

# Product Differentiation when Competing with the Suppliers of Bottleneck Inputs\*

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## Abstract

In this article, we analyze the product differentiation decision of a downstream entrant that purchases access to a bottleneck input from one of two vertically integrated incumbents, who will compete with him in the downstream market. We develop a three-stage model, where first an entrant chooses his product, then the entrant negotiates the access price with two incumbents, and finally the firms compete on retail prices. Contrary to what one might expect, both the entrant and the access provider prefer that the entrant chooses a product that is a closer substitute of the product of the access provider than of the product of the other incumbent. This occurs because the access provider interacts with the entrant both in the retail market and the wholesale market. We also consider the cases where both parties, rather than only the incumbents, make the access price offers, where the bargaining stage precedes the location stage, and where there is open access regulation.

**Key Words:** *Horizontal differentiation, Location, Access price.*

**JEL Classification:** L25, L51, L96.

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# 1 Introduction

In this article, we analyze the product differentiation decision of a downstream entrant that must purchase access to a bottleneck input from one of two vertically integrated incumbents, who will compete with him in the downstream market. The starting point of our analysis are the two following conjectures. First, the entrant prefers to have a product that is as differentiated as possible from the products of both of the incumbents. This leads to higher retail prices. Second, the access provider prefers that the product of the entrant is a closer substitute of the product of his rival incumbent than of his own product. This way, the retail revenues of the access provider are relatively less affected by entry than the retail revenues of the other incumbent. It turns out that both conjectures are incorrect.

Our problem applies to several markets. In mobile telephony, more than one firm own a license to use the radio-electric spectrum and a mobile telecommunications network, to which they can give access to downstream entrants, like mobile virtual network operators. In broadband access to the Internet, in some countries, both the telecommunications incumbent and cable television firms own local access networks capable of delivering telecommunications services, to which they can give access to downstream entrants, like Internet service providers.<sup>1</sup> In the film industry, several studios produce movies that can be exhibited at their own theaters or at independent theaters. In the airline industry, in some markets, there are several national or local carriers that can sell their handling services to foreign carriers.

We develop a three-stage model to analyze this problem. In the first stage, an entrant chooses his product. In the second stage, the entrant negotiates the access price with two incumbents. In the third stage, firms compete on retail prices. We model the industry as a horizontally differentiated product oligopoly on Salop's circle (Salop, 1979).<sup>2</sup>

Interestingly, both the entrant and the access provider prefer that the entrant chooses a product that is a closer substitute of the product of the access provider, than the product of the other incumbent. The reasoning underlying the initial conjectures is based only on the retail market, and ignores the wholesale market. However, the access provider interacts with the entrant in both the retail market, as a rival, and in the wholesale market, as a supplier. Consequently, the access provider has less incentives to cut his retail price after entry than the incumbent that does not provide access. Anticipating the different price responses by the

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<sup>1</sup>These telecommunications examples are discussed in detail in Brito and Pereira (2007a, 2007b).

<sup>2</sup>Horizontal differentiation has been used extensively to model telecommunications industries, which provide some examples of the type of problem we study. Recent examples are Laffont et al. (1998), Armstrong (1998), Foros and Hansen (2001), Biglaiser and DeGraba (2001), and Peitz (2005).

incumbents, the entrant chooses a product that is a closer substitute of the product of the access provider, than the product of the other incumbent. As for the access provider, given the retail prices, it is in his interest that the entrant offers a product that competes more closely with the product of his rival than with his own product. However, the product choice of the entrant affects not only the retail prices, but also the access price that the access provider can charge. If the entrant chooses a product that is a closer substitute of the product of the access provider than of the product of the other incumbent, the access provider can charge a high access price. The access provider takes the level of the access price into account when he sets the retail price. Whereas if the entrant chooses a product that is a close substitute of the product of the incumbent that does not provide access, the access provider must charge a low access price. Otherwise, the entrant competes with a cost disadvantage against a close rival that sets low retail prices, and therefore generates small wholesale revenues.

We consider three variations of the model, to check the robustness of our results. In the first variation, both parties make the access price offers, rather than only the incumbents. This variation captures the situation where both parties have bargaining power. The analysis shows that the bias of the entrant towards choosing a product that is a closer substitute of the product of the access provider than the product of the other incumbent increases. In the second variation, the order of the location and the bargaining stages is reversed. This variation captures the situation where at the bargaining stage, the entrant is unable to credibly commit to a type of product. The analysis shows that again the bias increases. In addition, it is in the entrant's interest to commit himself not to change his product after the access price is bargained. The third variation introduces open access regulation.<sup>3</sup> The analysis shows that open access regulation can induce a perfectly competitive outcome on the wholesale market, without imposing it. But more interestingly, open access regulation also affects the product choice of the entrant.

Our article relates to: **(i)** the literature on vertical foreclosure, and **(ii)** the literature on product choice in horizontal differentiation models. The first literature strand addresses the question of whether a vertically integrated firm can increase its profit by foreclosing the downstream market. The case in which the upstream market is monopolized, was reviewed by Rey and Tirole (2006). Ordover et al. (1990) and Hart and Tirole (1990) analyzed the case of oligopolistic vertical integration with an oligopolistic upstream market, focusing on the prof-

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<sup>3</sup>According to the Open Network Principle, all telecommunications firms should have access to the basic public telephone network, under the principles of: (i) non-discrimination, (ii) transparency and (iii) cost orientation. See, e.g., the Access Directive 2002/19/EC.

itability of vertical mergers. We depart from this literature by considering that the incumbents are vertically integrated from the beginning, and focus on their incentives to allow entry of new downstream competitors. More related to our article, Biglaiser and DeGraba (2001) and Peitz (2005) studied the effects of entry in industries with horizontal product differentiation, where the entrant buys access from a single incumbent.<sup>4</sup> Brito and Pereira (2007a) and Ordober and Shaffer (2006) analyzed the incentives to voluntarily provide access to a bottleneck input when there are multiple alternatives available for the entrant. In both cases, the entrant's product characteristics were exogenously defined, whereas we endogeneize the entrant's location.<sup>5</sup> The second literature strand addresses the question of the location choice. This literature goes back to the principle of minimum differentiation (Hotelling, 1929), and to the principle of maximum differentiation (D'Aspremont et al., 1979). More recently, Tabuchi and Thisse (1995) proved that, if allowed, firms should locate outside the Hotelling's main road. Hinloopen and Marrewijk (1999) and Woeckener (2002) extended the model to include binding reservation prices, and showed that this would, instead, lead firms to move in the direction of the centre. Hamilton et al. (1989) and Anderson and Neven (1991) found that the equilibrium can also involve agglomeration when Cournot competition is introduced in Hotelling's model. All these results were obtained with simultaneous choice of location. Neven (1987), Economides et al. (2002), and Goetz (2005) analyzed sequential location choice in the same model, while Fleckinger and Lafay (2003) analyzed the case in which firms move sequentially with the leader setting the price and the location first.<sup>6</sup> We contribute to this literature by analyzing the implications for the location decision of an entrant that competes at the retail level with vertically integrated incumbents from whom he must purchase access to a bottleneck input.

This paper is organized as follows. Section 2 presents the model, and section 3 characterizes the equilibrium. Section 4 presents three variations to the model. Section 5 concludes. All proofs are in the appendix.

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<sup>4</sup>See Vogelsang (2003) for a survey of the literature on access regulation for the telecommunications industry.

<sup>5</sup>Ordober and Shaffer (2006) considered only two possibilities of product positioning for the entrant: one leading to proportional cannibalization, and the other to host cannibalization.

<sup>6</sup>Location choice with other forms of differentiation has also been addressed in the literature. Ansari et al (1996) analyzed simultaneous location choice in two and three-dimensional variants of Hotelling's model and Valletti (2002) discussed product design in a duopoly with both horizontal and vertical differentiation.

## 2 The Model

In this section, we present the model, which is simple to convey the intuition of our results as clearly as possible. In section 4, we consider three variations of the model to check the robustness of our results.

### 2.1 Environment

Consider an industry where firms sell products horizontally differentiated on Salop's circle (Salop, 1979).<sup>7</sup> Initially there are two firms, the *incumbents*. A third firm, the *entrant*, wants to join the industry. Each incumbent owns a bottleneck input. To operate, the entrant has to buy access to the bottleneck input of one of the incumbents.

The game has three stages. In *stage 1*, the entrant chooses his location. In *stage 2*, the entrant negotiates an access price with the incumbents. In *stage 3*, the firms present in the market choose their retail prices simultaneously.

All relevant information becomes common knowledge as the game unfolds.

### 2.2 Consumers

There is a large number of consumers, formally a continuum, whose measure we normalize to 1. Consumers are uniformly distributed along a loop of unit length, and have quadratic transportation costs. Each consumer has a unit demand, and a finite reservation price such that all consumers purchase the product.<sup>8</sup>

### 2.3 Firms

We index the firms with subscript  $i = 0, 1, e$ , where firms 0 and 1 are the incumbents, and firm  $e$  is the entrant; and we index the access provider with superscript  $j = 0, 1$ .

The production of one unit of the final good involves the consumption of one unit of the bottleneck input, whose cost we normalize to 0. Denote by  $\alpha_i$  on  $\mathcal{R}_0^+$ , the unit access price of the bottleneck input of firm  $i = 0, 1$ , and denote by  $p_i$  on  $\mathcal{R}^+$ , the retail price of firm  $i = 0, 1, e$ . Denote by  $D_i$ , the demand of firm  $i$ , and denote by  $\pi_i^j$ , the profit of firm  $i$  when firm  $j$  is the

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<sup>7</sup>If a third firm enters in Hotelling's line (Hotelling, 1929), the prices of the incumbents will no longer affect each others' demands. We model the industry on Salop's circle to prevent this.

<sup>8</sup>If the market is fully covered, the entrant steals business from the incumbents.

access provider, with  $i = 0, 1, e$ , and  $j = 0, 1$ . Then,  $\pi_e^j = (p_e - \alpha_j)D_e$  and:

$$\pi_i^j = \begin{cases} p_i D_i & i \neq j \\ p_j D_j + \alpha_j D_e & i = j, \end{cases}$$

where  $p_i D_i$  are the retail profits and  $\alpha_j D_e$  are the wholesale profits.

[Figure 1]

As depicted in Figure 1, firm 0 locates at 0, firm 1 locates at 0.5, and firm  $e$  locates at  $l$  on  $(0, 0.5)$ .<sup>9</sup> A stage 1 *strategy* for the entrant is a decision on where to locate,  $l$ .

The bargaining stage unfolds as follows. First, the incumbents simultaneously make access price offers to the entrant. Afterwards, the entrant decides which offer to accept, if any. When indifferent between two offers, the entrant selects an offer at random. If both offers are rejected, the entrant stays inactive and receives a payoff of 0; if one offer is accepted, the entrant proceeds with the incumbents to stage 3. A stage 2 *strategy* for the incumbents is an access pricing rule that says which access prices they should offer to the entrant, given the entrant's location, and a stage 2 *strategy* for the entrant is an acceptance rule that says which offers he should accept, if any, given his location.<sup>10</sup>

A stage 3 *strategy* for the firms in the market is a retail pricing rule that says which prices they should charge, given the access price offers, the entrant's acceptance decision, and the entrant's location.

## 2.4 Equilibrium

A *subgame perfect Nash equilibrium in pure strategies* is a location choice by the entrant, a pair of access price rules for the incumbents, an acceptance rule for the entrant, and a profile of retail pricing rules for the firms in the market, such that:

**(E1)** each firm chooses a retail pricing rule to maximize profit, given the rivals' retail pricing rules, the incumbents' access pricing rules, the entrant's acceptance decision, and the entrant's location;

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<sup>9</sup>We assume maximum differentiation before entry. This could be the result of history. The incumbents entered when no further entry was deemed possible. Technological or regulatory changes, however, allowed the entry of a new firm. Relocation costs by the incumbents are assumed to be prohibitive.

<sup>10</sup>If  $l$  is interpreted as physical distance, it is conceivable that the incumbents make access price offers conditional on  $l$ . If  $l$  is interpreted as distance in preferences, it would be very hard to write contracts for price offers conditional on  $l$ , and almost impossible to enforce them.

