sellers who have market power and intertemporal capacity constraints face strategic buyers who make purchases over time. There may be a single buyer, as in the case of a government that purchases military equipment or awards construction projects, such as for bridges, roads, or airports, and chooses among the offers of a few large available suppliers. Or, there may be a small number of large buyers, such as in the case of airline companies that order aircraft or that of shipping companies that order cruise ships, where the supply could come only from a small number of large, specialized companies.<sup>1</sup> The capacity constraint may be due to the production technology: a construction company undertaking to build a highway today may not have enough engineers or machinery available to compete for an additional large project tomorrow, given that the projects take a long time to complete; a similar constraint is faced by an aircraft builder that accepts an order for a large number of aircraft. Or, the capacity constraint may simply correspond to the flow of a resource that cannot exceed some level: thus, if a supplier receives a large order today, he will be constrained on what he can offer in the future. This effect may be indirect, if the resource is a necessary ingredient for a final product (as often in the case of pharmaceuticals). More generally, cases like the ones mentioned above suggest a need to study dynamic oligopolistic price competition for durable goods under capacity constraints, when buyers are also strategic. Although this topic is both important and interesting, it has not been treated yet in the literature.

To obtain some first insights into the problem, consider the following simple setting. Take two sellers of some homogeneous product, say aircraft, to fix ideas. Each seller cannot supply more than a given number of aircraft over two periods. Suppose that there is only one large buyer in this market, this may be the Defense Department of some country, with a demand that exceeds the capacity of each seller but not that of both sellers combined. Let the period one prices be lower

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<sup>1</sup>Anton and Yao (1990) provide a critical survey of the empirical literature on competition in defense procurement - see also Burnett and Kovacic (1989) for an evaluation of relevant policies. In an empirical study of the defense market, Greer and Liao (1986, p.1259) find that “the aerospace industry’s capacity utilization rate, which measures propensity to compete, has a significant impact on the variation of defense business profitability and on the cost of acquiring major weapon systems under dual-source competition”. Ghemawat and McGahan (1998) show that order backlogs, that is, the inability of manufacturers to supply products at the time the buyers want them, is important in the U.S. large turbine generator industry and affects firms’ strategic pricing decisions. Likewise, production may take significant time intervals in several industries: e.g., for large cruise ships, it can take three years to build a single ship and an additional two years or more to produce another one of the same type. Jofre-Bonet and Pesendorfer (2003) estimate a dynamic procurement auction game for highway construction in California - they find that, due to contractors’ capacity constraints, previously won uncompleted contracts reduce the probability of winning further contracts.

for one seller than the other. Then, if the buyer's purchases exhaust the capacity of the low price seller, only the other seller will remain active in the second period and, unconstrained from any competition, he will charge the monopoly price. A number of questions arise. Anticipating such behavior, how should the buyer behave? Should he split his orders in the first period, in order to preserve competition in the future, or should he get the best deal today? Given the buyer's possible incentives to split orders, how will the sellers behave in equilibrium? Should sellers price in a way that would induce the buyer to split or not to split his purchases between the sellers? How do sellers' equilibrium profits compare with the case of only a single pricing stage? Does the buyer have an incentive to commit to not making purchases in the future? Are there incentives for the buyer to vertically integrate with a seller? When there is more than one buyer, are the equilibrium market shares of the (ex ante identical) sellers expected to be symmetric or asymmetric?

We consider a set of simple dynamic models with the following key features. There are two incumbent sellers who choose their capacities and a large number of potential sellers who can enter and choose their capacities after the incumbents. Capacity choices are, thus, endogenized and determine how much a firm can produce over the entire game. Next, sellers set first-period prices and then buyers decide how many units of the durable good they wish to purchase from each seller. The situation is repeated in the second period, given the remaining capacity of the firms; sellers set prices and buyers decide which firm to purchase from. We examine separately the cases of a single buyer (monopsony) and that of two or more buyers (oligopsony).

Our main results are as follows. First, entry is always blockaded - the capacity levels chosen by the incumbent sellers are such that there is no profitable entry by other sellers. Given these capacity levels, a pure strategy subgame perfect equilibrium fails to exist. This is due to a combination of two phenomena. On the one hand, a buyer has an incentive to split his order in the first period if the prices are close, in order to keep strong competition in the second period. This in turn, gives the sellers incentives to raise their prices. On the other hand, if prices get "high," each seller has a unilateral incentive to lower his price, and sell all his capacity. We characterize the mixed strategy equilibrium and show that it has two important properties. Buyers have a strict incentive to split their orders with positive probability: for certain realizations of the equilibrium prices (that do not differ too much) a buyer chooses to buy in the first period from both a high price seller and a low price seller. Further, we also show that the sellers make a positive economic rent above the profits of serving the buyer's residual demand, if the other seller sold all of his units. There are three main implications that follow from this result. First, buyers would like to commit to not make purchases

in the second period, so as to induce strong price competition in the first period (that is, a buyer is hurt when competition takes place in two periods rather than in one). This is consistent with the practice in the airline industry, where airlines have options to buy airplanes in the future. In particular, the common practice of airline companies when they are purchasing aircraft is to order a specific number (to be delivered over 2-4 years) and at the same time to agree on a significant number of “option aircraft”, with these options possible to be exercised over a specified time interval of say 5-7 years. So, airline companies choose not to negotiate frequently with the sellers and place new orders as their needs may increase over time, but instead they negotiate at one time in a way that covers their possible needs over the foreseeable future.<sup>2</sup> Second, a buyer has the incentive to instruct its purchasing agents to always buy from the lowest price firm. This is consistent with many government procurement rules that do not allow discretion to its purchasing officers. In other words, in equilibrium, a buyer is hurt by his ability to behave strategically over the two periods and would like to commit to myopic behavior, if possible. Furthermore, buyers have a strict incentive to vertically integrate with one of the suppliers. Finally, when there are multiple buyers, we find that it is highly likely that sellers’ market shares in both the first period and for the entire game can be quite asymmetric, even though sellers are *ex ante* identical.

This paper studies competition with strategic buyers and sellers under dynamic (that is, intertemporal) capacity constraints. It is broadly related to two literatures. First, the literature on capacity-constrained competition starts with the classic work of Edgeworth (1897) who shows that competition may lead to the “nonexistence” of a price equilibrium or, as is sometimes described, price “cycles”. Subsequent work that has studied pricing under capacity constraints includes Beckman (1965), Levitan and Shubik (1972), Osborne and Pitchik (1986) and Dasgupta and Maskin (1986b).<sup>3</sup> Other papers have studied the choice of capacities in anticipation of oligopoly competi-

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<sup>2</sup>This is common practice in the industry. For example, in 2002 EasyJet signed a contract with Airbus for 120 new A319 aircraft and also for the option to buy, in addition, up to an equal number of such aircraft for (about) the same price. While the agreed aircraft were being gradually delivered, in 2006 EasyJet exercised the option and placed an order for an additional 20 units to account for projected growth, with delivery set between then and 2008. Similarly, an order was placed in 2006 by GE Commercial Aviation to buy 30 Next generation 737s from Boeing and also to agree for an option for an additional 30 such aircraft.

<sup>3</sup>Kirman and Sobel (1974) prove equilibrium existence in a dynamic oligopoly model with inventories. Gehrig (1990, ch.2) studies non-linear pricing with capacity constrained sellers. Lang and Rosenthal (1991) characterize mixed strategy price equilibria in a game where contractors face increasing cost for each additional unit they supply.

tion<sup>4</sup> and the effect of capacity constraints on collusion.<sup>5</sup> All this work refers to capacity constraints that operate period-by-period, that is, there is a limit on how much can be produced or sold in each period that does *not* depend on past decisions. Dynamic capacity constraints, the focus of our paper, have received much less attention in the literature. Griesmer and Shubik (1963), with prices in a discrete set and Dudey (1992), allowing prices to vary continuously, study games where capacity-constrained duopolists face a finite sequence of identical buyers with unit demands. Under certain conditions, the equilibrium has sellers maximizing their joint profits. Ghemawat (1997, ch.2) and Ghemawat and McGaham (1998) characterize mixed strategy equilibria in a two-period duopoly (with one seller having initially half of the capacity of the other). Garcia, Reitzes, and Stacchetti (2001) examine hydro-electric plants that can use their capacity (water reservoir) or save it for use in a later period. Dudey (2006) presents conditions so that a Bertrand outcome is consistent with capacities chosen by the sellers before the buyers arrive. In the above mentioned papers, demand is modeled as static and independent across periods. The key distinguishing feature of our work is that the buyers (and not just the sellers) are strategic and the evolution of capacities across periods depends on the actions of both sides of the market.<sup>6</sup>

Second, the present paper is related to the body of research where both buyers and sellers are large and strategic.<sup>7</sup> In particular, it is related to other work that examines when a buyer influences the degree of competition among (potential) suppliers, as in the context of “split awards” and “dual-sourcing”. Rob (1986) studies procurement contracts that allow selection of an efficient supplier, while providing incentives for product development. Anton and Yao (1987, 1989, and 1992) consider models where a buyer can buy either from one seller or split his order and buy from two sellers, who have strictly convex cost functions. They find conditions under which a buyer will split his order

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<sup>4</sup>See e.g. Dixit (1980), Spulber (1981), Kreps and Scheinkman (1984), Davidson and Deneckere (1986), and Allen, Deneckere, Faith, and Kovenock (2000) to mention a few.

<sup>5</sup>See e.g. Brock and Scheinkman (1985), Lambson (1987), and Compte, Jenny and Rey (2002).

<sup>6</sup>In recent work, Bhaskar (2001) shows that, by acting strategically, a buyer can increase his net surplus when sellers are capacity constrained. In his model, however, there is a single buyer who has unit demand in each period for a perishable good, and so “order splitting” cannot be studied: the buyer can choose not to buy in a given period, thus receiving zero value in that period, but gets future units at lower prices. Our model allows both for a single and for multiple buyers. Buyers view the good as *durable* (receive value over both periods) and may wish to *split* their orders, possibly buying from the more expensive supplier in the first period. Our focus and set of results are, thus, quite different. Equilibrium behavior is such that the buyers get hurt as a result of their strategic behavior because it alters the pricing incentives for the sellers. In our model, in equilibrium a buyer never chooses not to buy any units in the first period.

