Duopoly Competition with Common Shareholders

Duarte Brito*
Universidade Nova de Lisboa

Luís Cabral*
New York University and CEPR

Hélder Vasconcelos*
Universidade Católica do Porto and *CEPR*

September 2008

Abstract

We develop a methodology for evaluating competition and welfare when shareholders hold (partial) positions in more than one competitor. We consider different models of product market competition and shareholder control. In each case, we derive consumer welfare as a function of shareholding Herfindahl indexes. We then apply our results to estimate the effect on consumer welfare of Portugal Telecom's (PT) divestiture of its subsidiary, PT Multimedia (PTM). Our results indicate that a sale to independent shareholders benefits consumers considerably compared to a sale to current shareholders, as proposed by PT. Moreover, the impact of the divestiture is drastically different depending on whether PT's share is sold to PT's shareholders or PTM's shareholders.

*Brito: Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa; dmb@fct.unl.pt. Cabral: Stern School of Business, New York University; lcabral@stern.nyu.edu. Vasconcelos: Universidade Católica Portuguesa (CEGE); hvasconcelos@porto.ucp.pt. This paper was written while the authors were working as consultants to the Portuguese Competition Authority (PCA). Discussions with Gary Biglaiser (Visiting Scholar at the PCA) and António Gomes of the PCA are gratefully acknowledged. All errors are the authors' sole responsibility.

1 Introduction

Consider a duopoly where the shareholders of one firm are also shareholders of its competitor. Clearly, if one shareholder owns (or controls) both firms, then effectively we have a monopoly situation. What about a situation where several shareholders hold partial shares in both competitors? Presumably, market performance will fall somewhere between the extremes of monopoly and duopoly with total independent firms. How close to each extreme?

Lest this be thought of as a mere curiosity, we present a specific example that reflects this scenario. Until November 2007, Portugal Telecom (PT) held a 58% share of PT Multimedia (PTM). The two firms operated in several markets as the two main "competitors" (sometimes sole competitors). Responding to government pressure that PT divest of its share in PT Multimedia, a sale did take place on November 7, 2007. The details of the sale followed a proposal by PT which the Portuguese government approved. Specifically, PT's shares in PTM were sold to PT's shareholders in the proportion of their holdings of PT stock (that is, a PT shareholder with k times more stock in PT than another shareholder received k times more of PTM stock).

In this paper, we develop a methodology for evaluating competition and welfare when shareholders hold (partial) positions in more than one competitor, either directly or through cross-holdings. We consider different models of product market competition and shareholder weight (that is, the degree to which the shareholder is able to influence the firm's decision). With respect to market competition, we examine both the cases of differentiated products (Hotelling model) and capacity competition (Cournot model). With respect to shareholder weight, we consider both share holdings and voting rights. In each case, we show that consumer welfare can be summarized by a series of Herfindahl-type indexes of shareholding concentration and correlation.

We then use our "sufficient statistic" result to evaluate the impact of various possible share transfers. Some lead to expected results. For example, an increase in common holdings (shareholder k owns shares in firms i and j) or in cross holdings (firm i owns shares in firm j) leads to a decrease in consumer welfare. Some other share transfer patterns lead to somewhat unexpected results. For example, the sale of firm i's cross-holdings in firm j to firm j's shareholders (in proportion to their shareholdings) increases consumer welfare; but a sale to firm i's shareholders (also in proportion to their shareholdings) decreases consumer welfare.

Finally, we apply our results to estimate the effect on consumer welfare of PT's divestiture of its subsidiary, PTM. We consider several possible procedures, including the plan proposed by PT and implemented in November 2007. Our results indicate that the actual divestiture is unlikely to have increased market competition. By contrast, a sale to independent shareholders would have benefited consumers considerably. Even a sale to PTM's shareholders (as opposed to PT's shareholders) would have increased competition, both compared to the initial scenario and the current one. In other words, while the exact numbers vary, the qualitative message of our calculations is that PT's proposal of stock divestiture did not significantly increase consumer welfare, whereas a sale to an independent or a series of independent shareholders would have increased consumer welfare.

Our paper is related to previous research on shareholding interlocks: Reynolds and Snapp (1986), Bresnahan and Salop (1986), Flath (1992). These papers are concerned with the case when firms own shares in rival firms, whereas we in addition consider the equally relevant case of shareholders who hold positions in more than one firm. Moreover, whereas the previous literature was limited to the case of homogenous product, we also consider the possibility of product differentiation.

The remainder of the paper is organized as follows. Section 2 lays out the formal framework. Section 3 presents a few general results. In Section 4, we apply these results to the case of Portugal Telecom. Section 5 concludes the paper.

2 Formal approach

Consider an industry with two firms and n shareholders. Shareholder k holds a share s_{ik} in firm i. In other words, shareholder k's payoff is given by $s_{1k}\,\Pi_1 + s_{2k}\,\Pi_2$, where Π_i is firm i's aggregate profit. Suppose moreover that firm i owns a share s_{j0} in firm j.[1] It follows that firm i's aggregate profit (including cross-holdings) is given by $\Pi_i = \pi_i + s_{j0}\,\pi_j$, where π_i is firm i's operating profit.

We assume that each firm maximizes a weighted sum of all of its shareholders' payoffs, with weights given by σ_{ik}. For the most part in this paper, we will take ownership shares as weights ($\sigma_{ik} = s_{ik}$), an assumption that

1. Strictly speaking, each firm has $n + 1$ shareholders, if we include the competing firm as a shareholder.

provides a natural and reasonable benchmark. In Section 4, we consider different alternative weights.