- (E2) the entrant chooses an acceptance rule to maximize profit, given the retail pricing rules, and his location;
- (E3) each incumbent chooses an access pricing rule to maximize profit, given the other incumbent's access pricing rule, the entrant's acceptance rule, the retail pricing rules, and the entrant's location;
- (E4) the entrant chooses his location to maximize profit, given the access pricing rules, his acceptance rule, and the retail pricing rules.

### 3 The Equilibrium

In this section, we construct the model's equilibria by backward induction. Hence, we characterize first the equilibrium prices, afterwards the equilibrium bargaining, and finally the equilibrium location.

#### 3.1 Stage 3: Retail Price Game

Next, we solve the price competition game for the cases where: (i) no entry occurred, and (ii) entry occurred.

##### 3.1.1 No Entry

We use superscript "n" to denote variables or functions associated with this case, which is the standard model of Salop (1979) with two firms. Equilibrium prices, market shares, and profits are given by:  $p_i^n = 250$ ,  $D_i^n = 0.5$ , and  $\pi_i^n = 125$ , for  $i = 0, 1$ .<sup>11</sup>

##### 3.1.2 Entry

We use superscript "j" to denote variables or functions associated with this case.

Denote by  $l_{i,i'}$ , the location of the consumer indifferent between purchasing from firm  $i$  or firm  $i'$ , and denote by  $l_i$ , the location on the loop of firm  $i$ . For the consumer indifferent between firms  $i$  and  $i'$ :

$$p_i + (l_{i,i'} - l_i)^2 = p_{i'} + (l_{i,i'} - l_{i'})^2,$$

or, equivalently,

$$l_{i,i'} = \frac{1}{2} \left[ \frac{p_{i'} - p_i}{l_{i'} - l_i} + (l_i + l_{i'}) \right].$$

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<sup>11</sup>Throughout the text, all monetary values are multiplied by 1,000.

We assume that all firms have a positive demand for their products. For firm  $i$ , located between firms  $i'$  and  $i''$ , demand is given by:

$$D_i(p_i, p_{i'}, p_{i''}) = l_{i,i''} - l_{i',i} = \frac{1}{2} \left[ \frac{p_{i''} - p_i}{l_{i''} - l_i} + \frac{p_{i'} - p_i}{l_i - l_{i'}} + (l_{i''} - l_{i'}) \right].$$

The first-order conditions for price for firms  $i, j$  and  $e$  are, respectively:

$$\begin{aligned} D_i + p_i \frac{\partial D_i}{\partial p_i} &= 0, \\ D_j + p_j \frac{\partial D_j}{\partial p_j} + \alpha_j \frac{\partial D_e}{\partial p_j} &= 0, \\ D_e + (p_e - \alpha_j) \frac{\partial D_e}{\partial p_e} &= 0. \end{aligned}$$

Denote by  $d$  on  $(0, 0.5)$ , the distance between the entrant and the access provider.<sup>12</sup> Let  $\alpha_j^u(d) := \frac{20d^3 - 20d^2 - 13d + 9}{4(5 - 6d)}$ .

The next Lemma presents the equilibrium retail prices.

**Lemma 1:** *Suppose that there is entry with access provided by firm  $j$  at access price  $\alpha_j$  on  $[0, \alpha_j^u(d)]$ . At the price competition game, in equilibrium, the firms charge retail prices:*

$$\begin{aligned} p_j^j(\alpha_j; d) &= \frac{2d^3 + 11d^2 - 9d - 12\alpha_j(1 - d)}{32d^2 - 16d - 12}, \\ p_i^j(\alpha_j; d) &= \frac{-4d^3 + 28d^2 - 7d + 4\alpha_j(2d - 3) - 3}{64d^2 - 32d - 24}, \\ p_e^j(\alpha_j; d) &= \frac{-20d^4 + 20d^3 + 13d^2 - 9d + 4\alpha_j(2d + 3)(d - 1)}{32d^2 - 16d - 12}. \end{aligned}$$

■

The condition  $\alpha_j \leq \alpha_j^u(d)$  ensures that all firms have non-negative market shares.

The incumbent that provides access, firm  $j$ , and the incumbent that does not provide access, firm  $i \neq j$ , are not symmetric. First, the distance between the entrant and each incumbent may differ. Second, the inspection of the first-order conditions shows that by hiking his retail price, firm  $j$  increases the entrant's sales,  $\frac{\partial D_e}{\partial p_j} > 0$ , and hence, its own wholesale revenues.<sup>13</sup> This additional incentive leads firm  $j$  to charge a retail price no smaller than the price of firm  $i$ , for

<sup>12</sup>Thus,  $d = l$  if the access provider is firm 0, and  $d = 0.5 - l$  if the access provider is firm 1.

<sup>13</sup>By hiking the retail price, the access provider increases his profit margin, and decreases his volume of sales. These are the usual effects caused by a price increase, and correspond to the first two terms of firm  $j$ 's first-order condition. However, there is a third effect. Part of the consumers that the access provider loses with the price increase move to the entrant, and thereby increase his wholesale revenues.



the same distance between the entrant and each incumbent.<sup>14</sup> We call *wholesale effect* to this upward pressure on the retail price of the access provider, caused by the fact that by hiking its retail price it increases his wholesale revenues. See also Brito and Pereira (2007a) and Chen (2001). The wholesale effect depends on: **(i)** the number of consumers that the access provider looses to the entrant when he raises his retail price, and **(ii)** the wholesale profit accruing to the access provider for each of those consumers. The first effect is decreasing with  $d$ , while the second effect is increasing with  $\alpha_j$ .

Depending on  $d$  and  $\alpha_j$ , any retail price ranking across the three firms may prevail.<sup>15</sup> A large  $\alpha_j$  means higher marginal costs for the entrant, and a large  $d$  means more differentiation between the product of the entrant and the product of the access provider.<sup>16</sup>

When entry occurs, the incumbents face three incentives to change their prices. First, the distance to the nearest competitor decreases, which gives an incentive to reduce prices. Second, the new closest competitor has no lower costs than the incumbents, which gives an incentive to increase prices. Third, the wholesale effect gives the access provider an incentive to increase his price. Consequently, with entry, the retail prices of any of the firms may be higher or lower than the retail prices of the incumbents without entry, depending on whether  $\alpha_j$  is high enough.

Entry impacts the profits of the access provider at both the retail and wholesale levels. At the retail level, the impact is negative because the retail market share of the access provider always decreases with entry, and in a way that more than compensates any eventual retail price increase. At the wholesale level, the impact is positive, and may more than compensate the retail effect, if  $\alpha_j$  is high enough and  $d$  small enough.

Denote by  $\alpha_j^m(d)$ , the access price firm  $j$  would set if it was the only firm able to provide access. We call  $\alpha_j^m(d)$  the *monopoly access price* of incumbent  $j$ . The next Remark collects some auxiliary results regarding the firms' profits that will be useful later.

**Remark 1:** Let  $\alpha_j$  belong to  $[0, \alpha_j^u(d)]$ .

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<sup>14</sup>Clearly,  $p_j^j(\alpha_j; d) > p_i^j(\alpha_j; 0.5 - d)$ , for  $\alpha_j > 0$ .

<sup>15</sup>For  $\alpha_j = 0$ , the entrant sets a lower price than the incumbents, because it faces a smaller demand and has the same costs. The incumbent more distant from the entrant charges the highest price. As  $\alpha_j$  increases, the prices of all firms increase, for any  $d$ , although by different amounts. The firm most sensitive to  $\alpha_j$  is the entrant, followed by firm  $j$ , the access provider, and, finally, by firm  $i$ .

<sup>16</sup>For  $\alpha_j = 0$ , all prices are concave in  $d$  and have a single maximum for  $d$  on  $(0, 0.5)$ . A small or a large  $d$  mean little differentiation between the entrant and one of the incumbents, leading to a downward pressure on equilibrium prices for these firms and also for the other incumbent. As  $\alpha_j$  increases, the prices of the entrant and of the access provider still have an interior single maximum for  $d$  on  $(0, 0.5)$ , while the price of the other incumbent may become strictly decreasing on  $d$ .

- (i) The profit function of incumbent  $i \neq j$ ,  $\pi_i^j(\cdot; d)$ , is increasing in the access price,  $\alpha_j$ .
- (ii) The profit function of the entrant,  $\pi_e^j(\cdot; d)$ , is decreasing in the access price,  $\alpha_j$ , and has a root for  $\alpha_j^u(d)$ .
- (iii) The profit function of the access provider,  $\pi_j^j(\cdot; d)$ , is concave in the access price,  $\alpha_j$ , and has a point of maximum with respect to the access price at  $\alpha_j^m(d)$ , where  $\alpha_j^m(d) < \alpha_j^u(d)$ .
- (iv) Value  $\alpha_j^m(d)$  is decreasing in  $d$ . ■

Remark 1: (i)-(iii) are straightforward. To understand the intuition of Remark 1: (iv) note that a large  $d$  means that the entrant sells a product that is more similar to the product of firm  $i$ , the incumbent that does not provide access. This leads firm  $i$  to set a lower retail price. As a consequence, the access provider sets a low access price to allow the entrant to compete with firm  $i$ , and thereby generate wholesale revenues.<sup>17</sup>

The next Remark presents some auxiliary results regarding the firms' profits when access is granted at the monopoly access price.

**Remark 2:** Let  $\alpha_j = \alpha_j^m(d)$ .

- (i) The profit of firm  $j$  is higher than if there is no entry,  $\pi_j^j(\alpha_j^m(d); d) > \pi_j^n$ , if and only if,  $d < 0.142$ .
- (ii) The profit of firm  $j$ ,  $\pi_j^j(\alpha_j^m(d); d)$ , is decreasing in  $d$ , for all  $d$ .
- (iii) The profit of firm  $j$ ,  $\pi_j^j(\alpha_j^m(d); d)$ , is lower than the profit of firm  $i$ , if  $d < 0.25$ .
- (iv) The profit of the entrant,  $\pi_e^j(\alpha_j^m(d); d)$ , is single-peaked in  $d$ , for  $d$  on  $(0, 0.5)$ , and has a point of maximum with respect to  $d$  at  $d = 0.331$ . ■

Due to the wholesale effect, the retail prices are higher when the entrant locates closer to the access provider, i.e., when  $d$  is small. This explains Remark 2: (i) and (ii). Remark 2: (iii) presents a sufficient condition for the incumbent that does not provide access to have a higher profit than the access provider. For a small  $d$ , the rivals of the incumbent that does not provide access, firm  $i$ , are located at a large distance, despite the entry by firm  $e$ . Consequently, firm  $i$  benefits from the retail prices being high. Adding Remark 2: (i) and (iii) implies that if entry is profitable for the access provider at the monopoly access price, then, the incumbent that does not provide access benefits from entry more than the access provider. The optimal location for the entrant, indicated in Remark 2: (iv), trades-off the level of the access price, which decreases

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<sup>17</sup>The access price elasticity of the wholesale demand is increasing in  $d$ .

with  $d$ , and the proximity to the lower priced competitor, which increases with  $d$ .

### 3.2 Stage 2: Bargaining Game

Next, we solve the bargaining game, given the equilibria of the price competition stage.

From Remark: 1 (ii), the profit function of the entrant,  $\pi_e^j(\cdot; d)$ , has a root at  $\alpha_j^u(d)$ . Access prices higher than  $\alpha_j^u(d)$  imply zero profits for the entrant. Consequently, we say that access price on  $[\alpha_j^u(d), +\infty)$  are *unprofitable*, and that access price offers on  $[0, \alpha_j^u(d))$  are *profitable*.

If the two access price offers are profitable, the entrant may not choose the lowest offer. Due to the wholesale effect, under equal circumstances, i.e., if  $\alpha_0 = \alpha_1$ , the entrant prefers to be supplied by the closest incumbent. Thus, if the incumbent that made the highest offer is close enough, the entrant may accept his offer.

If the offer of the rival is unprofitable, an incumbent best responds by offering the monopoly access price,  $\alpha_j^m(d)$ , or by making an unprofitable offer, depending on his distance to the entrant being smaller or larger than 0.142, respectively.<sup>18</sup> If the offer of the rival is profitable, it may be so high that it can be undercut with the monopoly access price, or it may be low enough to be undercut by a price lower than the monopoly access price. Undercutting a profitable offer of a rival is a best response for an incumbent only if the offer is relatively low, and if the entrant is closely located.