<sup>7</sup>Aspects of bilateral oligopoly have been studied, among other papers, in Horn and Wolinsky (1988), Dobson and Waterson (1997), Hendricks and McAfee (2000) and Inderst and Wey (2003).

and characterize seemingly collusive equilibria. Inderst (2006) examines a model similar to the work of Anton and Yao with multiple buyers. He demonstrates that having multiple buyers gives buyers a larger incentive to split their orders across sellers. Related studies on dual-sourcing are offered by Riordan and Sappington (1987) and Demski, Sappington and Spiller (1987). Our work differs in two important ways. The intertemporal links are at the heart of our analysis: the key issue is how purchasing decisions today affect the sellers' remaining capacities tomorrow. In contrast, the work mentioned above focuses on static issues and relies on cost asymmetries. Strategic purchases from competing sellers and a single buyer in a dynamic setting are also studied under "learning curve" effects; see e.g. Cabral and Riordan (1994) and Lewis and Yildirim (2002, and 2005 for switching costs). In our case, by buying from one seller a buyer makes that seller less competitive in the following period (and in fact inactive, when the seller is left with no capacity) - in the learning curve case, the more a buyer buys from a seller, the more competitive he makes that seller, as his unit cost decreases.<sup>8</sup>

The remainder of the paper is organized as follows. The model is set up in Section 2. Section 3 characterizes the equilibrium with one buyer and discusses a number of implications of the equilibrium properties. The duopsony case is presented in Section 4 - subsequently, the analysis is also generalized to the case of multiple buyers. We conclude in Section 5. Proofs not required for the continuity of the presentation are relegated to an Appendix.

## 2 The model

Buyers and sellers interact over two periods. We examine two market structures on the buyers' side, one where there is a single buyer (monopsony) and the other where there are two buyers (duopsony); we also demonstrate how our results generalize when there are more than two buyers. There are two identical incumbent sellers and many identical potential entrants on the seller side of the market. The product is perfectly homogeneous and perfectly durable over the lifetime of the model. All sellers and buyers have a common discount factor  $\delta$ .

Each buyer values each of the first two units  $V$  in *each* period that he has the unit and a third unit at  $V_3$  in only period 2. Thus, for the first two units that a buyer buys in period 1, he gets

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<sup>8</sup>In Bergemann and Välimäki (1996), sellers set prices and a buyer chooses which seller to purchase from, affecting how competitive each seller could be in subsequent periods. The action there comes from experimentation, not from capacities. Strategic competition with capacity constraints is also present in Yanelle's (1997) model of financial intermediation.

consumption value  $V$  in each period. We assume that  $V \geq V_3 > 0$ .<sup>9</sup>

At the start of the game the incumbent sellers simultaneously choose their capacities. The potential entrants observe the incumbents' capacity choices and then simultaneously choose whether to enter and their capacity choice if they enter. We assume that the cost of capacity for any seller is small,  $\varepsilon$ , but positive. The marginal cost of production is 0 if total sales across both selling periods one and two do not exceed capacity and infinite otherwise.<sup>10</sup> The capacity choice is the maximum that the seller can produce over the two periods. Thus, each seller has capacity at the beginning of the second period equal to his initial capacity less the units he sold in the first period.

Throughout the analysis, we will examine subgames where the capacity choice at the start of the game for each incumbent seller is equal to  $3N - 1$ , where  $N$  is the number of buyers (e.g. in monopsony each seller has 2 units and in duopsony each seller has 5 units) and that entry is always blockaded. We demonstrate that these are the equilibrium capacity choices when we work via backward induction in the analysis.

In each period, each of the sellers sets a per unit price for his available units of capacity.<sup>11</sup> Each buyer chooses how many units he wants to purchase from each seller at the price specified, as long as the seller has enough capacity. If the demand by buyers is greater than a seller's capacity, then they are rationed. The rationing rule that we use is that each buyer is equally likely to get his order filled. The rationed buyers can buy from the other seller as many units as they want.<sup>12</sup> We assume that sellers commit to their prices one period at a time and that all information is common knowledge and symmetric. We are looking for symmetric subgame perfect equilibria of the game.

Let us now discuss why we have adopted this modelling strategy. We analyze a dynamic bilateral

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<sup>9</sup>To clarify, the maximum gross value that a buyer could obtain over both periods and evaluated at the beginning of the first period is equal to  $2V(1+\delta) + \delta V_3$ . Our specification is consistent with growing demand. Note that, in general, the first and second units could have different values (say  $V_1 \geq V_2$ ). Also, we could allow the demand of the third unit to be random. It is straightforward to introduce either of these cases in the model, with no qualitative change in the results, only at the cost of some additional notation.

<sup>10</sup>It will be clear that if the marginal cost of producing an *additional* unit above the initial capacity level, 2 for monoposony, is sufficiently high, then the results still hold. Also, fixed costs would not change the results as long as fixed costs are below a seller's equilibrium profit. If the fixed costs were above a seller's equilibrium profit, then the firm would not enter.

<sup>11</sup>We focus on the core case where each seller sets a simple unit price, that is competition when there is no price discrimination among buyers or among units. The flavor of our results would be the same if discriminatory pricing was allowed with multiple buyers.

<sup>12</sup>Our results would not change qualitatively if the sellers could choose which buyer to ration, as long as each buyer has a positive probability of being rationed.



oligopoly game, where all players are “large” and are therefore expected to have market power. In such cases, one wants the model to reflect the possibility that each player can exercise some market power. By allowing the sellers to make price offers and the buyers to choose how many units to accept from each seller, all players have market power in our model. It follows that quantities and prices evolve from the first period to the second jointly determined by the strategies adopted by the buyers and the sellers. If, instead, we allowed the buyers to make price offers, then the buyers would have all the market power and the price would be zero. This would not be realistic, particularly in the case when there are at least as many buyers as sellers.<sup>13</sup> In fact, anticipating such a scenario, sellers would not be willing to pay even an infinitesimal entry cost and, thus, such a market would never open.<sup>14</sup> There are further advantages of this modeling strategy. First, it makes the results easy to compare between the monopsony and the oligopsony cases. Second, it makes our results more easily comparable with other papers in the literature, in particular the ones mentioned in the Introduction with intertemporal capacity constraints, where the prices are indeed set by the sellers. Third, there may be agency (moral hazard) considerations that contribute to why in practice we typically see the sellers making offers.<sup>15</sup> We note, that in any games with multiple players on both sides of the market there are many possible game forms. We have chosen a natural one as a starting point to examine the issues that we are investigating.

The interpretation of the timing of the game is immediate in case the sellers’ supply comes from an existing stock (either units that have been produced at an earlier time, or some natural resource that the firm controls). One simple way to understand the timing in the case where production

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<sup>13</sup>Inderst (2006) demonstrates that giving multiple buyers the right to make offers generates interesting results due to the convexity of the sellers’ cost functions for *all* units. In our model of constant marginal costs for all units where a seller has capacity in place, the buyer will always buy a unit at a price of 0 if the buyers can make offers.

<sup>14</sup>Of course, there are other structures that would allow buyers and sellers to each keep part of the market surplus, involving some form of multilateral bargaining. However these appear less robust and more complicated (and in particular, dependent on the exact modeling details) than the structure we have adopted here. In any game where the sellers would have some control over setting prices and the buyers over choosing where to buy from, it is expected that equilibrium behavior would reflect the same qualitative features we emphasize here.

<sup>15</sup>In general we see the sellers making offers, even with a single buyer, like when the Department of Defense (DOD) is purchasing weapon systems. The DOD may do this to solve possible agency problems between the agent running the procurement auction and the DOD. If an agent can propose offers, it is much easier for sellers to bribe the agent to make high offers than if sellers make offers, which can be observed by the regulator. This is because the sellers can bribe the agent to make high offers to each of them, but competition between the sellers would give each seller an incentive to submit a bid to make all the sells and it would be quite difficult for the agent to accept one offer that was much higher than another.

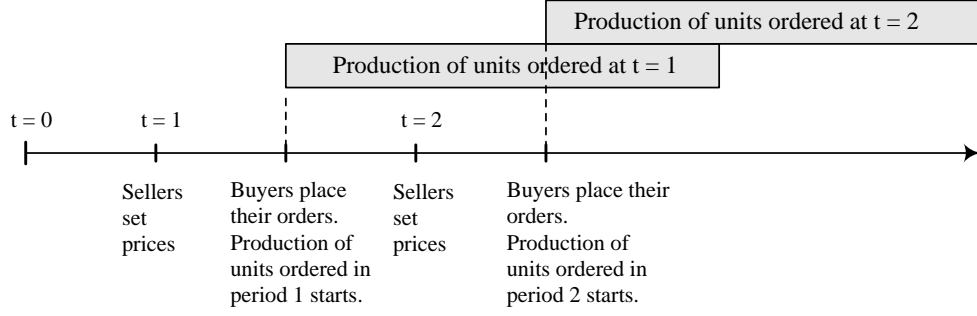


Figure 1: Timing

takes place in every period is illustrated in Figure 1. The idea here is that actual production takes time. Thus, orders placed in period 1 are not completed before period two orders arrive. Since each seller has the capacity to work only on a limited number of units at a time, units ordered in period one restrict how many units could be ordered in period two. In such a case, since our interpretation involves delivery after the current period, the buyers' values specified in the game should be understood as the present values for these future deliveries (and the interpretation of discounting should also be accordingly adjusted).

### 3 Monopsony

We first examine the single buyer case ( $N = 1$ ), that is, monopsony. We are constructing a subgame perfect equilibrium, and thus we work backwards by starting from period 2.

#### 3.1 Second period

There are several cases to consider, depending on how many units the buyer has bought from each seller in period one. We will use, throughout the paper, the convention of calling a seller with  $i$  units of remaining capacity seller  $i$ .

*Buyer bought two units in period 1.* If the buyer bought a unit from each of the sellers in period 1, then the price in period 2 is 0 due to Bertrand competition. If the buyer bought both units from the same firm, then the other firm would be a monopolist in period 2 and charge  $V_3$ . Thus, period 2 equilibrium profit of a seller that has one remaining unit of capacity is 0 and that of a seller with two remaining units of capacity is  $V_3$ .

