Small fish become big fish: merger in Stackelberg markets revisited

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Abstract

This paper analyzes, under convex costs, the price effects of mergers involving two Stackelber followers that become a leader, and revisits the "merger paradox". Contrary to what might be expected, prices are more likely to increase with cost convexity than with linear costs. Also, the incentive to free ride may reappear.

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1 Introduction

In their seminal work, Salant, Switzer, and Reynolds (1983) established a well known result, sometimes called the "merger paradox". In their specific setting, horizontal mergers involving less than 80 per cent of the firms in the industry are privately unprofitable and create an incentive to free-ride, as outsiders benefit from the merger. This is sometimes referred to as the "insiders' dilemma" and was first pointed out by Stigler (1950). As is well known, a merger in the Salant, Switzer, and Reynolds (1983) setting is equivalent to the exclusion of a rival, given that the firm that results from the merger is no different from its competitors or from each insider, taken individually.

Many papers have subsequently tried to solve this paradox by changing some of the original assumptions or by modelling the merger as having some impact on the insider firms. The latter could result from cost savings, changes in behavior or changes in the portfolio of brands controlled by the merged firm. To name just a few, Perry and Porter (1985) and Heywood and McGinty (2007a) considered quadratic costs instead of the cost linearity; Deneckere and Davidson (1985) used differentiated products and price competition instead of quantity competition; Kwoka (1989) allowed for the use of non-Cournot conjectures; Daughety (1990) considered the inclusion of players with asymmetric 'strategic power' - leaders and followers, instead of Cournot behavior, and Cheung (1992) allowed for more general demand function forms.

In some cases, however, these changes in the main assumptions, when taken individually, are insufficient to solve the merger paradox. Recently, some papers have analyzed mergers both under increasing marginal costs and Stackelberg competition – e.g. Heywood and McGinty (2007b) and (2008). This line of research follows Daughety (1990), Feltovich (2001), Huck et al (2001), Escrihuela-Villar and Faulí-Oller (2008) who have studied different types of mergers that may occur in Stackelberg competition with linear costs. This literature has addressed mergers between leaders, between leaders and followers and between followers. There are four possible merger cases in the literature: (i) merging leaders gives rise to a leader, (ii) merging leaders and followers gives rise to a leader; (iii) merging followers gives rise to a follower and (iv) merging followers gives rise to a leader. In all but the last case, the merger corresponds, once more, to the elimination of competitors and the firm that results from the merger is no different from some (or all) the insiders. Unsurprisingly, the merger paradox is still present.[1] The exception is the case where a merger between followers gives rise to a leader, which was first referred by Daughety (1990). Using linear costs, Daughety (1990) shows that the optimal structure in terms of welfare is obtained when the industry is composed of half leaders and half followers, so that any behavioral changing merger that approaches the industry to this

[1] With the same assumptions on demand and costs as Salant, Switzer and Reynolds (1983), mergers of type (i) and (iii) as defined above are unprofitable except if the number of insiders exceeds 80%, as shown by Hamada and Takarada (2007). Also, the merger between a leader and a follower results in a higher increase in profit for the leaders outside the merger.

balance is called a beneficial concentration.[2]

Assuming cost convexity, Heywood and McGinty (2008) have dealt with the case in which there is a single leader in the industry that merges with just one follower or with several followers, while Heywood and McGinty (2007b) consider mergers involving just two firms, both leaders or both followers, in which a two-follower merger gives rise to a new follower and a two-leader merger gives rise to a new leader. The case of the merger whereby two followers change behaviorally and become a leader is, however, not addressed.

In this paper we combine Daughety's behavioral changing merger with Heywood and McGinty (2007b) and (2008)'s cost convexity assumption.

Daughety (1990)'s analysis incorporates two effects. First, the number of firms in the industry is reduced as a follower is eliminated. Second, keeping the number of firms constant, a follower is turned into a leader. Heywood and McGinty (2007b)'s analysis incorporates the first effect above, does not include the second one, and introduces a third effect: one firm becomes more efficient than before. Hence, we complete the gap in the literature by analyzing the effects of a two-follower merger that gives rise to a leader under convex costs, which incorporates all three effects described above.

We conclude, perhaps surprisingly, that this type of merger may result in a price increase in circumstances under which prices would decline under cost linearity. Furthermore, we show that the free riding issue may reemerge under cost convexity, in situations where it would be absent under cost linearity.