Following our assumptions, firm i maximizes

$$\begin{aligned}\omega_i &= \sum_{k=1}^{n} s_{ik}\left(s_{ik}\,\Pi_i + s_{jk}\,\Pi_j\right) + s_{i0}\,\omega_j \\ &= \sum_{k=1}^{n} s_{ik}\left(s_{ik}\left(\pi_i + s_{j0}\,\pi_j\right) + s_{jk}\left(\pi_j + s_{i0}\,\pi_i\right)\right) + s_{i0}\,\omega_j \qquad (1)\end{aligned}$$

The above expression takes into account the fact that firm i effectively has $n+1$ shareholders: n "normal" shareholders and the competing firm, firm j.

An alternative way to think about firm i's maximization is to consider that, ultimately, there are n shareholders with interests in firms i and j, both direct and indirect interests. Taking this into account, shareholder k's weight in firm i's maximization is given by $s_{ik} + s_{i0}\,s_{jk}$, with the two terms measuring shareholder k's direct and indirect interest in firm i, respectively. Firm i's maximization problem then becomes

$$\begin{aligned}\omega_i &= \sum_{k=1}^{n}\left(s_{ik} + s_{i0}\,s_{jk}\right)\left(s_{ik}\,\Pi_i + s_{jk}\,\Pi_j\right) \qquad (2) \\ &= \sum_{k=1}^{n}\left(s_{ik} + s_{i0}\,s_{jk}\right)\left(s_{ik}\left(\pi_i + s_{j0}\,\pi_j\right) + s_{jk}\left(\pi_j + s_{i0}\,\pi_i\right)\right) \qquad (3)\end{aligned}$$

As it happens, the two approaches yield the same outcome, as the following results establishes.

Lemma 1 *The maximization of (1) is equivalent to the maximization of (2).*

Proof: Solving the system (1) yields

$$\omega_i = \sum_{k=1}^{n}\left(\frac{s_{ik} + s_{i0}\,s_{jk}}{1 - s_{i0}\,s_{j0}}\right)\left(s_{ik}\left(\pi_i + s_{j0}\,\pi_j\right) + s_{jk}\left(\pi_j + s_{i0}\,\pi_i\right)\right) \qquad (4)$$

This differs from (3) by a constant. ■

In what follows, we will work with the system (2).[2] It can be simplified into

2. One useful property of (4) is that the shareholder weights add up to 1. This is also true of (1) if we include cross-holdings as well.

$$\omega_i = \big(H_i + s_{i0}\, H\big)\big(\pi_i + s_{j0}\,\pi_j\big) + \big(H + s_{i0}\, H_j\big)\big(\pi_j + s_{i0}\,\pi_i\big) \tag{5}$$

where

$$\begin{aligned} H &= \sum_{k=1}^{n} s_{ik} s_{jk} \\ H_i &= \sum_{k=1}^{n} s_{ik}^2 \end{aligned}$$

Notice that the Herfindahl-type indexes H and H_i are not based on the usual market shares but rather on *ownership* shares. These indexes summarize the pattern of shareholdings in firms i and j. H_i measures the degree of concentration of shareholdings in firm i. It has the standard interpretation, with $H_i = 0$ corresponding to the extreme of complete dilution and $H_i = 1$ to the case of single shareholder (or $H_i = 10000$, if shares are measured between 0 and 100). The value of H plays a crucial role in the analysis of common holdings and market competition. If the shareholders of firm i are entirely independent of those of firm j, then $H = 0$. The opposite extreme corresponds to the case when the pattern of shareholdings in firms i and j are identical, that is, $s_{ik} = s_{jk}$. In that case, $H = H_i = H_j$.

We will now attempt an intuitive explanation for (5). First notice that it can be written in the simpler form

$$\omega_i = \big(H_i + s_{i0}\, H\big)\,\Pi_i + \big(H + s_{i0}\, H_j\big)\,\Pi_j$$

Why do we use Herfindal-type indexes, that is, indexes based on squares of shareholdings? The square terms correspond to the product of shareholdings as indexes of power and shareholdings as indicators of interests. In other words, s_{ik} indicates how much shareholder k is interested in firm i's success but it also measures the weight that shareholder k has in the decisions taken by firm i.

The reason why there are two terms multiplying Π_i and Π_j is that shareholder k's effective weight in firm i can be either direct or indirect. The direct influence corresponds to what we've discussed above: s_{ik} times s_{ik}, which leads to H_i. The indirect influence corresponds to shareholder k's indirect weight in firm i's decisions, $s_{jk}\, s_{i0}$, times his interest in firm i, s_{ik}, leading to s_{i0} times index H.

With respect to market competition, we consider two possibilities: Hotelling and Cournot competition. In the Hotelling case, we assume firms live at the ends of a segment of length one and that consumers are uniformly distributed along that segment. Consumers have a valuation v for each firm's product and buy exactly one unit from one of the firms. In addition to price, consumers pay a quadratic "transportation cost" $t\,d^2$, where d is the distance between the consumer and the firm's address. Given these assumptions, firm i's profit is given by

$$\pi_i = p_i \left(\frac{1}{2} + \frac{1}{2\,t} (p_j - p_i) \right)$$

In the Cournot case, we assume demand is linear: $p = a - (q_i + q_j)$, where q_i is firm i's output. Moreover, we assume symmetric, constant marginal costs. With no additional loss of generality, we assume zero costs and $a = 1$. Given these assumptions, firm i's profit is given by

$$\pi_i = (1 - q_i - q_j)\, q_i$$

In both cases, firm i chooses its strategic variable (price or quantity) to maximize ω_i.