Denote by  $\delta := \min \{l, 0.5 - l\}$  on  $(0, 0.25]$ , the distance between the entrant and the nearest incumbent.

The next Lemma presents the equilibria of the bargaining stage.

**Lemma 2:** *The bargaining stage has two equilibria.*

(i) *If  $\delta$  belongs to  $(0, 0.142]$ , then the nearest incumbent offers the monopoly access price,  $\alpha_j^m(\delta)$ , the distant incumbent makes an unprofitable offer, and the entrant accepts the offer of the nearest incumbent.*

(ii) *If  $\delta$  belongs to  $(0.142, 0.25]$ , then both incumbents make unprofitable offers, and the entrant rejects both offers.* ■

If the entrant is close to one of the incumbents, i.e., if  $\delta \leq 0.142$ , the nearest incumbent offers the monopoly access price,  $\alpha_j^m(\delta)$ . The other incumbent prefers not to undercut this offer, both because the entrant is a distant competitor, and because the entrant has high marginal

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<sup>18</sup>See Remark 2: (i).

costs. Furthermore, to outbid the rival, the distant incumbent would have to offer a very low access price.

If the entrant is far from both incumbents, i.e., if  $\delta > 0.142$ , both incumbents make unprofitable offers. Even if the entrant accepted the monopoly access price, the profit of the access provider would be smaller than if no entry occurred, as stated in Remark 2: (i).

It is also an equilibrium for both incumbents to offer  $\alpha_i = 0$ , and for the entrant to accept either offer. However, we rule out this equilibrium because playing  $\alpha_i = 0$  is weakly dominated by playing an unprofitable offer.<sup>19</sup>

### 3.3 Stage 1: Location Choice

Next, we solve the location stage, given the equilibria of the bargaining stage and the equilibria of the price competition stage.

The profit of the entrant is:

$$\pi_e(\alpha(\delta); \delta) = \begin{cases} \pi_e^i(\alpha_i^m(\delta); \delta) & \text{if } \delta \text{ is on } (0, 0.142] \\ 0 & \text{if } \delta \text{ is on } (0.142, 0.25] \end{cases}.$$

The next Proposition characterizes the optimal location for the entrant, in terms of his distance to the access provider, denoted by  $d^*$ .

**Proposition 1:** *The unique optimal location is such that  $d^* = 0.142$ .* ■

Since  $d^* = 0.142 < 0.25$ , the entrant locates closer to the access provider than to the incumbent that does not provide access. This bias of the entrant towards locating closer to the access provider is unexpected. The entrant ought to prefer to locate equidistantly from both incumbents. This would lead to higher retail prices. In addition, the access provider ought to prefer that the entrant locates closer to his rival incumbent than to himself. This way, the retail revenues of the access provider would be relatively less affected by entry than the retail revenues of the other incumbent. However, this reasoning is based only on the retail market, and ignores both the wholesale market, and the effect of the product differentiation choice on the access price.

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<sup>19</sup>If the incumbent  $i'$  makes an unprofitable offer, the incumbent  $i$  is better off by also making an unprofitable offer instead of  $\alpha_i = 0$ , because  $\pi_i^n > \pi_i^i(0, d)$ , for all  $d$ . If the incumbent  $i'$  makes any positive profitable offer, the incumbent  $i$  is better off by also making an unprofitable offer instead of  $\alpha_i = 0$ , because  $\pi_i^{i'}(\alpha_{i'}, d) > \pi_i^i(0, d)$ , for all  $d$  and all  $\alpha_{i'} > 0$ . Finally, if  $\alpha_j = 0$ , the incumbent  $i'$  is indifferent between making an unprofitable offer or offering  $\alpha_i = 0$ , because  $\pi_i^j(0, d) = \pi_i^i(0, d)$ .

To understand this bias, first we discuss the location incentives assuming that the access price is independent of the location choice. Afterwards, we discuss the impact of the location choice on the access price. Given a positive access price, the entrant benefits from locating closer to the access provider than to the other incumbent. Due to the wholesale effect, the access provider sets a higher retail price than the other incumbent, as discussed in section 3.1.2. However, the entrant does not prefer to locate very close to any of the incumbents. The entrant has higher costs than the incumbents. Thus, he prefers to sell a product that is sufficiently differentiated from the product of either of the incumbents.<sup>20</sup> The location of the entrant preferred by the access provider depends on the level of the access price. If the access price is low, the access provider prefers that the entrant locates closer to the other incumbent, but not too close. Otherwise, the competition between the entrant and the other incumbent leads to low retail prices. In the limit case of  $\alpha_j = 0$ , the optimal location in the access provider's perspective is  $d = 0.344$ . If the access price is high, the access provider prefers that the entrant locates close to him because the retail prices are higher, due to the wholesale effect. High retail prices lead to both higher retail and wholesale profits.<sup>21</sup>

Next we discuss how the location of the entrant affects the access price. The incumbent closest to the entrant offers the monopoly access price,  $\alpha_j^m(\delta)$ , provided that  $\delta \leq 0.142$ . As the monopoly price is decreasing in  $\delta$ , the firms have conflicting interests.<sup>22</sup> The access provider prefers to have the entrant as close as possible, to be able to set a higher access price. The entrant prefers to locate as far as possible from the access provider to obtain a lower access price. However, he has to choose a location that ensures he will receive a profitable offer, i.e., a location with  $\delta$  on  $(0, 0.142]$ .

The next Corollary summarizes the equilibrium of the whole game.

**Corollary 1:** *In equilibrium:  $(d^*, \alpha_j^m, p_j^j, p_i^j, p_e^j) = (0.142, 325, 323, 256, 340)$  and  $(\pi_j^j, \pi_i^j, \pi_e^j) = (125, 157, 1)$ .* ■

Corollary 1 suggests two observations. The first observation is that retail prices are higher when there is entry than when there is no entry. This follows from the combination of three aspects: first the wholesale effect, second the entrant has higher marginal costs than the incumbents, and third the large differentiation between the entrant and the incumbent with the

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<sup>20</sup>See Proposition 3 (i), below, for the exact optimal location of the entrant, taking the access price as given.

<sup>21</sup>This results from the fact that  $\frac{\partial^2 \pi_j^j(\alpha_j; d)}{\partial d \partial \alpha_j} < 0$ .

<sup>22</sup>See Remark 1: (iv).

lowest retail price. The possibility of the retail prices increasing with entry was already raised by Brito and Pereira (2007a). The second observation is that the access price exceeds the retail price of the access provider. This outcome fits the European Commission's definition of a margin squeeze: "A situation of a margin squeeze occurs where the incumbent's price of access combined with its downstream costs are higher than its corresponding retail price."<sup>23</sup> However, in this case, a retail price lower than the sum of the access price and the price of downstream costs neither prevents the entrant from competing, nor reflects any market foreclosure motivations of the access provider. The access provider could refuse to give access to entrant but, instead, prefers to allow entry.

We conclude this section by discussing the case of a monopolist incumbent, which helps to put our results in perspective.

**A Monopolist Incumbent** Suppose that there is one incumbent, firm 1, and an entrant, firm  $e$ . Firm 1 locates on 0, and firm  $e$  locates on  $l$  on  $(0, 0.5)$ . Denote by  $v$ , the consumers' reservation price, and by  $\alpha$  the access price. If there is no entry, firm 1 charges  $p_1^m = v - \frac{1}{4}$ , and earns  $\pi_1^m = v - \frac{1}{4}$ . If there is entry, firm 1 and firm  $e$  charge on the retail market, respectively,  $p_1 = p_e = \alpha + l(1 - l)$ , and firm 1 charges the access price  $\alpha^m := v + \frac{3l^2 - 2l - 1}{4}$ . Firm 1 and firm  $e$  earn, respectively,  $\pi_1 = \frac{4v + l^2 - 1}{4}$  and  $\pi_e = \frac{l(1 - l)}{2}$ . The optimal location for the entrant, both from his perspective and from the incumbent's perspective, is  $l^* = 0.5$ . This shows that the bias of the entrant towards locating close to the access provider depends not only on the wholesale effect, but also on there being competing incumbents. ■

## 4 Extensions

In this section, we present three variations of the model. The first variation allows both parties to make the access price offers, rather than only the incumbents. The second variation reverses the order of the location and the bargaining stages. The third variation introduces open access regulation.

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<sup>23</sup>See ONP Committee document ONPCOM 01-17.

## 4.1 Both Parties make Access Price Offers

The ability to make take-it-or leave-it offers gives the incumbents bargaining power.<sup>24</sup> This is an extreme situation. To analyze the impact on the product choice of the entrant of the parties' ability to make take-it-or leave-it offers, we develop a version of the bargaining game that allows either side of the market to make take-it-or leave-it offers.

Consider the model of section 2, except that the bargaining stage unfolds as follows. First, *nature* decides which party makes the access price offers, and which party decides to accept or reject them. With probability  $\lambda$  on  $[0, 1]$ , the entrant makes a public offer to each of the incumbents, and the incumbents decide if they accept or reject their offers. If both incumbents accept their offers, the offer yielding the highest profit for the entrant prevails; in case of a draw, the entrant selects an offer at random. With probability  $1 - \lambda$ , the incumbents simultaneously make access price offers to the entrant, and the entrant decides which offer to accept, if any. When indifferent between two offers, the entrant selects an offer at random. Independently of which party makes the offers, if both offers are rejected, the entrant stays inactive and receives a payoff of 0; if one offer is accepted, the entrant proceeds with the incumbents to stage 3. We take  $\lambda$  as a measure of the *bargaining power* of the entrant.<sup>25</sup>

The equilibrium of the price competition stage is as described in section 3.1.

To characterize the equilibrium of the bargaining game one has to: first analyze the case where the incumbents make the access price offers, second analyze the case where the entrant makes the access price offers, and third combine the two previous cases. The equilibrium of the case where the incumbents make the access price offers is described in section 3.2.

Denote by  $\alpha_j^\mu(d)$  the access price for which the incumbent  $j$  is indifferent between providing access and there being no-entry, i.e.,  $\pi_j^j(\alpha_j^\mu(d), d) \equiv \pi_j^n$ , with  $\alpha_j^\mu(d) \leq \alpha_j^m(d)$ .<sup>26</sup> We say that

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<sup>24</sup>In a one stage game where two players bargain over how to divide an object, i.e., in the ultimatum game, the bargaining power lies with who makes the offer. Our case is different because one of the sides of the market has two players. The number of players in each side of the market also affects the parties' bargaining power. One of the parties might have more bargaining power than the other because: it is less impatient, perhaps because it has a lower discount rate, or it is less risk averse, perhaps because the outcome of this venture has a smaller impact on its wealth. See Binmore and Harbord (2005) and Binmore et al. (1986) for a more thorough discussion of these issues.

<sup>25</sup>An ultimatum game between players  $A$  and  $B$  where nature chooses with probability  $\lambda$  if player  $A$  makes the offer, is equivalent to a Nash bargaining game where the bargaining powers of players  $A$  and  $B$  are  $\lambda$  and  $1 - \lambda$ , respectively.

<sup>26</sup>There are two roots for  $\pi_j^j(\alpha_j^\mu(d), d) \equiv \pi_j^n$ . Condition  $\alpha_j^\mu(d) \leq \alpha_j^m(d)$  singles out the lowest of the two. Function  $\alpha_j^\mu(d)$  is increasing with  $d$  and only exists for  $d \leq 0.142$ . Additionally,  $\alpha_j^\mu(0.142) = \alpha_j^m(0.142)$ . In case of indifference between providing access and there being no-entry we assume the incumbent provides access.

access price offers on  $[\alpha_j^\mu(d), +\infty)$  are *individually rational*, and that access price offers on  $[0, \alpha_j^\mu(d))$  are *non-individually rational*.<sup>27</sup>

From Remark 2, if the entrant makes an individually rational offer to one of the incumbents, then: **(i)** he cannot make another individually rational offer to the other incumbent, and **(ii)** the offer has a positive effect on the payoff of the other incumbent, who gains more than the access provider.<sup>28</sup>

The next Lemma presents the equilibria of the bargaining stage when the entrant makes the offers.