*Buyer bought one unit in period 1.* In this case, the buyer has demand for two units, one of

the sellers has a capacity of 1 unit, seller 1, while the other has a capacity of 2 units, seller 2. We demonstrate that there is no pure strategy equilibrium in period 2 by the following Lemma.

**Lemma 1** *If the buyer bought one unit in period 1 in the monopsony model, then there is no pure strategy equilibrium in period 2.*

**P roof.** First, notice that the equilibrium cannot involve seller 2 charging a zero price: that seller could increase his profit by raising his price (as seller 1 does not have enough capacity to cover the buyer's entire demand). Thus, seller 1 would also never charge a price of zero. Suppose now that both sellers charged the same positive price. One, if not both, sellers have a positive probability of being rationed. A rationed seller could defect with a slightly lower price and raise his payoff. Suppose that the prices are not equal:  $p_i < p_j \leq V_3$ . Clearly, seller  $i$  could increase his payoff by increasing his price since he still sells the same number of units. Similarly, seller  $i$  can improve his payoff by increasing his price if  $p_i < V_3 \leq p_j$ . Finally, if  $V_3 \leq p_i < p_j$ , seller  $j$  makes 0 profit and can raise his payoff by undercutting firm  $i$ 's price. ■

The fact that there is no pure strategy equilibrium here is a case of a more general phenomenon. The next Lemma is a general result for determining the equilibrium payoffs no matter how many buyers there are, when one of the sellers cannot cover the market in period 2 (while the other seller has enough capacity to cover the market). We note that throughout the paper all profit levels are gross of the capacity costs of  $\varepsilon$  per unit of capacity.

**Lemma 2** *Suppose that in period 2 the buyers have values for  $B$  units and the capacity of the low-capacity seller is  $C$ , with  $C < B$ . Then: (i) No pure strategy equilibrium exists. (ii) There is a unique mixed strategy equilibrium, where the high-capacity seller's profit is  $V_3(B - C)$  and the low capacity seller's profit is  $C \frac{V_3(B - C)}{B}$ . (iii) The support of the prices is from  $V_3(B - C)/B$  to  $V_3$  with distributions  $F_L(p) = \frac{B}{C} - \frac{V_3(B - C)}{pC}$  for the low and high capacity seller and  $F_H(p) = 1 - \frac{V_3(B - C)}{pB}$  for  $p < V_3$  with a mass point of  $\frac{B - C}{B}$  at  $p = V_3$  for the high capacity seller.*

Note that given the structure of demand and capacity, the situation described here will always be the case whenever we have asymmetric capacities in period 2: the low capacity seller's capacity will be strictly lower than the demand while the high capacity seller's capacity will be at least as high as the demand.<sup>16</sup>

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<sup>16</sup>This Lemma is immediately applicable for any number of buyers, since in period 2 a buyer will always buy from the low price seller.

We obtain two key insights from Lemma 2 that run throughout the paper. The first concerns the calculation of the equilibrium sellers' profits and the second regards the ranking of the sellers' price distributions. The high capacity seller can always guarantee himself a payoff of at least  $V_3(B - C)$ , since he knows that, no matter what the other seller does, he can always charge  $V_3$  and sell at least  $B - C$  units. This is the high-capacity seller's *security profit level*. This is because the low-capacity seller can supply only up to  $C$  of the  $B$  units that the buyer demands and the buyer is willing to pay up to  $V_3$ . This high-capacity seller's security profit puts a lower bound on the price offered in period 2. Given the high-capacity seller can sell at most  $B$  units (that is, the total demand), he will never charge a price below  $V_3(B - C)/B$ , since a lower price would lead to profit less than his security profit. Since the high-capacity seller would never charge a price below  $V_3(B - C)/B$ , this level also puts a lower bound on the price the low-capacity seller would charge and, as that seller has  $C$  units he could possibly sell, his profit is at least  $C \frac{V_3(B - C)}{B}$  his security profit.<sup>17</sup> Competition between the two sellers fixes their profits at their respective security levels.<sup>18</sup>

The second insight deals with the incentives for aggressive pricing. We find that the seller with larger capacity will price *less* aggressively than the seller with smaller capacity in period 2. The larger capacity seller knows that he will make sales even if he is the highest price seller, while the smaller capacity seller makes no sales if he is the highest price seller. So, the low capacity seller always has incentives to price more aggressively. More precisely, the high-capacity seller price distribution first-order stochastically dominates the price distribution of the low capacity seller. This general property has important implications for the quantities sold and the market shares over the entire game.

For the special case of interest here, with a single buyer, we have:<sup>19</sup>

**Lemma 3** *If the buyer bought one unit in period 1 in the monopsony model, then there is a unique mixed strategy equilibrium. Both sellers mix on the interval  $[V_3/2, V_3]$ . Seller 1's price distribution is  $F_1(p) = 2 - \frac{V_3}{p}$ , with an expected profit of  $V_3/2$ . Seller 2's price distribution is  $F_2(p) = 1 - \frac{V_3}{2p}$  for  $p < V_3$ , with a mass of  $1/2$  at price  $V_3$ , and expected profit equal to  $V_3$ . Seller 2's price distribution*

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<sup>17</sup>Note that while  $C \frac{V_3(B - C)}{B}$  is not strictly speaking the "security" profit of the low-capacity seller, it becomes that after one round of elimination of strictly dominated strategies.

<sup>18</sup>Mixed strategy equilibria in related settings are also characterized in Ghemawat and McGahan (1998) for the case where one seller has double the capacity of the other.

<sup>19</sup>We prove this result in Appendix A1. The arguments for the general case, Lemma 2, are similar and, thus, omitted.

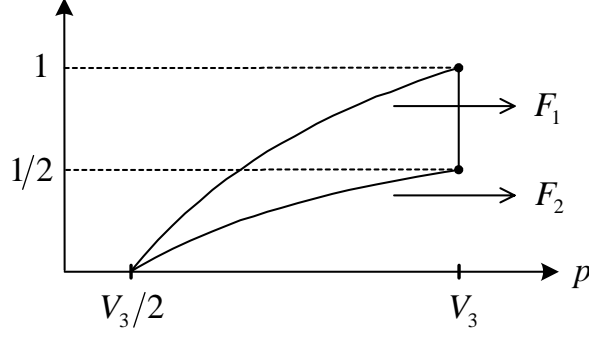


Figure 2: Mixed strategy equilibrium with asymmetric capacities

first order stochastically dominates seller 1's distribution.

Figure 2 illustrates the price distributions in this case.

We now examine the remaining period-two case (subgame).

*Buyer bought no units in period 1.* Each seller enters period 2 with 2 units of capacity, while the buyer demands 3 units. Using argument along the lines used for Lemma 2, each player's expected second-period equilibrium profit will be equal to  $V_3$ , the security profit of each seller. The equilibrium behavior in the second period is now summarized:

**Lemma 4** *Second period competition for a monopsonist falls into one of three categories. (i) If only one seller is active (the rival has zero remaining capacity), that seller sets the monopoly price,  $V_3$ , and extracts the buyer's entire surplus. (ii) If each seller has enough capacity to cover by himself the buyer's demand then the price is zero. (iii) If the buyer's demand exceeds the capacity of one seller but not the aggregate sellers' capacity, then there is no pure strategy equilibrium. In the mixed strategy equilibrium, a seller with two units of capacity has expected profit equal to  $V_3$  and a seller with one unit of capacity has expected profit equal to  $V_3/2$ .*

### 3.2 First period

Now, we go back to period 1. First, we demonstrate that the buyer will always buy two units in equilibrium and that there is no pure strategy equilibrium. We then characterize equilibrium payoffs and discuss the properties of equilibria.

**Proposition 1** *The buyer buys two units in period 1.*

We sketch the proof here; the formal proof is in Appendix A2. First, prices must be positive, since the seller knows that even if he does not sell a unit in period 1, he will make positive profits in period 2. We then show that a buyer will never buy three units in period 1. For the buyer to buy three units, he must buy two units from the low price seller at a positive price and one from the other seller. If he only buys one unit from each seller in period 1, the price for the third unit bought in period 2 is zero due to Bertrand competition; thus, the buyer will never buy three units. We next argue that the price never exceeds a bound such that the buyer prefers buying one unit from each seller as opposed to only one unit from the low price seller. This is because this price is greater than  $\delta V_3$ , which by Lemma 4 is greater than a seller's expected profit in period 2 if he makes no sales in period 1. Thus, two units will always be purchased in any equilibrium. An important implication for this result is that production is always efficient: the buyer always gets units when he needs them.

A feature of the equilibrium is the incentive of the buyer to split his order. This is captured by the following result.

**Lemma 5** *The buyer prefers to buy one unit from each seller as opposed to buying two units from the lowest price seller if the difference in prices is less than  $\delta V_3$ .*

This is an important result. The buyer prefers to split his order if the discounted price differential is lower than the discounted price of a third unit when facing a monopolist. The price of a third unit in the second period is zero when splitting an order in the first period, while if the buyer does not split an order it is  $V_3$ . This value is the expected discounted payoff to a seller of not selling a unit in period 1, which makes sense since the third unit will always be bought by the buyer so there is no efficiency loss.

The next Proposition demonstrates that there is no pure strategy equilibrium (symmetric or asymmetric) in the entire game.

**Proposition 2** *There is no pure strategy equilibrium in the monopsony model.*

**P roof.** See Appendix A3. ■

The non-existence of pure strategy equilibria is common in games with capacity constraints. In our model, the buyer's incentive to split his order as depicted in Lemma 5 when prices are within  $\delta V_3$  of each other creates the non-existence of pure strategy equilibria. If prices are within  $\delta V_3$  of

each other, each seller has an incentive to raise his price and still sell a unit. Thus, prices must be at least  $\delta V_3$  apart. If the gap is greater than  $\delta V_3$ , then the buyer will buy 2 units from the low price seller and none from the high price seller. In this case, the low price seller can raise his price and still sell two units. If the price difference is equal to  $\delta V_3$ , then one of the sellers will deviate from the putative equilibrium. If the low price seller's price is greater than  $\delta V_3/2$ , then the high price seller will undercut the low price seller's price and improve his payoff since the high price seller's profit in the putative equilibrium is  $\delta V_3$ . If the low price is below  $\delta V_3/2$ , then the low price seller can raise his price, since he can guarantee a payoff of  $V_3$  next period if he has no sales this period. If the low price is  $\delta V_3/2$ , then the buyer is indifferent between buying two units from the low price seller and splitting his order. The buyer will either buy two units from the low price seller, split his order, or mix between the two options. It is easy to show that either the low price seller will raise his price, if the buyer splits with positive probability, or the high price seller will lower his price if the buyer buys two units from the low price seller with positive probability.