3 Analytical results

Since maximization problems are invariant up to a multiplicative constant, without loss of generality we can write the firms' maximization problem as

$$\omega_i = \pi_i + \gamma_i\, \pi_j$$

where $\gamma_i \geq 0$ indicates the extent to which firm i takes the rival firm's operating profit into account. Specifically, from (5), we get

$$\gamma_i = \frac{(1 + s_{i0}\, s_{j0})\, H + s_{j0}\, H_i + s_{i0}\, H_j}{2\, s_{i0}\, H + H_i + s_{i0}^2\, H_j} \tag{6}$$

We are interested in understanding how changes in ownership (in particular, changes in s_{i0}, H_i and H) affect consumer welfare. For this purpose, the following result will prove useful.

Lemma 2 *Under Hotelling or Cournot competition, equilibrium prices are increasing in* γ_i*. Moreover, under Hotelling competition, if* $\gamma_i \approx \gamma_j$ *then average total cost (including both price and transportation cost) is increasing in* γ_i*.*

The proof of this and the remaining results in the paper may be found in the Appendix. Lemma 2 indicates that the values of γ_i can be sufficient statistics for the values of average price. In particular, increases in γ_i lead to increases in equilibrium prices. Under Hotelling competition, we require that the market not be too asymmetric. Under Cournot competition, no such restriction is required.

We can now perform a series of comparative statics exercises with respect to changes in ownership structure. Specifically, we are interested in changes in consumer welfare resulting from changes in H, H_i and s_{i0}. Some of these derivatives are simple partial derivatives, some directional derivatives (Apostol, 1969, p. 254). Each will have its own economic interpretation.

Consider first a change in H keeping H_i and s_{i0} constant, that is, the simple partial derivative of consumer welfare with respect to H. Such change occurs when shareholder x, who initially owns shares in firm i only, acquires the shares of shareholder y, who initially owned shares in firm j only.

Proposition 1 *If* $H_i = H_j$*, then consumer welfare is decreasing in* H*.*

This is a central theoretical result. In fact, this is the result that best corresponds to the paper's theme, duopoly competition with common shareholders. The message is quite simple: the more common shareholdings there are, the lower consumer welfare is. This is fairly intuitive. What is perhaps not so obvious is that the index H is a sufficient statistic of consumer welfare; in other words, given H_i and s_{i0}, there is a one-to-one monotonic mapping between H and consumer welfare, both under Hotelling and under Cournot competition.

A second possible exercise is to increase s_{i0} while keeping H and H_i constant (again, the simple partial derivative). To understand the meaning of this change, it is perhaps easier to think about a *decrease* in s_{i0}. If firm j divests of its share in firm i, those shares must be transferred to some other shareholders. If H and H_i are kept constant, then it must be that the s_{i0} shares are distributed to an infinite number of infinitesimal shareholders. It follows that the partial derivative of consumer welfare with respect to

s_{i0} may be interpreted as a purchase by firm j of shares in firm i held by infinitesimally small shareholders.[3]

Proposition 2 *If $H_i = H_j$ and $s_{i0} = s_{j0}$, then consumer welfare is decreasing in s_{i0}.*

The thrust of Proposition 2 is similar to Proposition 1: greater cross-holdings lead to lower consumer welfare. The main difference is that whereas Proposition 1 refers to common shareholdings — that is, shareholders who own shares in both firms — Proposition 2 corresponds to the possibility of one firm owning shares in a competing firm.

If the variation in s_{i0} is compensated by variations in the shareholdings by large shareholders, then we must take into account the resulting changes in H and H_i as well. Mathematically, this corresponds to a directional derivative. We consider two possibilities. One is that a variation in s_{i0} is compensated by proportional variations in all shareholdings, where the proportions of variation are given by the initial shareholdings in firm i. In this case, the vector of shareholdings variations is given by $ds_{ik} = -\frac{s_{ik}}{\sum_{\ell=1}^{n} s_{i\ell}}\, ds_{i0} = -\frac{s_{ik}}{1-s_{i0}}\, ds_{i0}$. A second possibility is that the variations are proportional to shares in firm j, so that $ds_{ik} = -\frac{s_{jk}}{\sum_{\ell=1}^{n} s_{j\ell}}\, ds_{i0} = -\frac{s_{jk}}{1-s_{j0}}\, ds_{i0}$. Naturally, as the values of s_{ik} change, so do the values of H_i and H.

Proposition 3 *Suppose that $H_i = H_j > 0$ and $s_{i0} = s_{j0} > 0$. Consider a small decrease in s_{i0}. If firm j's shares in firm i are sold to firm i's existing shareholders (in proportion to their shareholdings), then consumer welfare increases. If firm j's shares in firm i are sold to firm j's existing shareholders (in proportion to their shareholdings), then consumer welfare decreases.*

If Propositions 1 and 2 are fairly intuitive (and expected), the same cannot be said of Proposition 3. The reason is that we are lowering a direct cross shareholding (a good thing for consumers) but increasing the value of H (a bad thing for consumers, by Proposition 1) and increasing the value of H_i (which, as we shall see, is good for consumers).

3. Strictly speaking, this scenario is incompatible with our assumption that there is a finite number of shareholders, n. However, our results can be arbitrarily approximated by making n sufficiently large.