**Lemma 3:** *Let the entrant make the access price offers. The bargaining stage has two equilibria.*

**(i)** *If  $\delta$  belongs to  $(0, 0.142]$ , the entrant offers  $\alpha_j^\mu(\delta)$  to the closest incumbent, firm  $j$ , and makes a non-individually rational offer to other incumbent; the closest incumbent accepts his offer and the other incumbent rejects his offer.*

**(ii)** *If  $\delta$  belong to  $(0.142, 0.25]$ , the entrant makes non-individually rational offers to both incumbents, and both incumbents reject their offers. ■*

Assume that the entrant makes an individually rational offer to firm  $i$ , and a non-individually rational offer to firm  $i'$ . For firm  $i'$ , rejecting its offer is a dominant strategy. If firm  $i$  accepts its offer, firm  $i'$  is better off by rejecting its offer and benefiting from the increase in profits discussed in Remark 2: (iii). Besides, if firm  $i$  rejects its offer, firm  $i'$  is also better off by rejecting its offer and preventing entry. Thus, the equilibrium is for firm  $i$  to accept its individually rational offer. Assume now that the entrant makes non-individually rational offers to both incumbents. For the incumbent that received the offer that is less profitable for the entrant, accepting its offer is a weakly dominated strategy. Hence, the equilibrium is for both offers to be rejected. Consequently, and given that his profits are decreasing in the access price, the entrant offers the lowest individually rational access price to one of the incumbents, which he accepts.<sup>29</sup>

Denote by  $E[x]$ , the expected value of variable  $x$ . The following Corollary sums up the equilibrium of the bargaining stage.

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<sup>27</sup>I.e.,  $\alpha_j$  is individually rational for incumbent  $j$ , if and only if,  $\pi_j^j(\alpha_j, d) \geq \pi_j^n$ .

<sup>28</sup>The explanation for (i) is the same as for Remark 2: (i), and for (ii) is the same as for Remark 2: (iii).

<sup>29</sup>If two non-individually rational offers are made, the rejection of both is an equilibrium. However, the acceptance of the two offers may also be an equilibrium. We rule out this equilibrium because it is based on weakly dominated strategies, as can be seen in the proof of Lemma 3.



**Corollary 2:** *Let nature choose that the entrant makes the access price offers with probability  $\lambda$  on  $[0, 1]$ . The bargaining stage has two equilibria.*

(i) *If  $\delta$  belongs to  $(0, 0.142]$ , then access is provided by the closest incumbent, firm  $j$ , and the expected access price is  $E[\alpha_j(\lambda, d)] = \lambda\alpha_j^\mu(d) + (1 - \lambda)\alpha_j^m(d)$ .*

(ii) *If  $\delta$  belongs to  $(0.142, 0.25]$ , then there is no entry.* ■

The expected access price,  $E[\alpha_j(\lambda, d)]$ , is non-increasing in the bargaining power of the entrant,  $\lambda$ , because  $\alpha_j^\mu(d) \leq \alpha_j^m(d)$ .

Next we solve the location stage. Now, when the entrant chooses his location, he does not know yet which firm nature will choose to make the access price offers. We assume that the entrant is risk neutral. The expected profit of the entrant is:

$$E[\pi_e(\alpha(\delta); \delta, \lambda)] = \begin{cases} \lambda\pi_e^i(\alpha_i^\mu(\delta); \delta) + (1 - \lambda)\pi_e^i(\alpha_i^m(\delta); \delta) & \text{if } \delta \text{ is on } (0, 0.142] \\ 0 & \text{if } \delta \text{ is on } (0.142, 0.25] \end{cases}$$

Figure 2 illustrates the profit function of the entrant for the extreme cases of  $\lambda = 0$  and  $\lambda = 1$ , as well as for  $\lambda = 0.5$ .<sup>30</sup>

[Figure 2]

The next Proposition characterizes the optimal location for the entrant, in terms of his distance to the access provider, denoted by  $d^*(\lambda)$ .

**Proposition 2:** *Let nature choose that the entrant makes the access price offers with probability  $\lambda$  on  $[0, 1]$ .*

(i) *The location stage has a unique equilibrium given by  $d^*(\lambda)$  on  $(0.095, 0.142]$ .*

(ii) *For  $\lambda$  on  $(0.001, 1]$ , the optimal distance to the access provider,  $d^*(\cdot)$ , is decreasing in  $\lambda$ ; otherwise it is independent of  $\lambda$ .* ■

Since  $d^*(1) = 0.095 < d^*(0) = 0.142 < 0.25$ , as illustrated in Figure 2, the entrant locates closer to the access provider, independently of the parties' relative bargaining power.

If the entrant makes the access price offers, the equilibrium access price is  $\alpha_i^\mu(l)$ , which is increasing in  $d$ . Thus, by moving closer to the access provider, the entrant obtains a lower access

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<sup>30</sup>Let  $d \leq 0.142$ , otherwise there is no entry. If  $\lambda = 0$ , the entrant accepts the monopoly price from the nearest incumbent. From Remark 2: (iii), the profit function of the entrant is increasing in  $d$ . Thus, the entrant locates at  $d = 0.142$ . If  $\lambda = 1$ , the profit function of the entrant is concave in  $d$  with a point of maximum at  $d = 0.095$ . Finally, if  $\lambda = 0.5$ , the profit function of the entrant is concave in  $d$  with a point of maximum at  $d = 0.104$ .

price. The access provider is indifferent for any location, because the individually rationality constraint is binding. Thus, the larger the bargaining power of the entrant,  $\lambda$ , the closer he locates to the access provider.

The next Corollary summarizes the equilibrium of the whole game for the case where the entrant makes the offers.

**Corollary 3:** *Let the entrant make the access price offers. In equilibrium:  $(d^*, \alpha_j^\mu, p_j^j, p_i^j, p_e^j) = (0.095, 265, 274, 242, 286)$  and  $(\pi_j^j, \pi_i^j, \pi_e^j) = (125, 130, 3)$ . ■*

If the entrant makes the access price offers, he locates closer to the access provider, pays a lower access price, and retail prices are lower than when the incumbents make the access price offers. The retail prices are lower because the entrant has lower costs. When the incumbents make the access price offers, the retail prices are higher than when there is no entry, whereas when the entrant makes the access price offers, the incumbent that does not provide access sets a lower retail price than when there is no entry.

## 4.2 Bargaining the Access Price before Location

When bargaining with the incumbent, it is unclear whether the entrant can commit not to change his location afterwards. To analyze the impact on the product choice of the entrant of his inability to commit to a location before bargaining the access price, we develop a version of the model that reverses the order of the location and the bargaining stages.

Consider the model of section 2 except that the bargaining stage precedes the location stage.

The equilibrium of the price competition stage is as described in section 3.1.

The next Proposition characterizes the optimal location for the entrant, in terms of his distance to the access provider, denoted by  $d^{**}(a_j)$ .

**Proposition 3:** *Let the bargaining stage precede the location stage. Suppose that the entrant accepted a profitable offer from firm  $j$ ,  $\alpha_j \leq \alpha_j^u(0) = 450$ . In equilibrium:*

- (i) *The entrant locates at a distance from firm  $j$ ,  $d^{**}(a_j)$  on  $(0, 0.25]$ .*
- (ii) *The optimal distance from firm  $j$  is decreasing in the access price, with  $d^{**}(0) = 0.25$ .*

■

[Figure 3]

Each  $\alpha_j$  induces a unique location choice by the entrant,  $d^{**}(\alpha_j)$ . Figure 3 represents the optimal location as a function of  $\alpha_j$ , which cannot be solved explicitly for  $d^{**}(\alpha_j)$ . Hence, when  $\alpha_j$  is accepted, it is as if the incumbent  $j$ , chose the entrant's location indirectly through his offer  $\alpha_j$ . The next Lemma presents the equilibria of the bargaining stage.

**Lemma 4:** *Let the bargaining stage precede the location game. At the bargaining game, in equilibrium, one incumbent offers  $\alpha_j = 450$ , the other incumbent makes an unprofitable offer, and the entrant chooses the former offer.* ■

If the bargaining stage precedes the location stage, the entrant chooses the lowest profitable access price offered. The incumbents make their offers bearing this in mind, and also that it may be better not to be the access provider. Suppose that one of the incumbents offers a high access price. If the entrant accepts the offer, he locates close to the access provider. The other incumbent may prefer not to outbid this offer because he will benefit from having a high cost rival, although at a lower distance than before entry. An incumbent is better off by undercutting the rival only if the price access offer is low, and the entrant therefore locates close to the midpoint between the incumbents.

The profit functions of both of the incumbents are decreasing in  $d$ , where  $d$  was induced by the access provider when  $\alpha_j$  was set. If the optimal distance to the access provider is large, this means that the access price accepted at stage 1 is relatively low. This leads to lower profits for both incumbents. If the lowest offer is  $\alpha_j = 240$ , the entrant locates at  $d = 0.148$ . In this case, the access provider has the same profit as the other incumbent. For higher offers, the access provider will have a lower profit than the other incumbent. Thus, there is also an equilibrium in which both incumbents offer  $\alpha_j = 240$ , the entrant selects one offer randomly, and locates at  $d = 0.148$ . However, we rule out this equilibrium because it is unstable.<sup>31</sup>

The next Corollary summarizes the equilibrium of the whole game for the case where the bargaining stage precedes the location stage and the incumbents make the access price offers.

**Corollary 4:** *Let the bargaining stage precede the location stage. In equilibrium:  $(d^{**}, \alpha_j, p_j^j)$ ,*

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<sup>31</sup>Starting from these offers, if one of the incumbents offers a higher access price, the other one best responds by offering an even higher access price, because when offers exceed 240 it is better to be the incumbent not providing access. This overbidding pushes the access price towards 450. If one of the incumbents offers a lower access price, the other one best responds by undercutting the offer and so forth, until  $\alpha_i = 0$  is reached. This is also an equilibrium. However, we exclude it because playing  $\alpha_i = 0$  is a weakly dominated strategy.

$$p_i^j, p_e^j) = (0, 450, 450, 350, 450) \text{ and } (\pi_j^j, \pi_i^j, \pi_e^j) = (135, 245, 0). \quad \blacksquare$$

The bias of the entrant towards locating close to the access provider is also present in this version of the game, and is thus independent of the ordering of the location and bargaining stages. In fact, the bias is even stronger as the distance between the entrant and the access provider is now  $d^{**} = 0$ , whereas when the location stage precedes the bargaining stage it was  $d^* = 0.142$ . When the location stage precedes the bargaining stage, the entrant locates close to one of the incumbents in order to get a low access price offer from this firm. When the bargaining stage precedes the location stage, it is one of the incumbents that induces nearby entry, by offering a high access price.

If the bargaining stage precedes the location stage, the entrant has lower profits than when the location stage precedes the bargaining stage. Hence, it is in the entrant's interest to commit himself to a given location, before bargaining the access price. The explanation is simple. Suppose that one of the incumbents offers a high access price. If the entrant accepts this offer, then he locates close to the access provider, due to the wholesale effect. In the extreme case in which the entrant and the incumbent locate as close as possible, competition between the two firms pushes the retail prices down to the marginal cost of the entrant, the highest cost firm. The incumbent is able to sustain higher equilibrium retail prices and to capture all the entrant's retail profit, thus benefitting from this situation.<sup>32</sup>

### 4.3 Open Access Regulation

To ensure that entry occurs at a level of the access price that makes the entrant competitive, the sectorial regulator might impose open access regulation. To analyze the impact of open access regulation on both the product choice of the entrant and the outcome of the bargaining game, we develop a version of the model where the sectorial regulator imposes that the incumbents must offer access to their bottleneck input at an access price no higher than a given maximum level.

Consider the model of section 2, except that in stage 2 the incumbents make their access price offers subject to the regulatory constraint that both incumbents must offer access to their bottleneck input at an access price no higher than a given maximum level,  $\hat{\alpha}$ , i.e., both incumbents must offer  $\alpha_i \leq \hat{\alpha}$ ,  $i = 0, 1$ . We restrict attention to  $\hat{\alpha} < \alpha_i^m(0.25) = 261$ . This

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<sup>32</sup>Suppose the entrant makes the access price offers. The entrant makes an individually rational offer to one of the incumbents, which is accepted, and then locates at a distance of 0.128 from the access provider.

ensures that, for any location, at least one of the incumbents cannot set his monopoly price, or make an unprofitable offer.