Thus far, we have proved there is no pure-strategy equilibrium and the buyer always buys two units in period 1. Now we further characterize the (mixed-strategy) symmetric equilibria of the game.<sup>20</sup> First, we prove in Appendix A4 that the sellers' price distributions must be sufficiently wide, so that the buyer will accept in period 1 either 0, 1, or 2 units from a particular buyer. Figure 3 illustrates what can happen with the equilibrium prices distributed on an interval  $[\underline{p}, \bar{p}]$ . The figure demonstrates period 1 acceptances for the buyer as a function of the realizations of the prices of the two sellers. By Proposition 1, two units are always sold. If a seller sets a price between  $\underline{p}$  and  $\underline{p} + \delta V_3$ , he will sell either 1 or 2 units; the other seller will never undercut his price by more than  $\delta V_3$  so the seller will always sell at least one unit and if the other seller's price is greater than  $\delta V_3$  of his price he will sell two units. If the seller sets a price between  $\underline{p} + \delta V_3$  and  $\bar{p} - \delta V_3$  he will sell either 0, 1 or 2 units. If the prices are within  $\delta V_3$  of each other, the buyer will split his order between the sellers (Lemma 5). Otherwise, the buyer will buy two units from the low price seller. Finally, for prices between  $\bar{p} - \delta V_3$  and  $\bar{p}$ , the seller can never sell two units, since the other

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<sup>20</sup>Given that we have well-defined payoffs in each of the period-two subgames, we can guarantee existence of a (symmetric) mixed strategy Nash equilibrium in period-one prices and, consequently, existence of a subgame-perfect equilibrium in the entire game. We can use Theorem 5 in Dasgupta and Maskin (1986a): in the first period, discontinuities in each seller's profit function occur only at a small number of discrete own prices of each seller (even though these points do not have to be only where both sellers set equal prices). Individual profit functions are bounded and weakly lower semicontinuous in own prices and the sum of the profits is upper semicontinuous. Further, application of Theorem 6 establishes symmetry. Section 2 of Dasgupta and Maskin (1986b) illustrates how existence arguments can be applied in capacity-constrained price competition.

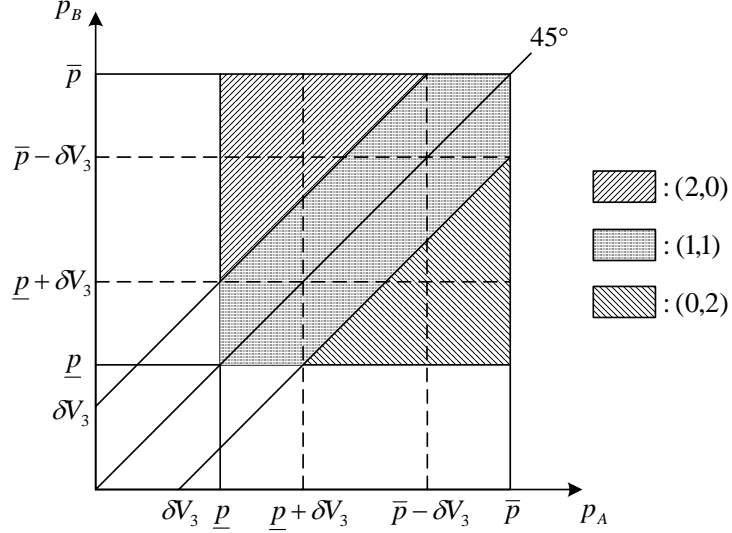


Figure 3: Period 1 acceptances as a function of prices.

seller's price will never be more than  $\delta V_3$  above his price. He will sell 1 unit if the prices are within  $\delta V_3$ , and 0 units otherwise. Since we show that  $\bar{p} - \underline{p} \geq 2\delta V_3$ , we establish the following important result.

**Proposition 3** *In the monopsony model, splitting of orders by the buyer between the two sellers occurs in equilibrium with positive probability: when the difference of the two prices is smaller than  $\delta V_3$ , the buyer buys one unit from each seller.*

In Appendix A4, we also prove that the lowest price,  $\underline{p}$ , offered by the sellers in a mixed strategy equilibrium of the monopsony model is greater than  $\delta V_3$ . It immediately then follows that:

**Proposition 4** *In the monopsony model, the expected profit of each seller is greater than  $\delta V_3$ .*

Thus, in equilibrium, the sellers receive rents above satisfying the residual demand after the buyer bought the other seller's capacity (or the static Bertrand competition),  $\delta V_3$ . Why is this the case? By Lemma 4, a seller knows that, if he makes no sales in period 1, his expected profit is  $\delta V_3$ . This gives a seller the incentive to raise his price above  $\delta V_3$  to take a chance of not making a sale in period 1, since by Lemma 5 a seller knows that even if he has the highest price he will make a sell as long as the price difference is less than  $\delta V_3$ . Since there is no cost of increasing his price above  $\delta V_3$  while there is a potential benefit, the seller can improve his payoff.



### 3.3 Initial capacity choices by sellers

Thus far, we have conducted the analysis in this Section assuming that each seller has 2 units of initial capacity. We now argue that these capacities are indeed the ones chosen in equilibrium by each incumbent and that there is no entry.

If the incumbents have a total of less than four units of capacity, then an entrant can profitably enter, since if there are four or fewer units of capacity among three sellers, the price in period 1 will always be strictly above 0; if each seller had only one unit of capacity, they would never charge a price less than  $\delta V_3$  in period 1, while if one seller had two units of capacity he will never charge a price less than  $\delta V_3/2$ . This creates a positive profit opportunity for an entrant. Entry to increase industry capacity will lower the profit of an incumbent who chose a capacity of one. A market configuration of either four sellers each having one unit of capacity or one seller having two units of capacity and two sellers each having one unit of capacity results in lower prices than when two sellers each have two units of capacity, since sellers with lower levels of capacity price more aggressively as demonstrated in Lemma 3.

On the other hand, if each incumbent chooses two units of capacity an entrant cannot profitably enter the market. This is because of the following argument. First, it is straightforward to demonstrate in any subgame in period 2 the price will equal 0. Given this fact, a firm knows it can make no profits from sales in period 2. Going back to period 1, the equilibrium has all sellers charging price 0, where no seller can profitably deviate by charging a higher price; all the buyer demand for three units can be satisfied by the other two sellers.

Finally, there is no possible way for a seller to improve his profit by increasing his capacity above 2 units. The price for a unit in period 2 is 0, unless the buyer bought two units from a seller with capacity 2. But, this means that the seller with a capacity greater than 2 units will have an equilibrium profit of  $\delta V_3$  less his capacity cost. If the seller with high capacity sold all three units in period 1, then these sales must be at a price of 0, since this would be the price in period 2 when both sellers would have a capacity of at least 1. Thus, the seller's profit falls by adding more than 2 units of capacity and we have:

**Proposition 5** *In the monopsony model, the equilibrium capacity choice by each incumbent is two units and there is no entry.*

### 3.4 Possible actions by the buyer to reduce sellers' rents

As we saw above (Proposition 4), in the equilibrium of the monopsony model each seller's profit exceeds  $\delta V_3$ . Note that our equilibrium is efficient and that the buyer's payment is equal to the total profit of the two sellers.<sup>21</sup> Therefore, in equilibrium the buyer obtains a gross value equal to  $2V(1 + \delta) + \delta V_3$  and pays an amount that exceeds  $2\delta V_3$ . That the equilibrium expected profit is greater than  $\delta V_3$  for each seller is an important property and we further discuss some of its implications in the following subsection. We illustrate three strategies that the buyer can use to reduce his expected payments and still preserve efficiency. First, the buyer benefits if he can commit to make all his purchases at once, effectively making the game collapse into a one-shot interaction. This is equivalent to the buyer having an option to buy units in period 2 at the period 1 prices. Second, we show that the buyer has an incentive to commit to (myopic) period-by-period minimization of his purchase costs. The buyer can accomplish this by hiring an agent and requiring the agent in each period to buy from the lowest price seller. Government procurement often works this way. Third, we demonstrate that the buyer will benefit by acquiring one of the sellers.

These three observations help to demonstrate the fundamental force underlying the equilibrium: due to strategic considerations, the buyer does not always purchase from the lowest price seller when he plans to make further purchases, giving sellers the incentive to raise their prices above the static equilibrium level. As the buyer is "hurt" by acting strategically across the two periods of the game, we show that there are actions he can take (e.g. through some unilateral policy commitments) to effectively change the game. In cases when such actions are possible, we thus identify reasons for which the buyer would like to choose them. We note that the unilateral incentives for a buyer to commit to either buying from the low price seller within the period or, alternatively, to not buying any units after the first period are valid regardless of whether the game form allows the buyers to take an action before or after the sellers choose their capacities. This is because the sellers will choose the same capacity levels in either game form.

Our first observation is:

**Corollary 1** *In the monopsony model, the buyer would like to commit to not buying any units in period 2.*

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<sup>21</sup>Whenever we talk about efficiency, we are ignoring capacity costs which can be made arbitrarily small. Thus, we are talking about efficient allocations net of capacity costs. Clearly, the most efficient capacity choice would be for total industry capacity to equal 3 times the number of buyers.