Why should two followers become a leader by merging remains an open question that we do not address in this paper. There is some literature on endogenous timing decisions that presents motives for some firms to move first. Among the motives presented are information asymmetry about demand, e.g. Mailath (1993), and cost asymmetry, e.g., van Damme and Hurkens (1999) or Branco (2010). In the above mentioned papers, the informed firms and the low cost firms tend to move first. Information pooling and cost savings are aspects that are clearly related to merger analysis.

The remaining of the paper is structured as follows. Section 2 presents the basic model and derives the equilibria before and after the merger. The effects of the merger on price and the analysis of the merger paradox are included in section 3. The last section concludes. All proofs are presented in the appendix.

2 The model

We assume a two-stage game: first all leaders simultaneously choose their output levels; then all followers simultaneously choose their output levels, after observing the leaders' choices. Initially, there are $l \geq 1$ leaders and $n \geq 3$ followers that compete in quantities over a linear demand $P = a - Q$, where

[2] Ino and Matsumuray (2009) extend Daughety's results to general demand and cost functions.

$Q = \sum_{j=1}^{l} q_j + \sum_{i=1}^{n} q_i$ and $a > 0$. Costs are convex and given by $\frac{cq^2}{2}$, with $c \geq 0$.[3] Under this setting we will analyze the effects of a merger between two followers that become a leader, that is, the merged firm acts as a leader after the merger. Setting either l or n to zero reduces our model to the standard Cournot.

The merged firm thus enjoys a leadership advantage, and a cost advantage. The merger gains deriving from cost convexity result from the possibility of allocating production across several plants. Keeping multiple plants provides a variable cost advantage over single-plant firms: the merged firm's cost function is $\frac{cq^2}{4}$. Note that although the cost function of each plant is unaffected by the merger, the slope of the merged firm's marginal cost is cut in proportion to the number of participants.

2.1 Before the merger

It is easy to show that, before the merger, the quantity produced by each leader and each follower is, respectively,

$$\begin{aligned} q_L^B &= \frac{(c+1)\,a}{2c+l+cl+cn+c^2+1} \\ q_F^B &= \frac{\left(2c+cn+c^2+1\right)a}{(2c+l+cl+cn+c^2+1)(c+n+1)}. \end{aligned}$$

The equilibrium price equals

$$P^B(l,n) = \frac{\left(2c+cn+c^2+1\right)(c+1)\,a}{(2c+l+cl+cn+c^2+1)(c+n+1)}$$

and individual profits are

$$\begin{aligned} \pi_L^B &= \frac{\left(3c+cn+c^2+2\right)(c+1)^2 a^2}{2\left(2c+l+cl+cn+c^2+1\right)^2 (c+n+1)} \\ \pi_F^B &= \frac{\left(2c+cn+c^2+1\right)^2 (c+2)\,a^2}{2\left(2c+l+cl+cn+c^2+1\right)^2 (c+n+1)^2} \end{aligned}$$

for each leader and for each follower, respectively.

Finally, consumer surplus is equal to

$$CS^B = \frac{(a-P^B)^2}{2}.$$

[3] We assume that fixed costs per plant are sufficiently small so that the merged firm has no incentive to shut down one of its plants. Hence, there are no fixed cost savings due to the merger.

In any case, fixed cost savings would be irrelevant in our analysis, because, as we will show, the merger is always profitable in their absence. Furthermore, prices and outsider profits are unaffected by fixed cost savings.

2.2 After the merger

When two followers merge and become a leader, there will be three types of players: $n-2$ followers and l leaders equal to the existing ones before the merger, and one additional leader - the merged firm - with lower costs. The corresponding equilibrium outputs are

$$\begin{aligned} q_M &= \frac{2\left(cn+c^2+1\right)(c+1)\,a}{\Psi(c,n,l)} \\ q_L^A &= \frac{\left(c+cn+c^2+2\right)(c+1)\,a}{\Psi(c,n,l)} \\ q_F^A &= \frac{\left(c+cn+c^2+2\right)\left(cn+c^2+1\right)a}{\Psi(c,n,l)\,(c+n-1)} \end{aligned}$$

where the merged firm produces q_M, the other leaders produce q_L^A each, the followers produce q_F^A each and

$$\begin{aligned} \Psi(c,n,l) \quad : &= c^2n^2 + nc\left(3c+l+cl+2c^2+5\right)+ \\ &+ (c+1)\left(3c+2l+cl+2c^2+c^3+c^2l\right)+4. \end{aligned}$$