Denote by  $d^r(\hat{\alpha})$ , the optimal distance to the access provider for the entrant when both incumbents are subject to offering access prices no larger than  $\hat{\alpha}$ . Let firm  $k = 0, 1$  be the closest incumbent to the entrant, and let  $\bar{\alpha}_k(d)$  denote the access price of firm  $k$  that firm  $i$  finds it profitable to undercut.

The equilibrium of the price competition stage is as described in section 3.1.

The next Lemma presents the equilibria of the bargaining stage.

**Lemma 5:** *Let the incumbents make access price offers subject to  $\alpha_i \leq \hat{\alpha}$ ,  $i = 0, 1$ . The bargaining stage has two equilibria.*

(i) *If  $\hat{\alpha}$  belongs to  $[\bar{\alpha}_k(d), \alpha_i^m(0.25)]$ , both incumbents offer  $\hat{\alpha}$ , and the entrant accepts the offer of the closest incumbent.*

(ii) *If  $\hat{\alpha}$  belongs to  $[0, \bar{\alpha}_k(d))$ , both incumbents offer 0, and the entrant is indifferent between accepting the offer of either of the incumbents.* ■

Suppose that the maximum access price is high, i.e.,  $\hat{\alpha} \geq \bar{\alpha}_k(d)$ . If an incumbent offers  $\hat{\alpha}$ , it is not profitable for the rival to outbid this access price, given that he will benefit from the entrant having higher marginal costs. Suppose now that the maximum access price is low, i.e.,  $\hat{\alpha} < \bar{\alpha}_k(d)$ . The incumbents engage in successive price cuts until the access price equals marginal cost. Thus, comparison with Lemma 2 shows that open access regulation can induce a perfectly competitive outcome on the wholesale market without imposing it.

The next Proposition presents the equilibria of the location stage.

**Proposition 4:** *Let the incumbents make access prices offers subject to  $\alpha_i \leq \hat{\alpha}$ ,  $i = 0, 1$ . The location stage has two equilibria:*

$$d^r(\hat{\alpha}) = \begin{cases} 0.25 & \text{if } \hat{\alpha} \text{ belongs to } [0, \bar{\alpha}_k(0.25)), \\ d^{**}(\hat{\alpha}) & \text{if } \hat{\alpha} \text{ belongs to } [\bar{\alpha}_k(0.25), \alpha_i^m(0.25)]. \end{cases}$$

■

If the maximum access price is high, i.e.,  $\hat{\alpha} \geq \bar{\alpha}_k(0.25) = 219$ , the equilibrium access price is higher than the marginal cost. As a consequence, the wholesale effect is present, and the incumbents have different incentives when setting their retail prices. As mentioned in section

3.2.1, the access provider charges a higher retail price than the incumbent that does not provide access. This makes the entrant locate closer to the access provider. The optimal distance is obtained as in Proposition 3, i.e.,  $d^r(\hat{\alpha}) = d^{**}(\hat{\alpha})$ .<sup>33</sup>

If the maximum access price is low, i.e.,  $\hat{\alpha} < \bar{\alpha}_k(0.25)$ , the equilibrium access price equals marginal cost. Consequently, the wholesale effect is null, and the entrant locates in the midpoint between the incumbents,  $d^r(\hat{\alpha}) = 0.25$ .

Open access regulation, as defined above, ensures entry. But more interestingly, open access regulation also affects the location of the entrant. The price ceiling decreases the equilibrium access prices, and thereby the wholesale effect. Consequently, it leads the entrant to locate equidistantly from both incumbents. Whether the entrant locates on the midpoint between the incumbents or closer to the access provider depends, therefore, on the magnitude of  $\hat{\alpha}$ .

The next Corollary summarizes the equilibrium of the whole game for the case where the incumbents make the access price offers subject to open access regulation, for  $\hat{\alpha} < \bar{\alpha}_k(0.25)$ .

**Corollary 5:** *Let the incumbents make the access price offers subject to  $\alpha_i \leq \hat{\alpha} < \bar{\alpha}_k(0.25)$ ,  $i = 0, 1$ . In equilibrium:  $(d^r(\hat{\alpha}), \alpha_k, p_k^k, p_i^k, p_e^k) = (0.25, 0, 109, 109, 86)$  and  $(\pi_k^k, \pi_i^k, \pi_e^k) = (36, 36, 30)$ .* ■

When the incumbents are subject to open access obligations, and the regulated access price is low, i.e.,  $\hat{\alpha} < \bar{\alpha}_k(0.25)$ , the entrant pays a zero access price, and the retail prices are lower than when the incumbents have no open access obligations, or when there is no entry.<sup>34</sup>

## 5 Concluding Remarks

In this article, we analyzed the product differentiation decision of a downstream entrant that must purchase access to a bottleneck input from one of two vertically integrated incumbents, who will compete with him in the downstream market. Our analysis draws attention to some unexpected consequences on the product choice of the entrant, caused by the circumstance that the access provider interacts with the entrant both in the retail and the wholesale markets. We

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<sup>33</sup>If  $\hat{\alpha} \geq \bar{\alpha}_k(0.25)$ , the entrant is offered  $\hat{\alpha}$  regardless of his location. Thus, the location choice does not influence the access price offers, and the optimal location is determined as in Proposition 3, i.e., as if the bargaining stage preceded the location choice.

<sup>34</sup>The analysis of asymmetric open access regulation, in which only one of the incumbents is subject to open access obligations, does not change qualitatively the results.

showed that the entrant prefers a product that is a closer substitute of the product of the access provider than the product of the other incumbent. Our results are robust with respect to: which party makes the access price offers, the order of the location and bargaining stages, and the existence of open access regulation, if the regulated access price is not low.

In all of our models, entry always occurs in equilibrium. This might give the impression that there are no foreclosure or competition problems in this context. This idea is misleading for two reasons. First, entry is always an equilibrium only if the entrant is free to locate where ever he wishes. However, it might be unfeasible for the entrant to locate close to either of the incumbents. The incumbents might have proprietary technologies, or it might be hard for the entrant to mimic the brand images of the incumbents. The second reason is that entry always occurs in our model, but at the expense of retail prices increasing with entry.

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## Appendix

In the appendix we prove the results in the main text. The proofs of Lemma 1, Corollaries 1 to 5, Proposition 4 and Remarks 1 and 2 are obvious, and therefore are omitted. We start by presenting some useful expressions omitted in the main text. For a given location and access price, the three firms equilibrium profits are as follows:

$$\begin{aligned}
\pi_j^j(\alpha_j; d) &= \frac{d(1-2d)(2d+1)(11d+2d^2-9)^2}{(1-2d)32(8d^2-4d-3)^2} + \\
&\quad + \frac{8(1-2d)(d+1)(44d^3-64d^2-9d+27)\alpha_j}{(1-2d)32(8d^2-4d-3)^2} + \frac{-16(36d^3-40d^2-13d+18)\alpha_j^2}{(1-2d)32(8d^2-4d-3)^2} \\
\pi_i^j(\alpha_j; d) &= \frac{1-d}{1-2d} \frac{(8d\alpha_j-12\alpha_j-7d+28d^2-4d^3-3)^2}{32(8d^2-4d-3)^2} \\
\pi_e^j(\alpha_j; d) &= \frac{d}{1-2d} \frac{(4(6d-5)\alpha_j+(2d-1)(10d^2-5d-9))^2}{32(8d^2-4d-3)^2}
\end{aligned}$$

If access is provided by incumbent  $j$  at monopoly price  $\alpha_j^m(d)$  we have:

$$\begin{aligned}
\alpha_j^m(d) &= \frac{-(36d+109d^2-126d^3-84d^4+88d^5-27)}{4(36d^3-40d^2-13d+18)} \\
\pi_j^j(\alpha_j^m(d); d) &= \frac{(108d+231d^2-270d^3-76d^4+56d^5-81)}{-32(36d^3-40d^2-13d+18)}
\end{aligned}$$

**Lemma 2:** Let  $\beta(l) := \frac{(2l-1)(3l+1)}{l(6l-5)}$ . In the presence of two profitable offers, the entrant located at  $l$  prefers to accept  $\alpha_0$  instead of  $\alpha_1$  if, and only if,

$$\pi_e^0(\alpha_0; l) > \pi_e^1(\alpha_1; l) \Leftrightarrow \alpha_0 < \beta(l)\alpha_1$$

Hence, the optimal acceptance rule of the entrant is:

$$AC_e(\alpha_0, \alpha_1; l) = \begin{cases} \text{none} & \text{if } \alpha_i^u(l) \leq \alpha_i, i = 0, 1 \\ \text{zero} & \text{if } \alpha_0 < \alpha_0^u(l) \wedge (\alpha_0 < \beta(l)\alpha_1 \vee \alpha_1 > \alpha_1^u(l)) \\ \text{one} & \text{if } \alpha_1 < \alpha_1^u(l) \wedge \left( \alpha_1 < \frac{\alpha_0}{\beta(l)} \vee \alpha_0 > \alpha_0^u(l) \right) \end{cases}$$

We now turn to the incumbents' best response functions. There are two possible ways of undercutting a rival's offer: (i) The offer may be so high that it may be undercut with the monopoly access price or (ii) it may require a lower price.

The first case occurs, for incumbent 0, when  $\alpha_1 > \frac{\alpha_0^m(l)}{\beta(l)}$ . It is preferable to undercut with the monopoly price if, and only if,  $\pi_0^0(\alpha_0^m(l), l) > \pi_0^1(\alpha_1, l) \Leftrightarrow \alpha_1 < \bar{\alpha}_1^m(l)$ , with

$$\begin{aligned}
\bar{\alpha}_1^m(l) &:= \frac{4l(2l+1)(11l+2l^2-9)(36l^3-40l^2-13l+18)}{16(2l+1)(l+1)(36l^3-40l^2-13l+18)} + \\
&\quad + \frac{4\sqrt{l(2l+1)(36l^3-40l^2-13l+18)(270l^3-231l^2-108l+76l^4-56l^5+81)(8l^2-4l-3)^2}}{16(2l+1)(l+1)(36l^3-40l^2-13l+18)}
\end{aligned}$$

We have  $\frac{\alpha_0^m(l)}{\beta(l)} > \bar{\alpha}_1^m(l)$  if and only if  $l > 0.215$ . In such case, even if it is possible to undercut with the monopoly price, firm 0 should not do so.

The second case occurs, for incumbent 0, when  $\alpha_1 > \frac{\alpha_0^m(l)}{\beta(l)}$ . Incumbent 0 prefers to undercut incumbent 1's offer if, and only if

$$\pi_0^0(\beta(l)\alpha_1, l) > \pi_0^1(\alpha_1, l) \Leftrightarrow \alpha_1 < \bar{\alpha}_1(l) := \frac{(36l^3 - 20l^2 - 21l + 9)(l+1)(6l-5)l}{-4(10l - 23l^2 - 24l^3 + 36l^4 + 3)}$$

We have  $\frac{\alpha_0^m(l)}{\beta(l)} < \bar{\alpha}_1(l)$  if and only if  $l < 0.215$ . In this case, firm 0 should always undercut the rival's price.

Thus, the access price best response correspondence of firm 0 is:<sup>35</sup>

$$\zeta_0(\alpha_1) = \begin{cases} \beta(l)\alpha_1 & \text{if } \alpha_1 < \min \left\{ \frac{\alpha_0^m(l)}{\beta(l)}, \bar{\alpha}_1(l) \right\} \\ \alpha_0^m(l) & \text{if } \frac{\alpha_0^m(l)}{\beta(l)} < \alpha_1 < \bar{\alpha}_1^m(l) \vee (\alpha_1 > \alpha_1^u(l) \wedge l < 0.142) \\ \alpha_0^u(l) & \text{if else} \end{cases}$$

With these best response functions we can now obtain the equilibrium.<sup>36</sup> Start by noting that:<sup>37</sup>

When  $l < 0.220$ , we have  $\alpha_1^m < \min \left\{ \frac{\alpha_0^m}{\beta}, \bar{\alpha}_1 \right\}$ : any offer  $\alpha_1$  equal to or below monopoly price will be undercut by firm 0 by setting  $\beta\alpha_1$ .

When  $0.220 < l < 0.25$  we have  $\bar{\alpha}_1 < \alpha_1^m < \frac{\alpha_0^m}{\beta}$ . Offers above  $\bar{\alpha}_1$  will not be undercut by firm 0. The others will be undercut by setting  $\beta a_1$ .