The equilibrium profit level described in Proposition 4 is larger than in the static equilibrium (when the buyer commits to buying all goods in period 1). This is by the following argument. We found in Lemma 4 that the second-period expected equilibrium profit if no units are sold in period one is  $V_3$ . If all competition took place in one period the sellers' expected payoff would be  $\delta V_3$ , since the strategic situation would be exactly the same as the last period with all sellers having full capacity (and the buyer's valuation for the third unit, as of period 1, equal to  $\delta V_3$ ). Thus, each seller's profit in the one-shot situation would be  $\delta V_3$ . Since the allocation is always efficient, lower seller profits implies a higher buyer profit. Thus, the buyer's profit is higher if he can commit to only buying once.<sup>22</sup>

The behavior described in Corollary 1 would require, of course, some vehicle of commitment that would make future purchases not possible. This is an interesting result and can be viewed as consistent with the practice of airlines placing a large order that often involves the option to purchase some planes in the future at the same price for firm orders placed now. Such behavior is sometimes attributed to economies of scale – our analysis shows that such behavior may emerge for reasons purely having to do with how sellers compete with one another. In particular, it is easy to see that a game where sellers set in period 1 prices for units purchased in periods 1 and 2 is exactly the same as if the game was only played in a single period.<sup>23</sup>

Our second observation is:

**Corollary 2** *The buyer would like to commit to myopic behavior and to make his purchases on the basis of static optimization in each period.*

Suppose that the buyer could commit to behaving myopically (that is, to not behaving strategically across periods). In other words, while valuations are the same as assumed in the model,

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<sup>22</sup>We note that a seller's capacity choice does not change if the buyer commits to only buying in period 1. First, if either seller has a capacity less than two units, then entry will occur and this will lower a seller's profits. Second, if both sellers have a capacity of at least 3, then the price would be 0. Finally, if one seller had a capacity of 3 and the other 2, then using Lemma 2, the expected profit of the seller with 3 units of capacity is  $\delta V_3 - 3\varepsilon$ , which is less than  $\delta V_3 - 2\varepsilon$ , his profit if he has two units of capacity and the other seller has 2 units of capacity. That is, if one seller has 2 units of capacity, then the other seller's equilibrium flow profit is  $\delta V_3$  regardless of whether he has 2 or 3 units of capacity.

<sup>23</sup>It is also easy to see that the buyer would be better-off if he could commit to *reduce* his demand to only two units. By committing to not purchasing a third unit (in any period), the value he obtains gets reduced by  $\delta V_3$ , while his payment gets reduced by an amount strictly higher than that (each seller's equilibrium profit drops from  $\pi > \delta V_3$  to zero). Of course, commitment to such behavior may be difficult: once the initial purchases have been made, the buyer would then have a strict incentive to "remember" his demand for a third unit.

now the buyer does not recognize the link between the periods and views his purchases in each period as a separate problem. Thus, the buyer within each period purchases a unit from the seller that charges the lowest price (as long as this price is below his reservation price). There are two possible ways to generate a pure strategy equilibrium for this model. First, in equilibrium each seller charges  $\delta V_3/2$  in the first period and the buyer purchases two units from one or the other seller. Then, the seller that has not sold his two units in the first period, charges a price of  $V_3$  in the second period and the buyer purchases one unit from that seller. Thus, total payment for the buyer is  $2\delta V_3$ . To establish that this is an equilibrium, first observe that the buyer indeed behaves optimally, *on a period by period basis*. Furthermore, neither seller has a profitable deviation. In period 1, if a seller lowers his price below  $\delta V_3/2$ , he then sells both units and obtains a lower profit. If he raises his price, he sells no units in the first period but obtains a profit equal to  $V_3$  in the second.<sup>24</sup>

The possibility that the buyer may split his order (he is indifferent, given the myopia assumption, between splitting his order and not splitting) may be viewed as a weakness of the equilibrium described just above. This can be easily addressed in the second possible way to establish an equilibrium, if we introduce a smallest unit of account,  $\Delta$ . The equilibrium has one seller charging  $\delta V_3/2 - \Delta$  and the other seller charging  $\delta V_3/2$  in the first period and the buyer buying two units from the low price seller. The seller that made no sales in the first period, charges  $V_3$  in the second period and the buyer purchases one unit from that seller. Thus, total payment in present value terms for the buyer is  $2\delta V_3 - 2\Delta$ .<sup>25</sup> Clearly, the equilibrium payoffs are essentially the same under both approaches.

What drives this result is that now a seller knows that if he sets a higher price than his rival he cannot sell a unit in period one (and can only obtain a second period profit of  $V_3$ ). The above comparison may provide a rationale for purchasing policies that large buyers have in place that require purchasing at each situation strictly from the lowest price seller. In particular, a government

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<sup>24</sup>As in the case when the buyer buys only in period 1, the capacity choices will still be 2 units each. First, note that any lower capacity will induce entry while if each seller had 3 units of capacity, the price would drop to 0. Second, if one seller has 3 units of capacity and the other 2 units, then the buyer would only buy 3 units from a seller with capacity of 3 if the price was 0, since this will be the price in period 2.

<sup>25</sup>To establish that this is an equilibrium, first observe that the buyer behaves optimally. Second, neither seller has a profitable deviation. Clearly, no seller can gain from lowering his price. If the low price seller raises his price to  $\delta V_3/2$ , the equilibrium can have the buyer splitting his order (as the prices would be equal) and lowering this seller's profit. Thus, there are no profitable deviations. We note that the equilibrium with a strategic buyer is *not* affected if there is a smallest unit of account, since the buyer will want to split his order as long as the gap between the two prices is less than  $\delta V_3$ .

may often assume the role of such a large buyer. It is often observed that, even when faced with scenarios like the one examined here, governments require that purchasing agents to absolutely buy from the low-price supplier, with no attention paid to the future implications of these purchasing decisions. While there may be other reasons for such a commitment policy (such as preventing corruption and bribes for government agents), our analysis suggests that by “tying its hands” and committing to purchase from the seller that sets the lowest current price, the government manages to obtain a lower purchasing cost across the entire purchasing horizon. We find, in other words, that delegation to such a purchasing agent that maximizes in a myopic way is beneficial, since it ends up intensifying competition among sellers.<sup>26</sup>

Suppose that a buyer can acquire a seller after he has chosen his capacity. A further implication of Proposition 4 is:

**Corollary 3** *In the monopsony model, the buyer has a strict incentive to acquire one of the sellers, that is, to become vertically integrated.*

This result is based on the following calculations. By vertically integrating, and paying the equilibrium profit of a seller when there is no integration,  $\pi$ , the total price that the buyer will pay is  $\pi + \delta V_3$  since the other seller would charge the monopoly price  $V_3$  for a third unit (sold in period 2). This total payment is strictly less than the total expected payment ( $2\pi$ ) that the buyer would otherwise make in equilibrium. Thus, even though the other seller will be a monopolist, the buyer’s payments are lower, since the seller that has not participated in the vertical integration has now lower profits.<sup>27</sup> We note here that a seller cannot gain by buying any units from the other seller. Since this is equal to the buyer’s valuation of the unit, there are no gains of trade between sellers.<sup>28</sup>

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<sup>26</sup>Strategic delegation has been also shown to be (unilaterally) beneficial by providing commitment to some modified market behavior in other settings (see e.g. Fershtman and Judd, 1987). In our case, the key is the separation from the subsequent period and the commitment to myopia.

<sup>27</sup>The capacity choices are still 2 units each. This is by the following reasoning. First, note that a seller is better-off being purchased than one who is not. Second, if one of the sellers chooses only one unit of capacity and the other two, the buyer will buy the larger capacity seller. Otherwise, if each seller chooses two units of capacity, the buyer could mix between which seller to purchase.

<sup>28</sup>In our analysis, sellers use linear prices. It should not be too surprising that the application of nonlinear pricing would lead to different results. This case would be relevant when a seller can price the sale of one unit separately from the sale of two units. In an earlier working paper, Biglaiser and Vettas (2004), we show that with a monopsonist under nonlinear pricing, there are unique pure strategy equilibrium payoffs with each seller making profit equal to  $\delta V_3$ . In period 1, both sellers charge  $\delta V_3$  for both a single unit and two units and the buyer buys either two or three units. The ability of each seller to price each of his units separately

## 4 Duopsony

Now, we study strategic issues raised when there are two buyers. Each buyer has the same demand as in the monopsony case, and each seller has now a capacity of five units. We will show that, in equilibrium, the same economic phenomena occur in duopsony as in monopsony and demonstrate that the analysis can be generalized for any number of buyers.

### 4.1 Second period

We first consider equilibrium behavior in the period-two subgames. Mixed strategy equilibria arise unless either both sellers can cover the market or one seller is a monopolist. The construction of equilibria is similar to that for the monopsony case and based on Lemma 2.<sup>29</sup> The key results from the period 2 analysis, required for our subsequent analysis of period 1, are summarized in the following Lemma.

**Lemma 6** *In the second period of a duopsony (i) the expected payoff for a seller with full capacity is  $V_3$  regardless of the other seller's capacity; (ii) if one seller has sold 4 units and the other seller no units, then the expected price is  $EP_2 > V_3/2$ ; (iii) if each seller has enough capacity to satisfy the market demand, then each seller's profit is 0.*

Part (ii) of the Lemma follows because, if demand is for two units, the high capacity seller, and subsequently the low capacity seller, will never charge a price less than  $V_3/2$  in the mixed strategy equilibrium. The equilibrium prices belong to  $[V_3/2, V_3]$ , with distribution functions  $F_1(p_1) = 2 - V_3/p_1$  for the seller with 1 unit of capacity and  $F_5(p_5) = 1 - V_3/2p_5$  with a mass point of  $1/2$  at  $V_3$ , for the seller with 5 units of capacity. Clearly, the expected equilibrium price is above  $V_3/2$ .

### 4.2 First period

Now, we go to period 1. As in the monopsony case, there will be no pure strategy equilibrium. At a mixed strategy equilibrium, we show that the expected profit for each seller is again strictly greater than  $\delta V_3$ , the profit in the one-shot game, and that the buyers split their orders between

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changes the strategic incentives, intensifies price competition and allows us to derive an equilibrium where the sellers make no positive rents.

<sup>29</sup>The calculation details are available from the authors upon request.

sellers with positive probability. As in the monopsony model, a buyer will always buy at least two units. That is, the sellers will always choose prices such that buyers prefer to buy in period 1 than to wait till period 2 for the first two units. Each seller chooses to have five units of capacity at the beginning of period 1.