The algebra involved in the proof of Proposition 3 is rather painful, but the line of argument is straightforward. By Lemma 2, we know that, at a symmetric equilibrium, the sum $\gamma_1 + \gamma_2$ is a sufficient statistic of consumer welfare. So we proceed by computing the total derivative of γ_i with respect to s_{i0}. It is given by

$$\frac{d\,\gamma_i}{d\,s_{i0}} = \frac{\partial\,\gamma_i}{\partial\,s_{i0}} + \frac{\partial\,\gamma_i}{\partial\,H}\,\frac{d\,H}{d\,s_{i0}} + \frac{\partial\,\gamma_i}{\partial\,H_i}\,\frac{d\,H_i}{d\,s_{i0}} + \frac{\partial\,\gamma_i}{\partial\,H_j}\,\frac{d\,H_j}{d\,s_{i0}}$$

Symmetry implies that

$$\frac{d\,(\gamma_i + \gamma_j)}{d\,s_{i0}} = \frac{\partial\,(\gamma_i + \gamma_j)}{\partial\,s_{i0}} + 2\,A\,\frac{d\,H}{d\,s_{i0}} + (B + C)\,\frac{d\,H_i}{d\,s_{i0}} + (B + C)\,\frac{d\,H_j}{d\,s_{i0}} \tag{7}$$

where

$$\begin{aligned} A &\equiv \frac{\partial\,\gamma_i}{\partial\,H} = \frac{\partial\,\gamma_j}{\partial\,H} \\ B &\equiv \frac{\partial\,\gamma_i}{\partial\,H_i} = \frac{\partial\,\gamma_j}{\partial\,H_j} \\ C &\equiv \frac{\partial\,\gamma_i}{\partial\,H_j} = \frac{\partial\,\gamma_j}{\partial\,H_i} \end{aligned} \tag{8}$$

Proposition 2 implies that the first term on the right-hand side of (7) is positive. Moreover, it takes the same value in both cases considered in Proposition 3. The key for the difference between the two cases must therefore be found in the remaining terms. Notice the cases considered in Proposition 3 imply no change in firm j's shares. It follows that $d\,H_j = 0$ and so the difference between the two cases considered in Proposition 3 reduces to

$$2\,A\,\frac{d\,H}{d\,s_{i0}} + (B + C)\,\frac{d\,H_i}{d\,s_{i0}} \tag{9}$$

Proposition 1 implies that $A > 0$. It can also be shown that $B + C < 0$: while a greater overlap of share ownership across firms is bad for consumers, an increase in concentration in a given firm is good for consumers. Intuitively, a strong shareholder in firm i counteracts the influence in firm i's decisions by shareholders with an interest in firm j.

A sale of shares from s_{i0} to existing shareholders implies an increase in H and in H_i. We thus have a "race" between the effect through H and the effect through H_i. The key to Proposition 3 is the following: a sale of firm

Table 1: Numerical example

	Scenario 0		Scenario 1		Scenario 2	
Shareholder	Firm 1	Firm 2	Firm 1	Firm 2	Firm 1	Firm 2
0	$1/3$	$1/3$	0	$1/3$	0	$1/3$
1	$1/3$	0	$1/3+1/6$	0	$1/3$	0
2	0	$1/3$	0	$1/3$	$1/6$	$1/3$
3	$1/3$	$1/3$	$1/3+1/6$	$1/3$	$1/3+1/6$	$1/3$
H	$1/9$		$1/6$		$2/9$	
H_i	$2/9$	$2/9$	$1/2$	$2/9$	$7/18$	$2/9$

i's shares to firm i's shareholders has primarily an impact on H_i, whereas a sale of firm i's shares to firm j's shareholders has primarily an impact on H.

To better understand the difference between the two possible sale patterns, consider a simple numerical example. There are three shareholders: shareholder 1 (resp. 2) owns a share $1/3$ in firm 1 (resp. 2); and shareholder 3 owns a share $1/3$ in each firm. Finally, each firm holds a $1/3$ share in its rival.

Table 1 summarizes these values under the Scenario 0 column. In the first row, we have firm shares in the rival firm. The next three rows show the holdings of each of our three shareholders. Finally, the bottom two rows give the values of H and H_i.

Consider now Scenario 1, a sale of firm 2's share in firm 1 to the existing three shareholders *in proportion to their current shareholdings in firm 1*. This implies the $1/3$ share is equally split between shareholders 1 and 3. Shareholder 2, who did not own shares in firm 1, remains with a zero shareholding. As a result of this share transfer, the value of H increases from $1/9$ to $1/6$ (a "small" increase) and the value of H_1 increases from $2/9$ to $1/2$ (a "large" increase). As shown above, the value of H_2 remains unchanged.

Consider now Scenario 2, a sale of firm 2's share in firm 1 to the existing three shareholders *in proportion to their current shareholdings in firm 2*. This implies the $1/3$ share is equally split between shareholders 2 and 3. Shareholder 2, who did not own shares in firm 1, now becomes a shareholder of firm 1 as well. As a result of this share transfer, the value of H increases from $1/9$ to $2/9$ (a "large" increase) and the value of H_1 increases from $2/9$

to 7/ 18 (a “small” increase).

In other words, selling firm 1’s shares to firm 1 existing shareholders increases H_1 a lot and H a little, whereas selling firm 1’s shares to firm 2 existing shareholders increases H a lot and H_1 a little. Increases in H are bad for consumers as they increase the degree of commonality of interests between firms. This explains why there is such a drastic difference in the welfare impacts of the two share sale patterns.

To conclude this section, we present a result on a possible share sale that closely mimics PT’s proposal. Unlike some of the previous results, which assume symmetry, we now consider the case when initial cross-holdings go one way: $s_{i0} > 0$ but $s_{j0} = 0$.

Proposition 4 *Suppose that $s_{j0} = 0$ and suppose that firm j’s share in firm i, s_{i0}, is sold to existing shareholders in proportion to their shareholdings in firm j, s_{jk}. Such share transfer is neutral in terms of consumer welfare.*

At a first glance, the above proposal merely converts shareholders indirect weight in firm i (through s_{i0}) into a direct weight (s_{ik}). Given equation (3), the neutrality in terms of consumer welfare seems unsurprising. However, if $s_{j0} > 0$ such proposal would have additional implications. To understand this, consider the case of a shareholder who, at the outset, owns no stock in firm i. To the extent that $s_{j0} > 0$, this shareholder now has both a direct and an indirect weight in firm j (direct through the initial holding in firm i; indirect through the newly acquired shares in firm j). So, while Proposition 4 is stronger than the previous ones in that it does not require symmetry and applies to non-marginal changes, it does require the special assumption $s_{j0} = 0$.