The post-merger equilibrium price is given by

$$P^A(l,n) = \frac{\left(c+cn+c^2+2\right)\left(cn+c^2+1\right)(c+1)\,a}{\Psi(c,n,l)\,(c+n-1)}$$

and profits and consumer welfare are as follows.

$$\begin{aligned} \pi_M &= \frac{\left(cn+c^2+1\right)^2\left(3c+cn+c^2+4\right)(c+1)^2 a^2}{\left(\Psi(c,n,l)\right)^2(c+n-1)} \\ \pi_L^A &= \frac{\left(c+cn+c^2+2\right)^3(c+1)^2 a^2}{2\left(\Psi(c,n,l)\right)^2(c+n-1)} \\ \pi_F^A &= \frac{\left(c+cn+c^2+2\right)^2\left(cn+c^2+1\right)^2(c+2)\,a^2}{2\left(\Psi(c,n,l)\right)^2(c+n-1)^2} \\ CS^A &= \frac{(a-P^A)^2}{2} \end{aligned}$$

Since there are no fixed cost savings, the profitability of the merger is given by $\triangle\pi_M(a,c,l,n) = \pi_M - 2\pi_F^B$. In turn, the incentive to free ride is given by $\pi_M - 2\pi_F^A$. Note that by setting $c=0$ we obtain Daughety's result, after the appropriate reparameterizations.

3 The effects of the merger

This section discusses the impact of the merger on consumers and on the merger paradox. In particular, we analyse how cost convexity affects the merger impact on consumer surplus, on profitability and on the incentive to free ride (insider's dilemma). Whenever appropriate, we decompose the merger impact in the three effects described in the Introduction, namely: reduction in the number of followers, transformation of a follower into a leader and increased efficiency for the leader resulting from the merger.

3.1 Consumer surplus

In our setting, price is a sufficient statistic for consumer surplus. The sign of $\Delta P = P^B(n,l) - P^A(n,l)$ tells us whether consumers are better off or worse off after the merger. The following Lemma establishes conditions necessary and sufficient for the merger to reduce consumer surplus.

Lemma 1: *The merger results in a price increase if and only if*

$$l > l^*(c) := \frac{(n-c-3)\left(2c+cn+c^2+1\right)\left(cn+c^2+1\right)}{(c+1)^2\left(c+cn+c^2+2\right)}.$$

■

In words, Lemma 1 states that the merger increases price if and only if there is a sufficiently large number of leaders. This condition is easier to verify when the initial number of followers is low. The explanation is the following. One of the consequences of this type of merger is to turn followers into leaders. Recall that Daughety (1990) established that the number of leaders that maximizes consumer surplus and welfare is half the number of firms. Equilibria with only leaders or only followers are welfare minimizing. Despite the fact that each leader produces more than each follower, it is not true that an industry made up of leaders would yield a higher social welfare. The incentive for the leader to produce more is reduced as the number of followers diminishes. This explains why an extreme number of leaders is welfare minimizing: if all firms were leaders, there would be no followers to be induced to decrease their production and the market would end up in a Cournot equilibrium. This same reasoning applies, although with some differences, to the case of convex costs. In this case, the optimal number of leaders, in terms of consumer surplus, is also positive and below the total number of firms.[4] Hence, if there are too many leaders, a merger of the type under analysis, by creating an additional leader, may increase price.

Note that $l^*(c)$ differs from half the number of firms in the industry. This happens because a merger has more implications than merely increasing the number of leaders for the same number of firms. In particular, it also reduces the total number of competitors (i.e., followers) and generates cost reductions for the participants. To understand all the forces at play we decompose the

[4] It can be shown that the number of leaders that maximizes consumer surplus in our setting is $N - \frac{(1+c)^2}{c}\left(\sqrt{1+\frac{c}{(1+c)^2}N} - 1\right)$, where N is total number of firms.

price variation into the three steps described in the Introduction, to isolate the different effects involved.