The equilibrium behavior in the first period is as follows. Like in the one buyer case, sellers mix with respect to the prices they charge. Now that we have two buyers, these buyers also have to mix given certain realizations of the prices: when the prices set by the sellers turn out (at the realization of the mixed strategies) to be different enough from each other, each buyer buys two units from the low price seller. But when these prices are not too far apart, each buyer plays a mixed strategy, randomizing between buying two units from the low price seller and splitting orders buying one unit from each seller.

Suppose that the prices offered by the sellers (as realizations of the mixed strategies played) are  $P_L$  and  $P_H$ , where  $P_L \leq P_H$ . As in the monopsony case, we can demonstrate that a buyer will buy only two units in period 1. Let  $\alpha$  be the probability in the symmetric equilibrium that a buyer will buy two units from the low price seller and  $(1 - \alpha)$  the probability that he will split his orders between the two sellers. It is never the case in equilibrium that a buyer will buy two units from the high price seller, since if a buyer splits his order between the two sellers, he will guarantee a price of 0 in period 2 for a third unit, since each seller will have at least two units of capacity and market demand is two. A buyer is indifferent between buying two units from the low price seller and splitting his order if

$$-2P_L - \alpha \delta EP_2 = -P_L - P_H$$

where, by Lemma 6,  $EP_2 > V_3/2$  is the expected price that the buyer will pay in period 2 if one seller sells 4 units in period 1 and the other seller sells 0. Solving for  $\alpha$  we obtain

$$\alpha = \frac{P_H - P_L}{\delta EP_2}. \quad (1)$$

Note that if the prices are equal ( $P_H = P_L$ ), the buyer always splits his order and if the price difference is greater than  $\delta EP_2$ , then the buyer always buys both units from the low price seller. This is because the value of splitting an order to obtain the unit in period 2 for a price of 0 instead of  $EP_2$  is not worth the additional cost of buying a unit from the high price seller.

The profit of a seller if he charges a lower price,  $P$ , than the other seller can be calculated as

$$\pi = \alpha^2 [4P + \delta V_3/2] + 2\alpha(1 - \alpha) [3P] + (1 - \alpha)^2 [2P]. \quad (2)$$

The first term is his profit if both buyers buy 2 units from him plus his expected profit in period 2,  $\delta V_3/2$ . The second term is his profit if one buyer buys two units from him and the other buyer buys one unit. This occurs with probability  $2\alpha(1 - \alpha)$  and by Lemma 6 the seller makes no profit in period 2. The final term is the seller's profit if both buyers buy one unit from each seller. The seller's profit is zero in period 2 in this case. Equation (2) simplifies to

$$\pi = \alpha^2 \delta V_3 / 2 + 2\alpha P + 2P. \quad (3)$$

Differentiating (3) with respect to  $P$ , we obtain

$$\frac{\partial \pi}{\partial P} = 2 \left[ 1 - \frac{P}{\delta EP_2} \right] + \alpha \left[ 2 - \frac{\delta V_3}{\delta EP_2} \right].$$

The important property to note is that this profit is strictly increasing in  $P$  for any  $P$  below  $\delta EP_2$  (using the fact that  $EP_2 > V_3/2$ , by Lemma 6).

Now we turn to the profit of the seller whose price is the higher of the two prices set. The profit of this high price seller is

$$\pi = \alpha^2 \delta V_3 + 2\alpha(1 - \alpha) [P] + (1 - \alpha)^2 [2P]. \quad (4)$$

The first term is his profit if both buyers buy 2 units from the other seller, using Lemma 6, his expected profit in period 2 is  $V_3$ . The second term is his profit if one buyer buys two units from the other seller and the other seller splits his order. This occurs with probability  $2\alpha(1 - \alpha)$  and by Lemma 6 the seller makes no profit in period 2. The final term is the seller's profit if both buyers buy one unit from each seller. The seller's period 2 profit is zero in this case.

Equation (4) simplifies to

$$\pi = \alpha^2 \delta V_3 - 2\alpha P + 2P. \quad (5)$$

Differentiating (5) with respect to  $P$  we have

$$\frac{\partial \pi}{\partial P} = 2 \left[ 1 - \alpha - \frac{P}{\delta EP_2} + \alpha \frac{\delta V_3}{\delta EP_2} \right].$$

The important property of this first order condition is that for any price  $P$  below  $\delta V_3/2$ , profit is strictly increasing in  $P$  for any  $\alpha \in [0, 1]$ .

Thus, we have just established that a seller's profit is always increasing in price, whether he is the low or high price seller, for any price less than  $\delta V_3/2$ . Since the low price seller has expected sales of greater than 2 units, a seller's expected profit is greater than  $\delta V_3$ , which is the one-shot equilibrium profit level.



Further, we need to argue that, if the low price seller charges a price less than  $\delta V_3/2$ , he would not want to have his fifth unit of capacity purchased in period 1 and would improve his payoff by raising his price. This is the case, since his payoff from not having this unit accepted,  $\delta EP_2$  is greater than the revenue from selling the unit at a price below  $\delta V_3/2$ .

Finally, as with the monopsony case, there will be no entry in the game and each incumbent will choose five units of capacity. To see this, note that if the total incumbent capacity is less than ten units, then an entrant can profitably enter the market (and in this case the profit of each incumbent would drop). Furthermore, it is straightforward to show that if an incumbent has more than five units, additional units will either go unsold or be sold for a price of 0. Finally, no entrant can profitably enter the market if each incumbent has 5 units, since the period 1 subgame has each firm charging 0. Thus, we have the following result

**Proposition 6** *In the duopsony model, sellers' expected profits are greater than in the corresponding static model, and buyers split their orders across sellers with positive probability.*

We observe a few things regarding the duopsony model. First, it is straightforward to show that there is no pure strategy equilibrium, symmetric or asymmetric. This is because at least one of the sellers always will have an incentive to deviate from any pair of putative equilibrium prices. Second, if prices are sufficiently close, within  $\delta EP_2$ , then buyers must split their orders with positive probability,  $0 < \alpha < 1$ . Note that this property differs from the monopsony model, where the buyer split his order with probability 1 if the prices were within  $\delta V_3$  (see Lemma 5), which is strictly greater than  $\delta EP_2$ . The reason why we see more order splitting for a *given* price differential in the monopsony model is that with two buyers each buyer is providing a “public good” to the other buyer by splitting his order, since if at least one buyer splits his order in period one, each seller will have enough capacity to satisfy the market in period 2.

As in the monopsony model, the following two corollaries follow directly from this Proposition.<sup>30</sup>

**Corollary 4** *The buyers have a strict incentive to commit not to buy in the future or to have a policy of only buying from the lowest price seller.*

Suppose that one of the buyers has a policy of either not buying in the future or buying only

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<sup>30</sup>As in the monopsony case, it is straightforward to demonstrate that the capacity choices will still be 5 units by each seller, whether one or both buyers adopt either a strategy to only buy from the lowest priced seller or to only buy in period 1.

from the lowest price seller. The other buyer would also adopt the same policy, since either policy will essentially make the game a one period game and limit each seller's expected profit to  $\delta V_3$ . This will limit the buyer's expected payment to  $\delta V_3$ , which is less than his expected payment in the two-period game. This follows from the result that in the static model, each seller's expected equilibrium profit is equal to  $\delta V_3$ .

**Corollary 5** *A buyer has a unilateral incentive to vertically integrate with a seller.*

Suppose that a buyer unilaterally buys a seller. This increases his expected profit because a buyer can buy a seller, by paying the equilibrium profit. This is the buyer's expected cost. He can then satisfy his demand for 3 units and have two extra units of supply and obtain an additional positive profit by selling to the other buyer.

It is interesting to note that the realized equilibrium market shares of the firms may be quite asymmetric both in terms of the quantity sold by each firm and their profits (or revenues): one seller may sell four units in period 1 and possibly one in period 2 for an expected profit of at least  $2.5\delta V_3$  (since both the first and second period prices are at least  $\delta V_3/2$ ), while the other sells at most two units in period 2 with an expected profit of  $\delta V_3$ . This result, which follows from there being only mixed strategy equilibria (due to the buyers' incentives to split orders), is complementary to other studies of asymmetries in the literature - see e.g. Saloner (1987), Gabszewicz and Poddar (1997) and Besanko and Doraszelski (2004). In these studies, capacity asymmetries are typically due to the selling firms' incentives to invest strategically in anticipation of a market competition stage. In contrast, in our analysis, the asymmetries are due to the fact that the buyers are large players and choose strategically which firms they should purchase from - the fact that they cannot fully coordinate their behavior in equilibrium, but need to mix, gives rise to asymmetric sellers' market shares.

### 4.3 Multiple buyers

Our analysis can be generalized to the case when there are  $N > 2$  buyers, each with the same demand as in the preceding analysis. Each incumbent seller now will choose a capacity of  $3N - 1$  units. The arguments needed to establish this capacity level for each seller are similar to those in the monopsony and duopsony cases.<sup>31</sup>

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<sup>31</sup>As before, a lower incumbent capacity level would invite entry (which would lower each incumbent's profit), whereas a higher level for each would lead to zero equilibrium profits.

As a benchmark, suppose that competition was taking place in a single period. Following arguments familiar from our analysis thus far, in this one-shot pricing game there is no pure strategy equilibrium. In the unique mixed strategy equilibrium, each seller makes an expected profit of  $\delta V_3$ , his security profit, and the support of the pricing distribution is  $\left[\frac{\delta V_3}{3N-1}, \delta V_3\right]$ , where the lowest price seller sells  $3N - 1$  units and the highest price seller sells a single unit.<sup>32</sup> Note that, again, this equilibrium is efficient and that each of the  $N$  buyers will have a gross surplus equal to  $2V(1 + \delta) + \delta V_3$  independently of  $N$  and make a net expected payment of  $2\delta V_3/N$ .<sup>33</sup>

In our model with  $N$  buyers and competition taking place over two periods, we can support equilibrium behavior that is similar to that in duopsony case. Again, sellers play a mixed strategy with respect to their prices and each buyer buys two units in the first period (and one in the second period): when the prices turn out to be far apart enough, were far is appropriately derived, all buyers buy in period 1 from the low price seller, while if prices are close enough each buyer randomizes between splitting his order and buying both units from the low price seller. Again seller expected profit will be above  $\delta V_3$ .