When  $0.25 < l < 0.385$  we have  $\bar{\alpha}_1 \leq \bar{\alpha}_1^m < \frac{\alpha_0^m}{\beta} < \alpha_1^m$ . Although monopoly access price offers by firm 1 can be successfully undercut with monopoly offers by firm 0 ( $\alpha_1^m > \frac{\alpha_0^m}{\beta}$ ), this will not occur ( $\alpha_1^m > \bar{\alpha}_1^m$ ). Lower offers will be undercut only if  $\alpha_1 < \bar{\alpha}_1$  by setting  $\beta\alpha_1$ .

When  $l > 0.385$  no offer by firm 1 will be undercut by firm 0 because  $\bar{\alpha}_1 < 0$  and  $\alpha_1^m > \bar{\alpha}_1^m$ .

In equilibrium, we cannot have a situation in which one firm prices above its monopoly access price and this offer is accepted. Reducing the offer would still result in its acceptance and would increase the profits of the access provider. It is straightforward to check that both offers equal to 0 is an equilibrium. We will now look for additional equilibria.

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<sup>35</sup>In some circumstances, firm 0 does not find it optimal to undercut any offer above  $\bar{\alpha}_1$ . By not undercutting we mean that it will best respond by setting any  $\alpha_0 > \beta\bar{\alpha}_1$ . However, playing any  $\alpha'_0 \in (\beta\bar{\alpha}_1, \alpha_0^u)$  is weakly dominated by playing  $\alpha_0^u$ . For any profitable  $\alpha_1 < \frac{\alpha'_0}{\beta}$  the payoff is the same regardless of firm 0 choosing  $\alpha'_0$  or  $\alpha_0^u$ . However, if  $\alpha_1 > \frac{\alpha'_0}{\beta} > \bar{\alpha}_1$  firm 0 will get  $\pi_0^0(\alpha'_0, l)$  when playing  $\alpha'_0$  or  $\pi_0^1(\alpha_1, l)$  when playing  $\alpha_0^u$ . The second payoff is clearly larger because with  $\alpha_1 > \bar{\alpha}_1$  firm 0 prefers not to undercut even if it could do so with its monopoly access price. Hence, whenever  $\alpha_1 > \bar{\alpha}_1$  we assume that firm 0's best response is  $\alpha_0^u$ , after elimination of weakly dominated strategies. The same holds for  $\bar{\alpha}_1^m$ .

<sup>36</sup>Firm 1's best response can be obtained replacing  $l$  with  $(\frac{1}{2} - l)$  and changing the subscript indexes.

<sup>37</sup>Every expression is a function of  $l$ . We have omitted the argument in the remainder of the proof.

(i) Let  $l$  belong to  $(0, 0.115)$ . Then  $\frac{1}{2} - l \in (0.385, 0.5)$ . There can be no equilibrium with a positive offer by firm 1 accepted, because firm 0 will undercut any offer. Firm 0 setting any positive offer below its monopoly price cannot be an equilibrium. Given that firm 1's best response is not to undercut, firm 1 would set a price strictly above  $\frac{\alpha_0}{\beta(l)}$ . In this case, firm 0 would benefit from increasing its price. Hence, firm 0 setting the monopoly price is the only equilibrium.

(ii) Let  $l$  belong to  $(0.115, 0.142)$ . Then  $\frac{1}{2} - l \in (0.358, 0.385)$ . There can be no equilibrium with a positive offer by firm 1 accepted, because firm 0 will undercut any offer. There can be no equilibrium with firm 0 making positive offers below  $\bar{\alpha}_0$  because these would be undercut by firm 1. Pricing above  $\bar{\alpha}_0$  leads to no undercutting by firm 1 and hence, for the same reason as in (i), firm 0 could increase its offer up to the monopoly price (note that  $\bar{\alpha}_0 < \alpha_0^m$  for  $l < 0.280$ ).

(iii) Let  $l$  belong to  $(0.142, 0.220)$ . Then  $\frac{1}{2} - l \in (0.280, 0.358)$ . There can be no equilibrium with a positive offer by firm 1 accepted because firm 0 will undercut any offer. There can be no equilibrium with firm 0 making offers below  $\bar{\alpha}_0$  because these would be undercut by firm 1. Pricing above  $\bar{\alpha}_0$  leads to firm 1 best responding by making an unprofitable offer. In this case, firm 0's best response would also be making an unprofitable offer.

(iv) Let  $l$  belong to  $(0.220, 0.25)$ . Then  $\frac{1}{2} - l \in (0.25, 0.280)$ . We will divide this case in four sub-cases involving strictly positive offers: i)  $\alpha_0 < \bar{\alpha}_0$  and  $\alpha_1 < \bar{\alpha}_1$ : This cannot be an equilibrium because the firm whose offer is not accepted should undercut the other firm. ii)  $\alpha_0 < \bar{\alpha}_0$  and  $\alpha_1 > \bar{\alpha}_1$ . For  $l < 0.25$ , firm 0's offer would be selected by the entrant. However, there can be no such equilibrium because firm 1 should undercut. iii)  $\alpha_0 > \bar{\alpha}_0$  and  $\alpha_1 < \bar{\alpha}_1$ . In this case it is not clear which offer will be accepted. If it is firm 1's, firm 0 should undercut and hence we cannot have an equilibrium. If it is firm 0's, then firm 1 would best respond by not undercutting, i.e., by making an unprofitable offer. But in such case, firm 0 would prefer not to make a profitable offer. iv)  $\alpha_0 > \bar{\alpha}_0$  and  $\alpha_1 > \bar{\alpha}_1$ . If firm  $i$ 's offer is the best for the entrant, then firm  $j$  would best respond by not undercutting, i.e., by making an unprofitable offer. But in such case, firm  $i$  would also prefer not to make a profitable offer. Making unprofitable offers is the only additional equilibrium if firms do not play weakly dominated strategies.

(v) For larger values of  $l$ , one can reverse the incumbent's roles in (i) to (iv) and obtain the symmetric results. ■

**Lemma 3:** Recall that the strategy space for incumbent  $i = 0, 1$  is  $\{accept, reject\}$  where *accept* means "accept offer  $\alpha_i$ " and *reject* stands for "reject offer  $\alpha_i$ ". Let us assume that the offers made are such that  $\pi_e^0(\alpha_0; l) \geq \pi_e^1(\alpha_1; l)$ .<sup>38</sup> Then, the game incumbent's play is

<sup>38</sup>I.e.,  $\alpha_0 \leq \beta(l)\alpha_1$ . In words, in case of acceptance of both offers, the entrant will select firm 0 as his access

represented in its normal form by:

		Firm 1:	
		<i>accept</i>	<i>reject</i>
Firm 0:	<i>accept</i>	$\pi_0^0(\alpha_0; l), \pi_1^0(\alpha_0; l)$	$\pi_0^0(\alpha_0; l), \pi_1^0(\alpha_0; l)$
	<i>reject</i>	$\pi_0^1(\alpha_1; l), \pi_1^1(\alpha_1; l)$	$\pi_0^n, \pi_1^n$

Start by noting that: (i)  $(accept, accept)$  is an equilibrium if and only if  $\pi_0^0(\alpha_0; l) \geq \pi_0^1(\alpha_1; l)$ . (ii)  $(accept, reject)$  is an equilibrium if and only if  $\pi_0^0(\alpha_0; l) \geq \pi_0^n$ . (iii)  $(reject, accept)$  is an equilibrium if and only if  $\pi_0^0(\alpha_0; l) \leq \pi_0^1(\alpha_1; l)$  and  $\pi_1^1(\alpha_1; l) \geq \pi_1^n$ . (iv)  $(reject, reject)$  is an equilibrium if and only if  $\pi_0^0(\alpha_0; l) \leq \pi_0^n$  and  $\pi_1^1(\alpha_1; l) \leq \pi_1^n$ . Additionally, if the entrant makes a profitable offer to an incumbent, it cannot make a profitable offer to its rival,

$$\pi_0^0(\alpha_0; l) \geq \pi_0^n \Rightarrow \pi_1^1(\alpha_1; l) < \pi_1^n,$$

and if an offer is profitable for firm 0, it will have a positive effect on firm 1's profit. This effect is such that firm 1 will gain more than firm 0:

$$\pi_0^0(\alpha_0; l) \geq \pi_0^n \Rightarrow \pi_1^0(\alpha_0; l) > \pi_0^0(\alpha_0; l) \geq \pi_0^n = \pi_1^n$$

Thus, if  $\pi_1^1(\alpha_1; l) \geq \pi_1^n$ , this implies that (i)  $\pi_0^0(\alpha_0; l) < \pi_0^n$  for all  $a_0$  and (ii)  $\pi_0^1(\alpha_1; l) > \pi_0^n$ . (i) and (ii) imply that  $\pi_0^1(\alpha_1; l) > \pi_0^0(\alpha_0; l)$ . Hence, if  $\pi_1^1(\alpha_1; l) \geq \pi_1^n$ ,  $(reject, accept)$  is an equilibrium. Furthermore, given that  $\pi_0^1(\alpha_1; l) > \pi_0^0(\alpha_0; l)$  and  $\pi_0^0(\alpha_0; l) < \pi_0^n$ , the other alternatives cannot be equilibria. This case is only possible if  $l \geq 0.5 - .142$ .

If  $\pi_0^0(\alpha_0; l) \geq \pi_0^n$ ,  $(accept, reject)$  is an equilibrium. As  $\pi_0^0(\alpha_0; l) \geq \pi_0^n \Rightarrow \pi_1^1(\alpha_1; l) < \pi_1^n$ ,  $(reject, reject)$  and  $(reject, accept)$  cannot be equilibria. As  $\pi_1^1(\alpha_1; l) < \pi_1^n$  it is easy to check that choosing *accept* is a weakly dominated strategy for firm 1. Hence we rule  $(accept, accept)$  out as an equilibrium. This case is only possible if  $l \leq 0.142$ .

Finally, if  $\pi_0^0(\alpha_0; l) < \pi_0^n$  and  $\pi_1^1(\alpha_1; l) < \pi_1^n$ , it is easy to check that firm 1 choosing *accept* is a weakly dominated strategy. If we eliminate this strategy, the only equilibrium is  $(reject, reject)$ . This occurs for any pair of offers if  $l \in (0.142, 0.5 - 0.142)$ .

The entrant's profits are:  $\pi_e^0(\alpha_0; l)$  if  $\pi_0^0(\alpha_0; l) \geq \pi_0^n$ ;  $\pi_e^1(\alpha_1; l)$  if  $\pi_1^1(\alpha_1; l) \geq \pi_1^n$  and 0 otherwise. As  $\pi_e^0(\alpha_0; l) \geq \pi_e^1(\alpha_1; l)$  the entrant will choose  $\alpha_0^\mu(l)$  such that  $\pi_0^0(\alpha_0^\mu(l); l) = \pi_0^n$ , i.e., it will choose the lowest possible access price that still makes entry profitable for the access provider and make an(y) unprofitable offer for the other incumbent. Naturally, there is also

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provider.

the symmetric case in which the access provider is firm 1. Summing up, the entrant will locate at distance  $d \leq 0.142$  from incumbent  $j$  and offer

$$\alpha_j^\mu(d) = \frac{(1-2d)(d+1)(44d^3 - 64d^2 - 9d + 27)}{4(36d^3 - 40d^2 - 13d + 18)} + \frac{\left((8d^2 - 4d - 3)\sqrt{(2d-1)(d+1)(65d + 6d^2 - 132d^3 + 56d^4 - 9)}\right)}{4(36d^3 - 40d^2 - 13d + 18)}$$

■

**Proposition 1 and Proposition 2:** Assume initially that the incumbents make the offers, i.e.,  $\lambda = 0$ . As seen in Lemma 2, entry will occur at the monopoly access price. The entrant will choose incumbent  $j$  and locate at a distance  $d$  from this firm such that  $d$  maximizes  $\pi_e^j(\alpha_j^m(d); d)$ , subject to  $d \leq 0.142$ . For  $d \in [0, 0.142]$ ,  $\pi_e^j(\alpha_j^m(d), d)$  is increasing with  $d$  and hence the entrant will choose  $d^* = 0.142$ .