We then have $\Delta P = \Delta_1 P(c) + \Delta_2 P(c) + \Delta_3 P(c)$ where $\Delta_1 P(c) = P^B(l, n-1) - P^B(l, n)$ captures the suppression of a follower, $\Delta_2 P(c) = P^B(l+1, n-2) - P^B(l, n-1)$ captures the transformation of a follower into a leader, and $\Delta_3 P(c) = P^A(l+1, n-2) - P^B(l+1, n-2)$ captures the efficiency effect associated with the cost reduction that the new leader enjoys.[5]

The third effect above is a direct consequence of the cost reductions that result from cost convexity. Given that this effect pushes prices downwards, one may conclude that the existence of cost convexity makes it more likely that the merger in question decreases price. The effect of c, however, is uncertain. Table 1 presents the threshold c^*, obtained numerically, such that for $c > c^*$ we have $l^*(c) < l^*(0)$. Thus, if marginal costs are sufficiently steep, there is an interval for l where price increases with convex costs but decreases in the case of linear costs ($c = 0$).

	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=10$	$n=15$	$n=25$
c^*	0	0.638	1.440	2.233	3.022	5.378	9.292	17.107

Table 1: Critical value for c such that $l^*(c) < l^*(0)$

In other words, the condition for price to increase presented in Lemma 1 may be easier to verify in the presence of cost convexity, provided that this is sufficiently large.[6]

From now on our analysis will be focused on the interval $[l^*(c), l^*(0)]$. This corresponds to the counterintuitive case where the merger increases price with cost convexity but decreases it with linear costs. Note that for $n \leq 4$ we have $l^*(0) < 1$ and, hence, it is impossible to have a value $l \geq 1$ that belongs to this interval. For $n > 4$, there is always the possibility that $l \in [l^*(c), l^*(0)]$.

If introducing a positive c creates an additional effect that pushes price downwards, but, nevertheless, it is more likely that price will increase, the explanation must come from the impact of c on the other two effects, namely the suppression of a follower and the transformation of a follower into a leader. In fact, it is possible that cost convexity may change these two effects, making them both more conducive to a price increase, as the following example illustrates for the case of $n = 5$ and $l = 1$. Table 2 presents the decomposition of the price change that results from the merger:

[5] With a linear demand, under Cournot competition, a two-firm merger always increases prices, both with linear or quadratic costs. However, the percentual price increase is lower when costs are quadratic and, hence, when the insiders benefit from a reduction in marginal costs. In this case there are only two effects: the supression of a rival and the reduction in the costs of one of the firms.

[6] Furthermore, increasing the degree of cost convexity may relax the condition for price to increase. In fact, a sufficient condition for $\frac{\partial l^*}{\partial c} < 0$ is that $c > 1$. For lower values of c we can either have $\frac{\partial l^*}{\partial c} < 0$ or $\frac{\partial l^*}{\partial c} > 0$.

	1st effect	2nd effect	3rd effect	Total
$c = 0$	+1.67%	-1.67%	0	0
$c = \frac{5}{2}$	+4.25%	-0.57%	-3.33%	+0.35%

Table 2: Decomposition of the price variation for $n = 5$ and $l = 1$, measured in percentage of parameter a.

The following subsections analyze these effects separately, highlighting the role of cost convexity. We will show that for $l \in [l^*(c), l^*(0)]$ the sign of the three effects is as presented in Table 3.

	1st effect	2nd effect	3rd effect	Total
$c = 0$	$(+)$	$(-)$	0	$(-)$ or 0
$c > 0$	$(+)$	$(-)$	$(-)$	$(+)$

Table 3: Sign of the three effects on price for $l \in [l^*(c), l^*(0)]$

3.1.1 1st Effect

It is easy to show that $\Delta_1 P(c)$ is positive for every combination of the parameters. This result is not surprising, as this effect simply corresponds to less firms and lower competition.

Inspection of the impact of c on $\Delta_1 P(c)$ is not fully conclusive but it allows for the possibility that a positive c amplifies the price increase associated with a reduction in the number of followers (see Table 2).

Lemma 2: *Let $c \leq n^2 - n - 1$. Then, the absolute increase in price due to the suppression of a follower is larger with convex costs than with linear costs. Let $c > n^2 - n - 1$. Then, the absolute increase in price due to the suppression of a follower may be larger or smaller with convex costs than with linear costs.* ■

Reducing the number of followers affects the market outcome by (i) reducing followers' aggregate output for each level of leaders' production and (ii) reducing followers' output contraction in response to an increase in the production of leaders. Anticipating these changes, leaders will increase production in response to the first effect and decrease production in response to the second effect. The net effect is an increase in the leaders' production and a decrease in total quantity. According to Lemma 2, how this net effect is influenced by c is ambiguous.