4 Application

We now apply the above theoretical results to the particular case discussed in the introduction: the divestiture of PTM by PT, Portugal’s largest telecommunications operator. At the time the divestiture took place, there were a few large shareholders who owned shares both in PT and PTM. Moreover, PTM was partly owned by PT. Under pressure from the Portuguese government, PT agreed to sell its shares in PTM. PT management’s proposal, which was accepted by the Portuguese government, was to sell its share in

Table 2: Large shareholders in PT and PTM (percentages as of April 2007).

Shareholder	Shares PT	Shares PTM	Voting rights PT	Voting rights PTM
PT	—	58.43	—	19.88
PTM	0.00	—	0.00	—
Telefonica	9.96	0.00	9.96	0.00
Grupo Espírito Santo	7.77	6.96	7.77	13.83
Brandes Investments Partners	7.41	0.00	7.41	0.00
Ongoing Strategy Investments	5.35	0.00	5.35	0.00
Grupo Caixa Geral de Depósitos	5.11	11.26	5.11	19.88
Telmex	3.41	0.00	3.41	0.00
Paulson & Co. Inc.	2.34	0.00	2.34	0.00
Merrill Lynch International	2.20	0.00	2.20	0.00
Fidelity	2.09	0.00	2.09	0.00
Barclays Group	2.06	0.00	2.06	0.00
Capital Group Companies	2.04	0.00	2.04	0.00
Grupo Visabeira	2.01	0.00	2.01	0.00
Controlinveste / Joaquim Oliveira	2.00	3.77	2.00	7.49
Grupo BPI	0.00	5.16	0.00	10.26
Cofina, SGPS, S.A.	0.00	2.23	0.00	4.43
Total	53.75	29.38	53.75	75.77

PTM to PT's shareholders, in proportion to their holdings at the time the divestiture took place.

Table 2 lists the main shareholders in PT and PTM at the time the divestiture was announced (April 2007). As can be seen, there are a few relatively large shareholders. Moreover, aside from the fact that PT owns a share in PTM, there is a significant overlap in ownership of both firms.

In our empirical application, we will consider various possible shareholder weights. It thus helps to write the firms' objective functions in more general terms:

$$\omega_i = \sum_{k=1}^{n} \sigma_{ik} \Big(s_{ik} \big(\pi_i + s_{j0}\, \pi_j \big) + s_{jk} \big(\pi_j + s_{i0}\, \pi_i \big) \Big) + \sigma_{i0}\, \omega_j$$

where σ_{ik} denotes denotes the weight given by firm i to shareholder k's payoff. This can be rewritten as

$$\omega_i = K_i \left(\pi_i + s_{j0}\, \pi_j \right) + K_{ij} \left(\pi_j + s_{i0}\, \pi_i \right) + \sigma_{i0}\, \omega_j \tag{10}$$

where

$$\begin{aligned} K_i &= \sum_{k=1}^{n} \sigma_{ik}\, s_{ik} \\ K_{ij} &= \sum_{k=1}^{n} \sigma_{ik}\, s_{jk} \end{aligned}$$

Solving (10) and normalizing results in

$$\omega_i = \pi_i + \frac{K_{ij} + K_i\, s_{j0} + K_j\, \sigma_{i0} + K_{ji}\, s_{j0}\, \sigma_{i0}}{K_i + K_{ij}\, s_{i0} + K_{ji}\, \sigma_{i0} + K_j\, s_{i0}\, \sigma_{i0}}\, \pi_j$$

In Section 2, we assume that each shareholder's weight is given by his share, that is, $\sigma_{ik} = s_{ik}$. In our empirical application, in addition to this case, we consider two additional possibilities: $\sigma_{ik} = v_{ik}$, where v_{ik} is shareholder k's voting rights in firm i; and $\sigma_{ik} = s_{ik}^2$. As we mentioned in Section 2, $\sigma_{ik} = s_{ik}$ provides a natural and reasonable benchmark. Using voting rights as weights, $\sigma_{ik} = v_{ik}$, reflects the view that important decisions are directly or indirectly taken by voting at shareholder meetings. Finally, the idea of $\sigma_{ik} = s_{ik}^2$ is that large shareholders may wield disproportionately large power.

Ideally, the best scenario in terms of consumer welfare is for the ownership of PT and PTM to be totally independent. We thus consider two benchmark situations: the current situation and the ideal of total separation. The question is then, how close to these benchmarks are the various proposals to implement PT's divestiture. Specifically, we consider the following alternatives:

1. Current situation.

2. Sale of PT's share in PTM according to PT's proposal, that is, sale to PTM shareholders in proportion to their initial shares in PT.

3. Sale of PT's share in PTM to PTM shareholders in proportion to their initial shares (in PTM).

4. Sale of PT's share in PTM to small shareholders different from the current shareholders.

5. Sale of PT's share in PTM to a large shareholder different from the current shareholders.

6. Complete separation of shareholdings.

Denote PT by firm 1 and PTM by firm 2. In all cases, PTM holds no share in PT, that is, $s_{10} = 0$. Our focus is on the value of s_{20}, positive in the current situation and zero in all other scenarios. Specifically, we proceed as follows. For each scenario, we compute the values of s_{20}, K_{ij} and K_i (in the case when $\sigma_{ik} = s_{ik}$, the latter become H and H_i, respectively). We then compute measures of consumer welfare and determine their percent difference with respect to the two extreme scenarios: the current situation and complete separation of shareholding. Regarding product market competition, we consider both the assumptions of Hotelling and Cournot competition. Finally, regarding the weight given each shareholder, we consider three alternative possibilities: (a) weights proportional to shareholdings; (b) weights proportional to voting rights; (c) weights proportional to the square of shareholdings.