If  $\lambda = 1$ , meaning that the entrant makes the access price offers, the equilibrium access price is  $\alpha_j^\mu(d)$ , and the entrant's profit function,  $\pi_e^j(\alpha_j^\mu(d); d)$ , is single-peaked in  $d$ , for  $d < 0.142$ . The interior profit maximizing location,  $d^*$ , can be calculated numerically and is  $d^* = 9.4809 \times 10^{-2}$ . Summing up, entrant profits and access price in both cases are:

$$\begin{aligned} \max \pi_e^j(\alpha_j^m(d); d) &= 1.0729 \text{ at } d^* = 0.142 \text{ and } \alpha_j^m(d^*) = 325 \\ \max \pi_e^j(\alpha_j^\mu(d); d) &= 2.8268 \text{ at } d^* = 9.4809 \times 10^{-2} \text{ and } \alpha_j^\mu(d^*) = 265 \end{aligned}$$

The expected profit is a convex combination of these two cases and it is maximized either at  $d = 0.142$  or at  $d^*(\lambda) \in [9.4809 \times 10^{-2}, 0.142)$ , the solution to

$$\lambda \frac{\partial (\pi_e^j(\alpha_j^\mu(d); d))}{\partial d} + (1 - \lambda) \frac{\partial (\pi_e^j(\alpha_j^m(d); d))}{\partial d} = 0$$

We have a corner solution, i.e.,  $d^* = 0.142$ , if and only if  $\lambda \pi_e^j(\alpha_j^\mu(d); d) + (1 - \lambda) \pi_e^j(\alpha_j^m(d); d)$  is increasing in  $d$ , i.e., if

$$\lambda \left. \frac{\partial (\pi_e^j(\alpha_j^\mu(d); d))}{\partial d} \right|_{d=0.142} + (1 - \lambda) \left. \frac{\partial (\pi_e^j(\alpha_j^m(d); d))}{\partial d} \right|_{d=0.142} > 0 \Leftrightarrow \lambda < 1.2554 \times 10^{-3}$$

Otherwise, with an interior solution  $d^*(\lambda)$ , we have that

$$\frac{\partial d}{\partial \lambda} = - \frac{\frac{\partial (\pi_e^j(\alpha_j^\mu(d); d))}{\partial d} - \frac{\partial (\pi_e^j(\alpha_j^m(d); d))}{\partial d}}{\frac{\partial \left( \lambda \frac{\partial (\pi_e^j(\alpha_j^\mu(d); d))}{\partial d} + (1 - \lambda) \frac{\partial (\pi_e^j(\alpha_j^m(d); d))}{\partial d} \right)}{\partial d}} < 0$$

■

**Proposition 3:** We start by showing that if the entrant has accepted a profitable offer from firm  $j$ ,  $\alpha_j < \alpha_j^m(d)$ , it will locate at a distance  $d^{**}(\alpha_j)$  from firm  $j$  such that

$$\alpha_j = \frac{-(-2d^{**} + 1)(4d^{**} - 1)(9d^{**} + 2d^{**2} - 80d^{**3} + 80d^{**4} + 27)}{-296d^{**} + 992d^{**2} - 1472d^{**3} + 768d^{**4} + 60}$$

Assuming that the entrant has accepted a profitable offer  $\alpha_j < \alpha_j^u(d)$ , it is straightforward to show that

$$\begin{aligned} \frac{\partial \pi_e^j(\alpha_j; d)}{\partial d} &= \frac{4(6d - 5)\alpha_j - 13d - 20d^2 + 20d^3 + 9}{32(8d^2 - 4d - 3)^3(2d - 1)^2} \times \\ &[(1 - 2d)(4d - 1)(9d + 2d^2 - 80d^3 + 80d^4 + 27) + 4(248d^2 - 74d - 368d^3 + 192d^4 + 15)\alpha_j] \end{aligned}$$

Given that the first term is negative, the sign of the derivative is given by the opposite sign of the last term.

Note that the last term is increasing in  $\alpha_j$  for any  $d$  because  $(248d^2 - 74d - 368d^3 + 192d^4 + 15) > 0$ . Evaluating the last term at  $\alpha_j = 0$ , it is positive for  $d > 0.25$ . Hence, we have established that  $\pi_e^j(\alpha_j; d)$  is decreasing in  $d$  for  $d > 0.25$  and for any  $\alpha_j$ . We will now analyze  $\pi_e^j(\alpha_j; d)$  for  $d < 0.25$ . It is easy to check that the function  $\pi_e^j(\alpha_j; d)$  is increasing with  $d$  at  $d = 0$  and decreasing at  $d = 0.25$ :

$$\begin{aligned} \left. \frac{\partial \pi_e^j(\alpha_j; d)}{\partial d} \right|_{d=0} &= \frac{(20\alpha_j - 9)^2}{288} > 0 \\ \left. \frac{\partial \pi_e^j(\alpha_j; d)}{\partial d} \right|_{d=0.25} &= \frac{(32\alpha_j - 11)\alpha_j}{28} < 0 \text{ for all } \alpha_j < \alpha_j^m(0.25) \end{aligned}$$

We will now analyze the entrant's profit second derivative in  $d$ :

$$\begin{aligned} \frac{\partial^2 \pi_e^j(\alpha_j; d)}{\partial d^2} &= \\ &\frac{(12852d^2 - 5643d - 3864d^3 - 7336d^4 + 22400d^5 - 25600d^7 + 12800d^8 + 2592)(2d - 1)^3}{16(8d^2 - 4d - 3)^4(1 - 2d)^3} + \\ &+ \frac{8(8088d^2 - 5736d - 4800d^3 + 1088d^4 + 1280d^5 + 1341)(1 - 2d)^3\alpha_j}{16(8d^2 - 4d - 3)^4(1 - 2d)^3} + \\ &+ \frac{32(4406d - 19852d^2 + 50952d^3 - 84352d^4 + 88448d^5 - 52992d^6 + 13824d^7 - 345)\alpha_j^2}{16(8d^2 - 4d - 3)^4(1 - 2d)^3} \end{aligned}$$

It is tedious but straightforward to show that  $\frac{\partial^3 \pi_e^j(\alpha_j; d)}{\partial d^2 \partial \alpha_j} > 0$ . Evaluated at  $\alpha_j^m(d)$  the entrant's profit second derivative in  $d$  is negative for  $d < 0.25$ . Hence, for  $\alpha_j \leq \alpha_j^m(d)$  and  $d < 0.25$  the entrant's profit is concave in  $d$  and has a maximum implicitly defined by

$$(1 - 2d)(4d - 1)(9d + 2d^2 - 80d^3 + 80d^4 + 27) + 4(248d^2 - 74d - 368d^3 + 192d^4 + 15)\alpha_j = 0$$

Finally, note that

$$\frac{\partial d^{**}}{\partial \alpha_j} = - \frac{4(248d^2 - 74d - 368d^3 + 192d^4 + 15)}{8\alpha_j(248d - 552d^2 + 384d^3 - 37) + 60d^2 - 328d - 2304d^3 + 5600d^4 - 3840d^5 + 153}$$

Both numerator and denominator are positive for  $d < 0.25$ . Hence, optimal location is decreasing in  $\alpha_j$ . ■

**Lemma 4:** Choosing a given  $\alpha_j$  leads to a location choice by the entrant,  $d^{**}(\alpha_j)$ , that is uniquely defined. Figure 3 represents the optimal location as a function of  $\alpha_j$ , which cannot be solved explicitly for  $d^{**}(\alpha_j)$ .

[Figure 3]

Hence, it is as if incumbent 0, by offering  $\alpha_0$ , picks the entrant's location directly in case  $\alpha_0$  is accepted. Note that  $\pi_0^0(\alpha_0; l) < \pi_1^0(\alpha_0; l)$  for  $l < 0.148$  and that  $\pi_0^0(\alpha_0; l) > \pi_0^n$  for  $l < 0.128$ . For  $l = 0.148$  we have  $\alpha_0 = 240$  and for  $l = 0.128$  we have  $\alpha_0 = 287$ .

It is straightforward to show that the following are equilibria of this game:

- (i) Both firms setting  $\alpha_i = 0$  is always an equilibrium of this game.
- (ii) both firms offering  $\alpha_0 = \alpha_1 = 240$ .
- (iii) a firm making an offer  $\alpha_i = 450$  and the other making an unprofitable offer,  $\alpha_j > \max_d \alpha_j^u(d)$ .

The other cases are, for two profitable positive offers: (i)  $\alpha_0 = \alpha_1 > 240$ . By raising access price one firm becomes the incumbent that does not provide access and increases its profit. (ii)  $\alpha_0 = \alpha_1 < 240$ . By undercutting, either firm becomes the access provider and increases its profit. (iii)  $\alpha_0 < \alpha_1 \leq 240$ . Firm 1 increases profits by setting  $\alpha_1 = \alpha_0 - e$ . (iv)  $\alpha_0 > \alpha_1 \geq 240$ . Firm 1 increases profits by setting  $\alpha_1 = \alpha_0 + e$ . (v)  $\alpha_0 < 240 \leq \alpha_1$ . Firm 0 increases profits by setting a slightly higher price. (vi)  $\alpha_0 \leq 240 < \alpha_1$ . Firm 0 increases profits by setting a slightly higher price. ■

**Lemma 5:** Consider the case of incumbent 0. Depending on  $l$ , the following may occur:  $\alpha_0^m(l) > \hat{\alpha}$  if  $l$  is ‘small’ or the opposite if  $l$  is large. Let us initially assume the first case. As far as firm 0 is concerned, there are three relevant cases for its rival's offer: (i) it can be too large, i.e.,  $\alpha_1 > \alpha_1^u(l)$ , meaning that the entrant will not find it profitable, (ii) it can take intermediate values, i.e.,  $\frac{\hat{\alpha}}{\beta(l)} < \alpha_1 < \alpha_1^u(l)$ , such that it would be accepted by the entrant, but can be undercut by firm 0 with  $\hat{\alpha}$  and (iii) it can be so low that it is necessary to price below  $\hat{\alpha}$  to undercut it, i.e.,  $\alpha_1 < \frac{\hat{\alpha}}{\beta(l)}$ . Undercutting, however, is the best response only if the rival's offer is relatively low, i.e.,  $\alpha_1 < \bar{\alpha}_1(l, \hat{\alpha})$  or  $\alpha_1 < \bar{\alpha}_1(l)$ , respectively for cases (ii) and (iii) above. On the other hand, if  $\alpha_0^m(l) < \hat{\alpha}$ , the best response function is similar to the case with no



regulation with the exception that, when  $\hat{\alpha} < \alpha_0^u$ , incumbent 0 cannot make unprofitable offers. Hence, the access price best response correspondence of firm 0 is:

$$\zeta_0(\alpha_1) = \begin{cases} \beta(l)\alpha_1 & \alpha_1 < \min \left\{ \frac{\hat{\alpha}}{\beta(l)}, \bar{\alpha}_1(l) \right\} \\ \hat{\alpha} & \text{else} \end{cases}$$

if  $\hat{\alpha} < \alpha_0^m(l)$ ; or

$$\zeta_0(\alpha_1) = \begin{cases} \beta(l)\alpha_1 & \alpha_1 < \min \left\{ \frac{\alpha_0^m(l)}{\beta(l)}, \bar{\alpha}_1(l) \right\} \\ \alpha_0^m(l) & \frac{\alpha_0^m(l)}{\beta(l)} < \alpha_1 < \bar{\alpha}_1^m(l) \vee (\alpha_1 > \alpha_1^u \wedge \hat{\alpha} < \alpha_0^u) \\ \min \{ \hat{\alpha}, \alpha_0^u(l) \} & \text{else} \end{cases}$$

if  $\hat{\alpha} > \alpha_0^m(l)$ .