As before, we cannot have an equilibrium where all buyers buy always from the low price seller, regardless of the price difference. Suppose that all the other  $N - 1$  buyers are expected to buy from the low price seller. Then, if the period 1 prices are close enough to each other,  $P_H - P_L \leq \delta V_3$ , a buyer has a unilateral incentive to split his order instead of buying both units from the low price seller. If he splits his order, then each seller will have enough residual capacity in period 2 (that is, at least  $N$  units) and, as a result, the buyer will get his period 2 unit for free. If he buys both units from the low price seller in period 1, he and all other buyers will have to pay  $V_3$  in period 2 for a unit. Another way to view this is that if one buyer splits his order with probability 1, then all other buyers buy 2 units from the low price seller. Thus, the buyer who splits his order provides a public good for all the other buyers.

In the characterization of the equilibrium for the two buyer model above, we have assumed that buyers did not coordinate their strategies. We have demonstrated that this induced positive rents above  $\delta V_3$  for the sellers. We observe that even if buyers could perfectly coordinate their purchases,

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<sup>32</sup>In equilibrium, each seller prices according to the distribution function  $F(p) = 1 - \frac{\delta V_3}{p(3N-1)}$  with support  $\left[\frac{\delta V_3}{3N-1}, \delta V_3\right]$ .

<sup>33</sup>Note that the net surplus for each buyer is strictly increasing with  $N$ . The reason why each buyer benefits by having more buyers in the market is that the sellers then become more aggressive, since if they are the low price seller they can sell more units and obtain higher profits.

sellers will get positive rents above  $\delta V_3$ . This is because it is not credible for all buyers to refuse with probability one to not buy from a higher price seller if the price differential is small enough. This gives a seller an incentive to deviate from any equilibrium where expected profits are less than or equal to  $\delta V_3$ . Thus, our results hold even if buyers can use correlated strategies when deciding on which seller to make purchases from in period 1. Finally, as in the one and the two buyer cases, the buyers would benefit if they could commit to not buying in the future and to have a policy to always buy from the current lowest price supplier.

## 5 Conclusion

Capacity constraints play an important role in oligopolistic competition. In this paper, we have examined markets where both sellers and buyers act strategically. Sellers have intertemporal capacity constraints, as well as the power to set prices. Buyers decide which sellers to buy from, taking into consideration that their current purchasing decisions affect the intensity of sellers' competition in the future. Capacity constraints imply that a pure strategy equilibrium fails to exist. Instead, sellers play a mixed strategy with respect to their pricing, and the buyers may split their orders. Importantly, we find that the sellers enjoy higher profits than what they would have in an one-shot interaction (or, equivalently, the competitive profit from satisfying residual demand). The buyers are hurt, in equilibrium, by their ability to behave strategically over the two periods, since this behavior allows the sellers to increase their prices above their rival's and still sell their products. Thus, the buyers have a strict incentive to commit not to buy in the future, or to commit to myopic, period-by-period maximization (perhaps by delegating purchasing decisions to agents), as well as to vertically integrate with one of the sellers.

While we have tried to keep the model as simple as possible, our qualitative results appear robust to possible alternative formulations. Perhaps the most important ones refer to how the capacity constraints function. In the model, if a seller sells one unit today, his available capacity decreases tomorrow by exactly one unit. In some of the cases for which our analysis is relevant, like the ones mentioned in the Introduction, it may be that the capacity decreases by less than one unit, in particular, if we adopt the view that each unit takes time to build and, thus, occupies the firm's production capacity for a certain time interval. Similarly, instead of the unit cost jumping to infinity once capacity is reached, in some cases it may be that the unit cost increases in a smoother way: cost curves that are convex enough function in a way similar to capacity constraints. We believe the spirit of our main results is valid under such modifications, as long as the crucial property that,

by purchasing a unit from a seller, you decrease this seller's ability to supply in the subsequent periods holds.

This is, to our knowledge, one of the first papers that considers capacity constraints and buyers' strategic behavior in a dynamic setting. A number of extensions are open for future work. While non trivial, these present theoretical interest and, at the same time, may make the analysis more directly relevant for certain markets. First, one may wish to examine the case where the products offered by the two sellers are differentiated. Is there a distortion because buyers strategically purchase products different from their most preferred ones, simply with the purpose of intensifying competition in the future? A second interesting extension is allowing for non-linear prices. In an earlier working paper, Biglaiser and Vettas (2004), we have worked out the equilibrium in the monopsony case and found that the sellers obtained no additional rents. Interestingly, we found that the sellers still retain rents in the duopsony case. It is an open question what happens when there are more than two buyers and capacity is endogenous. In particular, there may be no pure strategy equilibrium in the capacity levels of firms. Finally, in our model, sellers set prices in each period. Alternative price determination formulations are also possible. For instance, sellers may be able to make their prices dependent on the buyers' purchasing behavior e.g. by offering a lower price to a buyer that has not purchased in the past - or does not currently purchase - a unit from the rival seller. Our general setting may allow us to examine such "loyalty discount" discriminatory schemes.

# Appendix

## Appendix A1: Proof of Lemma 3.

First, we argue that the players choose prices in the interval  $\left[\frac{V_3}{2}, V_3\right]$ . Suppose that seller 2, asked a price  $p$  less than  $V_3/2$ . If seller 1 charges a price less than  $p$ , then seller 2 will sell 1 unit, while if seller 1 charges a price higher than  $p$ , seller 2 sells 2 units. Seller 2 could improve his payoff no matter what prices seller 1 asks by asking for  $V_3 - \epsilon$  for  $\epsilon$  very small and selling at least one unit for sure, since  $V_3 - \epsilon > 2p$ . Since seller 2 will charge a price of at least  $V_3/2$ , then so will seller 1; otherwise, seller 1 could increase his price and still guarantee a sell of 1 unit. Thus, both sellers charge at least  $V_3/2$ . Now, we argue that price will be no more than  $V_3$ . Take the highest price  $p$  offered in equilibrium greater than  $V_3$ . First, assume that there is not a mass point by both sellers at this price. This offer will never be accepted by the buyer, since he will always buy the second unit from the lower price seller and his valuation for a third unit is  $V_3 < p$ . The seller could always improve his payoff by charging a positive price less than  $V_3/2$ . Second, if there is a mass point by both sellers, then at least one of them is rationed with positive probability and a seller can slightly undercut his price and improve his payoff. Thus, all prices will be between  $V_3/2$  and  $V_3$ .

Now, we argue that the expected equilibrium period 2 payoffs are  $V_3/2$  for seller 1 and  $V_3$  for seller 2. Given that the equilibrium prices are between  $V_3/2$  and  $V_3$ , we know that the profit for seller 1 is at least  $V_3/2$  and for seller 2 at least  $V_3$ . First, we argue that it can never be the case that *both* sellers will have an atom at the highest price  $p_H$ ; later we further show that seller 2 will have a mass point at  $p_H$ . If both did, then there is a positive probability of a seller being rationed, and a seller could improve his payoff by slightly lowering his price. Thus, a seller asking  $p_H$  knows that he will be the highest price seller. If he is seller 1 he will not make a sell, while if he is seller 2 he will make a sell of one unit. If seller 2 charges  $p_H$  he knows that his payoff will be  $p_H$ , thus  $p_H$  must equal  $V_3$ . If the lowest price offered in equilibrium,  $p_L$ , were greater than  $V_3/2$ , then seller 2 could improve his payoff by offering  $p_L - \epsilon > V_3/2$ , with the buyer buying two units from the seller and, thus, improve his payoff above  $V_3$ . Thus, the lowest price is  $V_3/2$ . Since both sellers must offer this price, seller 1's expected payoff must be  $V_3/2$ .

We now derive the equilibrium price distributions. Let  $F_i$  be the distribution of seller  $i$ 's price offers. Seller 1's price distribution is then determined by indifference for seller 2:

$$p[F_1(p) + 2(1 - F_1(p))] = V_3, \quad (\text{A1.1})$$

since seller 2's expected payoff is  $V_3$  by the earlier argument. Seller 2's payoff is calculated as

follows. When seller 2 charges price  $p$ , then with probability  $F_1(p)$  seller 1's price is lower and seller 2 sells one unit, while with probability  $1 - F_1(p)$  seller 1's price is higher and seller 2 sells both his units. Solving equation (A1.1), we obtain:

$$F_1(p) = 2 - \frac{V_3}{p}.$$

Seller 2's price distribution is a little more complicated. For  $p < V_3$ , it is determined by

$$p[1 - F_2(p)] = \frac{V_3}{2}. \quad (\text{A1.2})$$

Seller 1 sells one unit if his price is lower than the rival's and this happens with probability  $1 - F_2(p)$ ; otherwise, he sells no units. This equals seller 1's expected profit  $V_3/2$  by Lemma 3. (A1.2) implies

$$F_2(p) = 1 - \frac{V_3}{2p}.$$

There is a mass of  $1/2$  at price  $V_3$ . Simple arguments establish that the equilibrium pricing distributions must be continuous and the only mass point may be located at  $V_3$  for seller 2.

## Appendix A2: Proof of Proposition 1.

First, we define some notation and buyer payoffs. Then, we proceed to prove the Proposition in a series of Lemmas. Suppose that the prices in period 1 are  $p_H$  and  $p_L$ , with  $p_H \geq p_L$ . Note that pricing in period one could, in principle, be determined via either pure or mixed strategies. In the former case,  $p_H$  and  $p_L$  are the prices set by the two sellers, whereas in the latter these are realizations of the mixed strategies. We use Lemma 4 in computing the payoffs.

The buyer's payoff if he buys one unit from each of the firms in period 1 is

$$W_1 \equiv 2V(1 + \delta) - p_H - p_L + \delta V_3.$$

In this case, the buyer gets one unit for free in the following period by Lemma 4.

The buyer's payoff if he buys both units from firm  $L$  is

$$W_2 \equiv (1 + \delta)2V - 2p_L.$$

In this case, the buyer faces a monopolist and pays  $V_3$  in period 2.