Table 3 presents the results under the Hotelling market competition assumption. Consider first the case when weights are given by shareholdings. As expected from Proposition 4, PT's proposal leads to no change in consumer welfare. Scenarios 3 and 4 are very similar in terms of consumer welfare outcome. This is due to the fact that in Scenario 3, 70.6% of PT's share in PTM are de facto sold to small, independent shareholders. In other words, the scenarios' outcomes are similar because the scenarios themselves are similar.

Finally, we see that a sale to an independent, large shareholder (Scenario 5) fares better than the sale to a large number of small shareholders. In fact, such transfer is almost equivalent, in terms of consumer welfare, to total separation of ownership. Intuitively, the idea is that the large independent shareholder's power works as an effective counterbalance to the remaining common shareholders (especially Grupo Caixa Geral de Depósitos and Grupo Espírito Santo).

Table 3: Change in average price with respect to Scenario 1 and in proportion to the difference between Scenarios 6 and 1 under the assumption of Hotelling competition.

	Shareholder weight		
Scenario	s_{ik}	v_{ik}	s_{ik}^2
1. Current Situation	0	0	0
2. PT's proposal	0	−96.8	5.8
3. Sale to PTM shareholders	77.8	61.9	71.7
4. Sale to independent shareholders	78.5	63.4	71.4
5. Sale to independent shareholder	93.9	85.2	93.0
6. Total separation	100	100	100

If a shareholder's weight is given by its voting rights, then PT's proposal actually decreases consumer welfare. The reason for this is that, because of legal caps on voting rights, PT's voting rights in PTM (in the current situation) are considerably lower than its shareholdings. By transferring shares to large shareholders, PT is actually increasing the concentration of voting rights, since the shares it transfers will correspond to greater voting rights than in the initial situation. In other words, by relinquishing 1% of voting rights, PT is increasing PT's shareholders voting rights by more than 1%.

Table 4 presents the results under the Cournot market competition assumption. As can be seen, the results are qualitatively equivalent to those under Hotelling competition.

Our analysis suggests that different assumptions regarding the nature of shareholder control and market competition lead to different values of the estimated impact of PT's divestiture. However, our qualitative results are remarkably robust in the sense that the action plan proposed and implemented by PT hardly improved the conditions for market competition, if at all. By contrast, selling PT's share in PTM to independent shareholders would have considerably improved consumer welfare through increased competition. An additional improvement would have been obtained by completely separating the set of shareholders in PT and PTM.

Table 4: Change in consumer welfare with respect to Scenario 1 and in proportion to the difference between Scenarios 6 and 1 under the assumption of Cournot competition.

	Shareholder weight		
Scenario	s_{ik}	v_{ik}	s_{ik}^2
1. Current Situation	0	0	0
2. PT's proposal	0	-0.2	0.1
3. Sale to PTM shareholders	36.2	35.9	39.1
4. Sale to independent shareholders	39.6	39.2	37.9
5. Sale to independent shareholder	65.3	60.8	69.9
6. Total separation	100	100	100

Throughout our analysis, we made different assumptions regarding the conditions for effective control. One way in which our analysis can be criticized is that, historically, some shareholders have effectively controlled PT. In fact, as recent experience suggests, 20 or 30% of a firm's shares may suffice to effectively control a firm's destiny, namely in terms of selecting its management and vetoing changes in its statutes. And a few shareholders have been able to, directly or indirectly, secure such control. We conjecture that the main message of our analysis would not change — in fact would be strengthened, if anything — if we were to consider the possibility of a few shareholders holding effective control.

One argument by PT is that the sale that took place in November 2007 only lead to a temporary overlap in share ownership. However, to the extent that the joint shareholders are able to secure effective control of PT and PTM, it is highly unlikely that they will sell such shares. In fact, the sale would reduce the shareholder value of PT and PTM together. As such, it is highly unlikely that an outside shareholder would be willing to pay more for the stock than the current shareholders would require. This is a version of what Gilbert and Newbery (1982) refer to as the efficiency effect: a monopolist has more to lose from accommodating a competitor than a competitor has to gain from entering. In the present context, we do not have the extremes of monopoly and duopoly, but the same qualitative message holds.

5 Concluding remarks

We developed a methodology for evaluating competition and welfare effects when shareholders hold partial positions in more than one competitor. We showed that consumer welfare can be summarized by a series of shareholding Herfindahl-type indexes. This characterization facilitates the evaluation of the welfare impact of various possible asset transfers amongst shareholders.

We then applied our results to estimate the effect on consumer welfare of PTs divestiture of PTM. Our analysis suggests that the action plan proposed and implemented by PT hardly improved the conditions for market competition, if at all.