As  $\hat{a} > \alpha_0^m(l)$  implies  $l > 0.25$  and, with  $l > 0.25$ , we have  $\frac{\alpha_0^m(l)}{\beta(l)} > \bar{\alpha}_1^m(l)$  and  $\frac{\alpha_0^m(l)}{\beta(l)} > \bar{\alpha}_1(l)$  we can further simplify the best response function to

$$\zeta_0(\alpha_1) = \begin{cases} \beta(l)\alpha_1 & \alpha_1 < \bar{\alpha}_1(l) \\ \alpha_0^m(l) & \alpha_1 > \alpha_1^u \wedge \hat{a} < \alpha_0^u \\ \min \{ \hat{a}, \alpha_0^u(l) \} & \text{else} \end{cases}$$

Inspection of the best response functions reveals that there are the following candidates for equilibrium:<sup>39</sup>

- (i)  $(\hat{\alpha}, \hat{\alpha})$  with acceptance of firm  $i$ 's offer; (ii)  $(\hat{\alpha}, \frac{\hat{\alpha}}{\beta})$  with acceptance of firm 1's offer;<sup>40</sup> (iii)  $(\alpha_0^m, \hat{\alpha})$  with acceptance of firm 0's offer;<sup>41</sup> (iv)  $(0, 0)$  with acceptance of any offer.

We will start by showing under which conditions  $(\hat{\alpha}, \hat{\alpha})$  with acceptance of  $\hat{\alpha}$  by firm 0 is an equilibrium of this game. A necessary condition is that  $l < 0.25$ . This ensures that  $\hat{\alpha} < \alpha_0^u$  and that the entrant chooses firm 0's offer. Firm 0 does not want to lower its price if and only if  $\hat{\alpha} \leq \alpha_0^m$  which is always true for  $l \leq 0.25$ . For  $l < 0.25$  we have  $\hat{\alpha} < \beta\alpha_1^m$ , meaning that firm 1 cannot undercut with its monopoly price, but only with  $\alpha_1 = \frac{\hat{\alpha}}{\beta}$ . Firm 1 does not want to undercut its rival's offer if and only if  $\pi_1^0(\hat{\alpha}, l) > \pi_1^1(\frac{\hat{\alpha}}{\beta}, l) \Leftrightarrow \hat{\alpha} > \bar{\alpha}_0$ . Hence,  $(\hat{\alpha}, \hat{\alpha})$  with acceptance of firm 0's offer is an equilibrium if (i)  $l \leq 0.25$  and (ii)  $\hat{\alpha} > \bar{\alpha}_0$ . The same conditions hold, with the obvious differences, for  $(\hat{\alpha}, \hat{\alpha})$  with acceptance of firm 1's offer.

We now show that offers of  $(\hat{\alpha}, \frac{\hat{\alpha}}{\beta})$  with acceptance of firm 1's offer cannot be an equilibrium. Start by noting that  $\frac{\hat{\alpha}}{\beta}$  is a possible offer if and only if  $\beta > 1$  which implies  $l < 0.25$ . This also

<sup>39</sup>We exclude  $(\alpha_0^m, \alpha_1^m)$  and  $(\alpha_0^u, \alpha_1^u)$  because these are ruled out by the assumption that  $\hat{\alpha} < \alpha_1^m(0.25)$ . Note that  $\alpha_i^m, \alpha_i^u$  and  $\beta$  are all functions of  $l$ .

<sup>40</sup>The symmetric case is  $(\beta\hat{\alpha}, \hat{\alpha})$  with acceptance of firm 0's offer;

<sup>41</sup>Or the symmetric case,  $(\hat{\alpha}, \alpha_1^m)$  with acceptance of firm 1's offer;

implies that  $\hat{\alpha} < \alpha_0^m$  and hence, firm 0 will not find it profitable to decrease its offer if and only if  $\pi_0^0(\hat{\alpha}, l) < \pi_0^1(\frac{\hat{\alpha}}{\beta}, l) \Leftrightarrow \hat{\alpha} > \beta\bar{\alpha}_1$ . Firm 1, does not find it profitable to increase its price if  $\pi_1^1(\frac{\hat{\alpha}}{\beta}, l) > \pi_1^0(\hat{\alpha}, l) \Leftrightarrow \hat{\alpha} < \bar{\alpha}_0$ . It is impossible to verify both  $\hat{\alpha} > \beta\bar{\alpha}_1$  and  $\hat{\alpha} < \bar{\alpha}_0$  for  $l < 0.25$ . The same argument rules out  $(\beta(l)\hat{\alpha}, \hat{\alpha})$  with acceptance of firm 0's.

Offers of  $(\alpha_0^m, \hat{\alpha})$  with acceptance of  $\alpha_0^m < \hat{\alpha}$  cannot be an equilibrium of this game. Note that  $\alpha_0^m < \hat{\alpha}$  implies that  $l > 0.25$ . The entrant would accept  $\alpha_0^m$  if  $\alpha_0^m < \alpha_0^u$  and  $\alpha_0^m < \beta\hat{\alpha} \Leftrightarrow \hat{\alpha} > \frac{\alpha_0^m}{\beta}$ . But this is impossible if  $l > 0.25$ . Finally, it is straightforward to check that  $(0, 0)$  is always an equilibrium of this game. ■

## A Figures

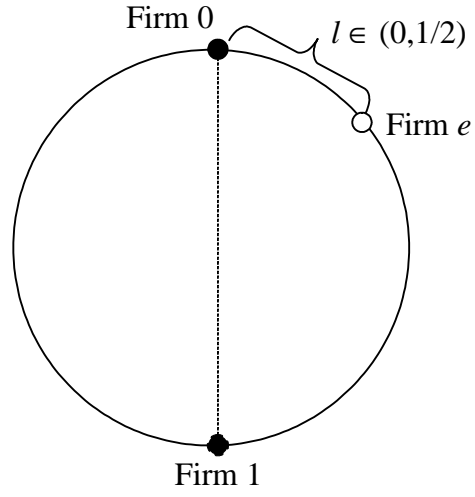


Figure 1: Firm's locations on the loop.

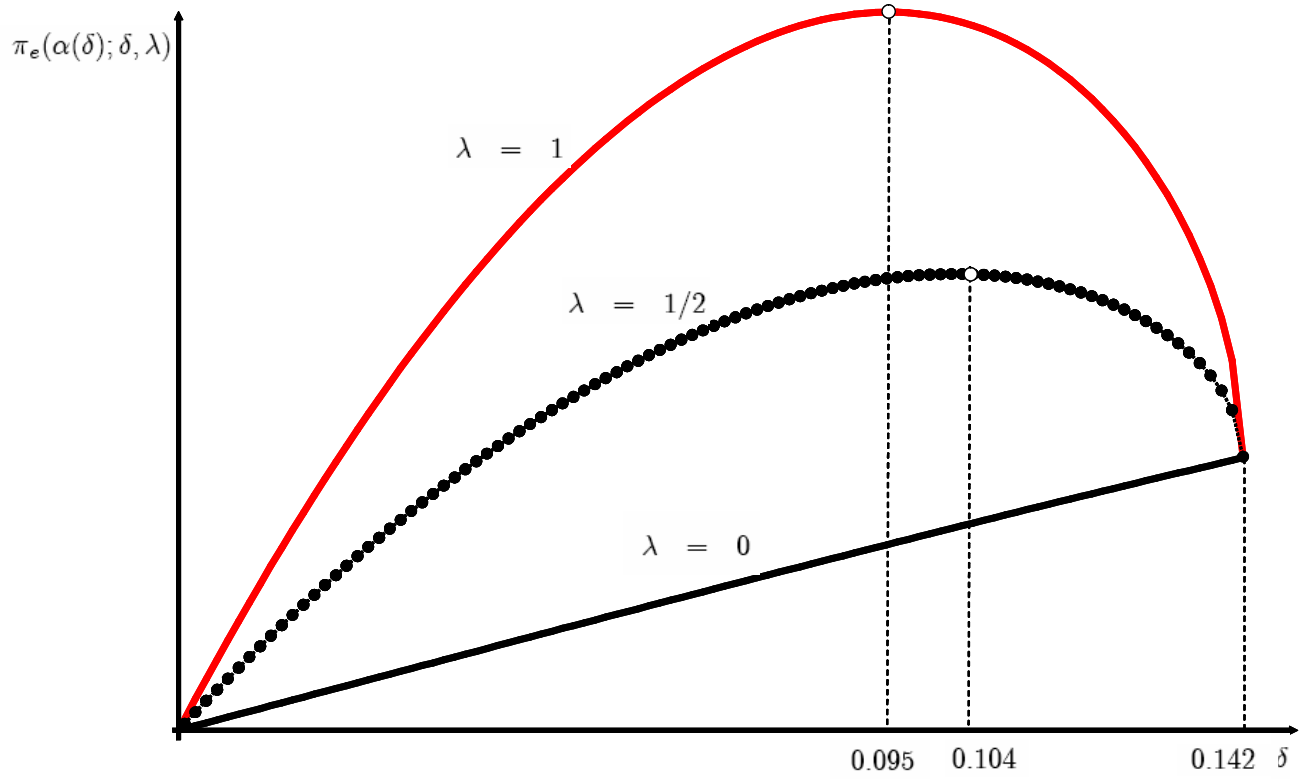


Figure 2: Entrant's profits.

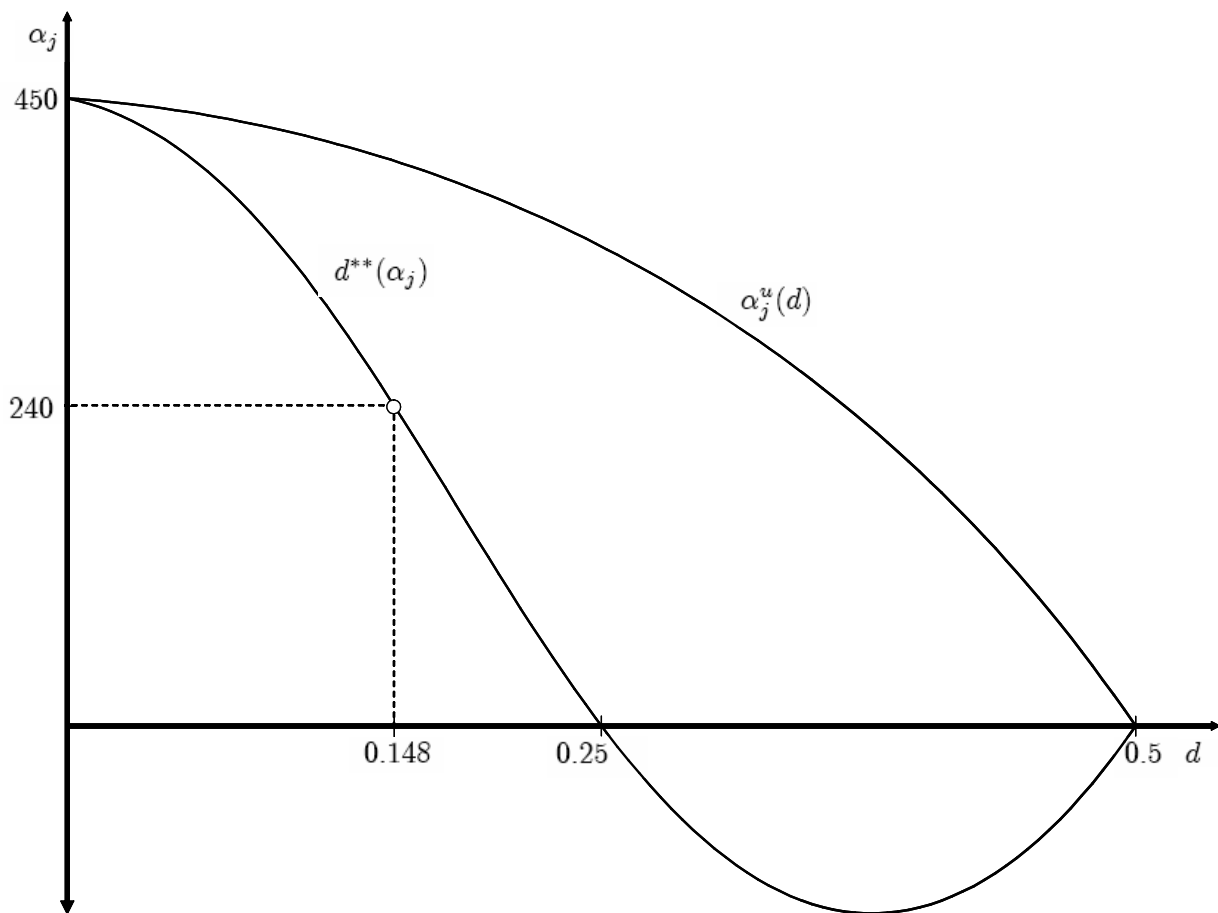


Figure 3: Entrant's optimal location as a function of the accepted access price.