The buyer's expected payoff if he buys only one unit from firm  $L$  is

$$W_3 \equiv (1 + \delta)V - p_L + \delta[V + V_3 - E \min[p_1, p_2] - Ep_2].$$

In the following period, the buyer will buy two additional units. He will pay the lowest price offered in the following period for the second unit and will buy the third unit from the seller who has two units of capacity in period 2, since either he is the low price seller or the other seller has no more capacity.

The buyer's expected payoff if he buys two units from the lowest price seller and one from the highest price seller is

$$W_4 \equiv 2V(1 + \delta) - 2p_L - p_H + \delta V_3.$$

Now, the series of Lemmas. These are numbered (A1-A5) independently from the Lemmas in the main body of the paper.

**Lemma A1** The sellers set strictly positive prices in period 1.

**P roof.** If a seller set a price of 0, then either the buyer would buy two units from that seller or one from each of the sellers. In either case, the seller makes 0 profit. If the buyer buys two units from that seller, then the seller can sell no more in period 2. If the seller sells one unit, then the buyer must have bought one unit from the other seller, since  $W_1 > W_3$  at a 0 price in period 1. A seller could raise his price, sell no units in period 1, and improve his payoff by Lemma 3. ■

It follows directly that:

**Lemma A2** The buyer never buys three units in period 1.

**P roof.** By Lemma A1, both prices are positive. Since  $W_1 > W_4$ , if  $p_L > 0$  then buying two units always dominates buying three units. ■

We now argue in the following three Lemmas that no price will be above  $V + \delta [E \min[p_1, p_2] + Ep_2] \equiv p^C$ , from which the Proposition will be proven.

**Lemma A3** The buyer prefers to buy one unit from each of the sellers instead of only one unit from the low price seller if  $p_H \leq p^C$ . Thus the buyer will not buy any units from a seller charging  $p_H > p^C$ , when  $p_H > p_L$ .

**P roof.** Compare  $W_1$  and  $W_3$ . ■

**Lemma A4** In any equilibrium, each price offered by each seller in period 1 is an offer which results in his selling at least one unit with positive probability.

**P roof.** Let  $\bar{p}$  be the highest price offered in any (possibly mixed strategy) equilibrium by a seller. Suppose that in equilibrium  $\bar{p}$  is never accepted. By Lemma 4 the seller's expected payoff



of making this offer is  $\delta V_3$ . Let  $\underline{p}$  be the lowest price offered in equilibrium by the other seller. By Lemmas 3 and A1, if the difference between the two prices is less than  $\delta V_3$ , then the highest price will always be accepted, as long as it is less than  $p^C$ . By Lemma 1,  $\underline{p} > 0$ . A player offering  $\bar{p}$  can defect and offer a price  $p$  that is the minimum of  $\{\underline{p} + \delta V_3, p^C\}$  and know that it will be accepted, and increase his payoff. Thus, no offer is made that is always rejected. ■

**Lemma A5** In any equilibrium, no seller will offer a price above  $p^C$ .

**P roof.** Let  $\bar{p}_i$  be the highest price offered in any equilibrium by seller  $i$ . Suppose that  $\bar{p}_i \geq \bar{p}_j$ ,  $i \neq j$ , and  $\bar{p}_i > p^C$ . If  $\bar{p}_i > \bar{p}_j$ , then seller  $i$ 's offer will never be accepted by Lemma A3. Then seller  $i$ 's payoff is  $\delta V_3$ . Seller  $i$  can clearly improve his payoff by making an offer of  $\delta V_3 + \epsilon$  (note that  $\delta V_3 + \epsilon < p^C$ ). Suppose now that  $\bar{p}_i = \bar{p}_j \equiv \bar{p}$ . There could not be a mass point at  $\bar{p}$  by each seller, since only one unit will be bought and that seller could increase his payoff by a slight undercut in price. If there is no mass at  $\bar{p}$ , then there is no possibility that the offer will be accepted. But, this contradicts Lemma A4. ■

Thus, we have proved Proposition 1.

### Appendix A3: Proof of Proposition 2.

**P roof.** suppose we have a pure strategy equilibrium with prices  $p_H$  and  $p_L$ , where  $p_H \geq p_L$ . A pure strategy equilibrium could exist only if both sellers offered  $p^C$  and the buyer was purchasing a unit from each seller. At any other price, at least one of the sellers could defect and improve his payoff. To see this, we need to look at various cases. First, suppose that the lower offer in equilibrium,  $p_L$ , is greater than  $\delta V_3$ . If  $p_L > p_H - \delta V_3$ , then the buyer will split his order by Lemma 5. Seller  $L$  could improve his profit by increasing his offer. If  $p_L < p_H - \delta V_3$ , then the buyer will buy both units from seller  $L$ . Seller  $H$  will have a payoff of  $\delta V_3$ . Seller  $H$  can improve his payoff by making an offer that is accepted. If  $p_L = p_H - \delta V_3$ , then either seller  $L$  is not selling 2 units or seller  $H$  is not selling any units. One of the sellers has an incentive to defect. To see this, suppose that  $\alpha \in [0, 1]$  is the probability that the buyer splits his order between the sellers. Then the payoff to seller  $L$  is  $\pi_L = \alpha p_L + (1 - \alpha)2p_L$ . The payoff to seller  $H$  is  $\pi_H = \alpha p_H + (1 - \alpha)\delta V_3$ , which equals  $\pi_H = \alpha p_L + \delta V_3$  by the assumption that  $p_L = p_H - \delta V_3$ . But seller  $H$ 's payoff must be at least as large as  $p_L + \delta V_3 - \epsilon$  for all positive  $\epsilon$ , since he could always guarantee an acceptance by lowering his price by  $\epsilon$ . Thus,  $\alpha$  would have to equal 1. But, if  $\alpha = 1$ , then seller  $L$  could improve his payoff by raising his price.

Now, suppose that  $p_L \leq \delta V_3$ . If  $p_L > p_H - \delta V_3$ , then the buyer will split his order by Lemma 5.

Seller  $L$  could improve his profit by increasing his offer. If  $p_L < p_H - \delta V_3$ , then the buyer will buy both units from seller  $L$ . If  $\frac{\delta V_3}{2} < p_L$ , then seller  $H$  can improve his payoff by making an offer that is accepted. If  $p_L < \frac{\delta V_3}{2}$ , then seller  $L$  could raise his offer to  $p_H$ , the buyer then splits his order and the low seller's profit increases. As before, if  $p_L = p_H - \delta V_3$ , then one of the sellers could do better by defecting.

An equilibrium with both sellers offering  $p^C$  could arise only if  $p^C \geq 2(p^C - \delta V_3)$  or if  $2\delta V_3 \geq p^C$ . This is because a defection by a seller that gets the buyer to buy two units from the seller will reduce his profits. Otherwise a seller would defect. This is equivalent to  $2\delta V_3 \geq V_2 + \delta [E \min[p_1, p_2] + E p_2]$ . But, this condition can never hold, since both  $p_1$  and  $p_2$  are greater than  $V_3/2$ . ■

#### Appendix A4: Proof of Proposition 3.

**P roof.** We know that  $\bar{p} - \underline{p} \geq \delta V_3$ ; otherwise a seller could increase his payoff by moving mass from lower parts of the price distribution to higher parts and still get accepted. Suppose that  $\bar{p} - \underline{p} < 2\delta V_3$  and for now assume that the equilibrium price distribution is continuous. Define three regions as follows: region 1 where  $p \in [\underline{p}, \bar{p} - \delta V_3]$ , region 2 where  $p \in [\bar{p} - \delta V_3, \underline{p} + \delta V_3]$  and region 3 where  $p \in [\underline{p} + \delta V_3, \bar{p}]$ . A price offered in region 1 will be accepted for either 1 or for 2 units. A price in region 2 will always be accepted for 1 unit. A price in region 3 will be accepted either for a single unit or no units. But, if there is an offer in region 2, then a seller can always improve his payoff by moving all the probability mass in region 2 to a price of  $\underline{p} + \delta V_3$ . Thus, there would be a gap in the price offer distribution.

Suppose that there was a gap in the price offer distribution in region 2. Then prices offered in region 1 would all be moved to the top of region 1 at a price of  $\bar{p} - \delta V_3$ , since whether the offer is accepted either once or twice is independent of the price in region 1. But, if sellers move up all their mass to  $\underline{p} + \delta V_3$ , then the price distribution would reduce to  $\delta V_3$ , but then any price in the interior distribution is inferior to either a price at the bottom or the top of the distribution. Thus, we would have a two-point distribution. But this cannot be an equilibrium. Suppose one player made an offer of  $\underline{p}$  and the other at  $\bar{p}$ . Then either the buyer accepts 2 units at the low price or splits his order. In the former case, the high seller could increase his payoff by reducing his offer slightly, while in the latter case the low seller could increase his payoff by a price reduction.

Let  $\pi$  be the equilibrium payoff. The equilibrium pricing equations are as follows. If  $p < \underline{p} + \delta V_3$ ,

$$p[2 - F(p + \delta V_3)] = \pi. \quad (\text{A4.1})$$

If  $\underline{p} + \delta V_3 < p < \bar{p} - \delta V_3$ ,

$$\delta V_3 F(p - \delta V_3) + P[2 - F(p + \delta V_3) - F(p - \delta V_3)] = \pi. \quad (\text{A4.2})$$

If  $p > \bar{p} - \delta V_3$ ,

$$\delta V_3 F(p - \delta V_3) + P(1 - F(p - \delta V_3)) = \pi. \quad (\text{A4.3})$$

Some further important facts about the equilibrium follow.

Substituting  $p = \underline{p}$  and  $p = \underline{p} + \delta V_3$  into (A4.1), setting the two resulting values equal to each other and manipulating the equation, we obtain

$$\delta V_3 [2 - F(\underline{p} + 2\delta V_3)] = \underline{p} [F(p + 2\delta V_3) - F(p + \delta V_3)].$$

Since  $2 - F(\underline{p} + 2\delta V_3) \geq 1$  and  $F(\underline{p} + 2\delta V_3) - F(\underline{p} + \delta V_3) < 1$ ,  $\underline{p} > \delta V_3$ . ■

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