Appendix

Proof of Lemma 2: Consider the case of Hotelling competition. Algebraic calculations show that, in equilibrium,

$$p_i = \frac{3+\gamma_i}{3-\gamma_i-\gamma_j-\gamma_i\,\gamma_j}\;t \tag{11}$$

From this we get

$$\begin{aligned}\frac{\partial\,p_i}{\partial\,\gamma_i} &= \frac{2\,(3+\gamma_j)}{(3-\gamma_i-\gamma_j-\gamma_i\,\gamma_j)^{\,2}}\;t\;>\;0\\ \frac{\partial\,p_i}{\partial\,\gamma_j} &= \frac{(3+\gamma_i)\,(1+\gamma_i)}{(3-\gamma_i-\gamma_j-\gamma_i\,\gamma_j)^{\,2}}\;t\;>\;0\end{aligned}$$

Average price, including transportation costs, is given by

$$\bar{p} = \sum_{i=1}^{2}\;\int_0^{\alpha_i}\big(p_i + t\,\alpha^2\big)\,d\alpha \;=\; \sum_{i=1}^{2}\alpha_i\big(p_i + \tfrac{1}{3}\,t\,\alpha_i^3\big)$$

where $\alpha_i = \frac{1}{2} + \frac{1}{2\,t}\,\big(p_j - p_i\big)$ denotes the distance from firm i to the indifferent consumer's address. Substituting (11) for price and simplifying, we get

$$\left.\frac{\partial\,\bar{p}}{\partial\,\gamma_i}\right|_{\gamma_i\,=\,\gamma_j\,=\,\gamma} = \frac{1}{2\,(1-\gamma)^{\,2}}$$

Consider now the case of Cournot competition. Equilibrium output is given by

$$q_i = \frac{1-\gamma_i}{3-\gamma_i-\gamma_j-\gamma_i\,\gamma_j}$$

Substituting into the price equation, we get

$$p = \frac{1-\gamma_i\,\gamma_j}{3-\gamma_i-\gamma_j-\gamma_i\,\gamma_j}$$

Finally,

$$\frac{\partial\,p}{\partial\,\gamma_i} = \left(\frac{1-\gamma_i}{3-\gamma_i-\gamma_j-\gamma_i\,\gamma_j}\right)^{2} > 0$$

which completes the proof. ■

Proof of Proposition 1: From (6), we get

$$\left.\frac{\partial \gamma_i}{\partial H}\right|_{H_i = H_j} = \frac{(1 - s_{i0}^2)\,(1 - s_{i0}\, s_{j0})\, H_i}{\big(H_i\,(1 + s_{i0}^2) + 2\, H\, s_{i0}\big)^2} > 0 \tag{12}$$

The result then follows from Lemma 2. ■

Proof of Proposition 2: Given our symmetry assumption, at the margin consumer welfare is proportional to $d(\gamma_1 + \gamma_2)$. Computation establishes that

$$\left.\frac{\partial\,(\gamma_1 + \gamma_2)}{\partial\, s_{i0}}\right|_{\substack{s_{i0} = s_{j0} \\ H_i = H_j}} = \frac{2\,(1 - s_{i0}^2)\,(H_i^2 - H^2)}{\big(2\, s_{i0}\, H + (1 + s_{i0}^2)\, H_i\big)^2} \tag{13}$$

which is positive, since $H \leq H_i$. The results then follows from Lemma 2. ■

Proof of Proposition 3: Given that shareholdings in firm j remain constant, we have $\frac{d\,H_j}{d\,s_{i0}} = 0$. From (7), the total derivative of $\gamma_i + \gamma_j$ with respect to s_{i0} is then given by

$$\frac{d\,(\gamma_i + \gamma_j)}{d\,s_{i0}} = \frac{\partial\,(\gamma_i + \gamma_j)}{\partial\, s_{i0}} + 2\,A\,\frac{d\,H}{d\,s_{i0}} + (B + C)\,\frac{d\,H_i}{d\,s_{i0}} \tag{14}$$

where A, B, C are given by (9). From (13), we get the first term on the right-hand side:

$$\left.\frac{\partial\,(\gamma_1 + \gamma_2)}{\partial\, s_{i0}}\right|_{\substack{s_{i0} = s_{j0} \\ H_i = H_j}} = \frac{2\,(1 - s_{i0}^2)\,(H_i^2 - H^2)}{\big(2\, s_{i0}\, H + (1 + s_{i0}^2)\, H_i\big)^2}$$

From (12), we get

$$\left.\frac{\partial \gamma_i}{\partial H}\right|_{\substack{s_{i0} = s_{j0} \\ H_i = H_j}} = \left. A\right|_{\substack{s_{i0} = s_{j0} \\ H_i = H_j}} = \frac{(1 - s_{i0}^2)^2\, H_i}{\big(2\, s_{i0}\, H + (1 + s_{i0}^2)\, H_i\big)^2}$$

Computation establishes that

$$\left.\frac{\partial \gamma_i}{\partial H_i}\right|_{\substack{s_{i0} = s_{j0} \\ H_i = H_j}} = \left. B\right|_{\substack{s_{i0} = s_{j0} \\ H_i = H_j}} = -\frac{(1 - s_{i0}^2)\big(H + s_{i0}\, H_i\big)}{\big(2\, s_{i0}\, H + (1 + s_{i0}^2)\, H_i\big)^2}$$

$$\left.\frac{\partial \gamma_i}{\partial H_j}\right|_{\substack{s_{i0}\,=\,s_{j0}\\ H_i\,=\,H_j}} = \left. C \right|_{\substack{s_{i0}\,=\,s_{j0}\\ H_i\,=\,H_j}} = \frac{s_{i0}\,(1-s_{i0}^2)\big(s_{i0}\,H + H_i\big)}{\big(2\,s_{i0}\,H + (1+s_{i0}^2)\,H_i\big)^2}$$

(Notice in passing that, as claimed in the text, at a symmetric situation we have $B + C < 0$.)

The terms $\frac{d\,H}{d\,s_{i0}}$ and $\frac{d\,H_i}{d\,s_{i0}}$ depend on the particular scenario we consider. If firm j's shares in firm i, s_{i0}, are sold to firm i's shareholders then

$$ds_{ik} = -\frac{s_{ik}}{1-s_{i0}}\;ds_{i0}$$

It follows that

$$dH_i = \sum_{k=1}^{n} 2\,s_{ik}\,ds_{ik} = \sum_{k=1}^{n} 2\,s_{ik}\left(-\frac{s_{ik}}{1-s_{i0}}\;ds_{i0}\right) = -\frac{2\,H_i}{1-s_{i0}}\;ds_{i0}$$

$$dH = \sum_{k=1}^{n} s_{jk}\,ds_{ik} = \sum_{k=1}^{n} s_{jk}\left(-\frac{s_{ik}}{1-s_{i0}}\;ds_{i0}\right) = -\frac{H}{1-s_{i0}}\;ds_{i0}$$

Substituting all of these terms into (14) and simplifying we get

$$\left.\frac{d\,(\gamma_i+\gamma_j)}{d\,s_{i0}}\right|_{\substack{s_{i0}\,=\,s_{j0}\\ H_i\,=\,H_j}} = \frac{2\,(1-s_{i0}^2)\big(H_i^2 - H^2\big)}{\big(2\,s_{i0}\,H + (1+s_{i0}^2)\,H_i\big)^2}$$

Since at a symmetric situation $H < H_i$, we conclude that $\frac{d\,(\gamma_1+\gamma_2)}{d\,s_{i0}} > 0$. The first part of the result then follows from Lemma 2.

If firm j's shares in firm i, s_{i0}, are sold to firm j's shareholders then

$$ds_{ik} = -\frac{s_{jk}}{1-s_{j0}}\;ds_{i0}$$

It follows that

$$dH_i = \sum_{k=1}^{n} 2\,s_{ik}\,ds_{ik} = \sum_{k=1}^{n} 2\,s_{ik}\left(-\frac{s_{jk}}{1-s_{j0}}\;ds_{i0}\right) = -\frac{2\,H}{1-s_{j0}}\;ds_{i0}$$

$$dH = \sum_{k=1}^{n} s_{jk}\,ds_{ik} = \sum_{k=1}^{n} s_{jk}\left(-\frac{s_{jk}}{1-s_{j0}}\;ds_{i0}\right) = -\frac{H_j}{1-s_{j0}}\;ds_{i0}$$

Substituting all of these terms into (14) and simplifying we get

$$\left.\frac{d\,(\gamma_i+\gamma_j)}{d\,s_{i0}}\right|_{\substack{s_{i0}\,=\,s_{j0}\\ H_i\,=\,H_j}} = -\frac{2\,s_{i0}\,(1-s_{i0}^2)\big(H_i^2 - H^2\big)}{\big(2\,s_{i0}\,H + (1+s_{i0}^2)\,H_i\big)^2}$$

Since at a symmetric situation $H < H_i$, we conclude that $\frac{d(\gamma_1+\gamma_2)}{d s_{i0}} < 0$. The second part of the result again follows from Lemma 2. ■

Proof of Proposition 4: Denote by a $*$ superscript values post share sale. Then we have

$$s^*_{ik} = s_{ik} + s_{i0}\, s_{jk} \tag{15}$$

whereas $s^*_{jk} = s_{jk}$ and $s^*_{i0} = s^*_{j0} = s_{j0} = 0$. Before the share transfer, firm i's objective function is given by

$$\begin{aligned} \omega_i &= \sum_{k=1}^{n} \left(\frac{s_{ik} + s_{i0}\, s_{jk}}{1 - s_{i0}\, s_{j0}}\right) \big(s_{ik}\,(\pi_i + s_{j0}\,\pi_j) + s_{jk}\,(\pi_j + s_{i0}\,\pi_i)\big) \\ &= \sum_{k=1}^{n} \big(s_{ik} + s_{i0}\, s_{jk}\big)\big(s_{ik}\,\pi_i + s_{jk}\,(\pi_j + s_{i0}\,\pi_i)\big) \end{aligned} \tag{16}$$

where we use the fact that $s_{j0} = 0$. After the share transfer, we have

$$\omega^*_i = \sum_{k=1}^{n} s^*_{ik}\big(s^*_{ik}\,\pi_i + s^*_{jk}\,\pi_j\big)$$

where we use the fact that $s^*_{i0} = 0$ (and $s^*_{j0} = s_{j0} = 0$). Substituting (15) for s_{ik}, we have

$$\omega^*_i = \sum_{k=1}^{n} \big(s_{ik} + s_{i0}\, s_{jk}\big)\Big(\big(s_{ik} + s_{i0}\, s_{jk}\big)\pi_i + s_{jk}, \pi_j\Big) \tag{17}$$

where we use the fact that $s^*_{jk} = s_{jk}$. Immediate inspection reveals that the right-hand side of (17) is equal to the right-hand side of (16).

A similar argument applies for firm j. ■

References

[1] Apostol, Tom M (1969) *Calculus*, Vol II, Second Edition, New York: Wiley.

[2] Bresnahan, Timothy and Steven Salop (1986) "Quantifying the Competitive Effects of Production Joint Ventures," *International Journal of Industrial Organization* **4**, 155–175.

[3] Flath, David (1992), "Horizontal Shareholding Interlocks," *Managerial and Decision Economics* **13**, 75–77.

[4] Gilbert, R.J. and Newbery, D.M. (1982) "Pre-emptive Patenting and the Persistence of Monopoly", *American Economic Review*, **72**, 514-26.

[5] Reynolds, Robert J, and Bruce R Snapp (1986), "The Competitive Effects of Partial Equity Interests and Joint Ventures," *International Journal of Industrial Organization* **4**, 141–153.