In what follows we will first discuss how c affects the reduction in followers' output after a follower is suppressed and then how it affects the leaders' response.

Note that if we take just the followers' problem, this is equivalent to a Cournot game with demand $a - Q_L - Q_F$. The equilibrium is given by

$$Q_F = \frac{n}{n+1+c}\left(a - Q_L\right). \tag{1}$$

Now, if n decreases by 1 the change in Q_F is

$$\Delta Q_F(c) = \left(\frac{n-1}{n+c} - \frac{n}{n+1+c}\right)(a - Q_L)$$

We have $\Delta Q_F(c) < \Delta Q_F(0) < 0$ if and only if $c < n^2 - n - 1$. This means that although ΔQ_F is always negative, a positive (but small) c will result in a higher contraction in the quantity of followers. To understand this, note that the change in followers' output results from the suppression of a firm and also from the output expansion by the remaining followers. Hence,

$$\Delta Q_F(c) = -\frac{1}{n+1+c}(a - Q_L) + (n-1)\left(\frac{1}{n+c} - \frac{1}{n+1+c}\right)(a - Q_L)$$

Clearly, the higher is c the lower the output of the firm that is suppressed. But, at the same time, the smaller will be the output expansion by the surviving rivals.[7] On the one hand, output falls less when a follower exits the market when c is positive as compared with $c = 0$. On the other hand, due to the steepness of their marginal cost functions, the surviving followers increase their output less significantly when c is positive as compared with $c = 0$. This explains why cost convexity may lead to a larger reduction in followers' production for a given Q_L.

Inspection of (1) shows that c influences the impact of n on $\partial Q_F/\partial Q_L$ in the same way as it affects the impact of n on Q_F. A reduction in the number of followers reduces the way followers respond to changes in the leaders' aggregate production by $\Delta R(c) := R(n-1, c) - R(n, c) < 0$ where $R(n, c) := |\partial Q_F/\partial Q_L| = \frac{n}{n+1+c}$. We have $\Delta R(c) < \Delta R(0)$ if and only if $c < n^2 - n - 1$. Hence, with a low c followers become less responsive to the leaders' aggregate output after the exclusion of a follower.

We now turn to the leaders' response. As discussed above, when a follower exits the market the leaders face a higher residual demand, that makes them produce more. But, at the same time, followers become less responsive to the leaders' production, which makes leaders produce less. When costs are linear, these opposite signed effects cancel each other and the leaders' aggregate production remains unchanged with n. Under convex costs and with $c < n^2 - n - 1$ both effects becomes stronger and the first one dominates. Hence, following a reduction in n the leaders will increase their production when costs are convex, while they will keep it constant under linear costs. Nevertheless, it turns out that this output expansion does not compensate the stronger reduction in the followers' output and, hence, price increases more under convex costs.

3.1.2 2nd Effect

The sign of $\Delta_2 P(c)$ is given by the sign of expression $n - l - 2c - 2cl - cn - 2c^2 - c^2 l + cn^2 + c^2 n - 2$, which is positive for

$$l > l^{**}(c) := (n-2)\,\frac{c + cn + c^2 + 1}{(c+1)^2}.$$

This means that for a transformation of a follower into a leader to improve consumer surplus, the number of existing leaders as compared to existing followers must be low enough. The above relationship generalizes the result of

[7] The net effect is always negative, but is not monotonous in c.

Daughety (1990) when $c = 0$. It turns out that, given the total number of firms, the consumer surplus maximizing number of leaders is larger when costs are convex than when costs are linear, i.e, $l^{**}(c) > l^{**}(0)$. Hence, cost convexity makes it less likely that price increases with the merger due to this effect.[8] Note that $l^{**}(c) > l^{*}(c)$, so a sufficient condition for price to rise with the merger is that turning a follower into a leader increases price, regardless of the other two effects.

When $c > c^{*}$ we have $l^{*}(c) < l^{*}(0) < l^{**}(0) < l^{**}(c)$. Therefore, whenever l belongs to $[l^{*}(c), l^{*}(0)]$ it is also true that $l < l^{**}(0) < l^{**}(c)$. This means that, in the interval under analysis, the second effect always pushes price downwards both with linear and convex costs. Although the second effect is negative in both cases, one can illustrate, with examples, that it may either be of a larger magnitude under linear or under convex costs.

3.1.3 3rd Effect

It can be easily shown that $\Delta_3 P(c)$ is negative for every combination of the parameters, that is, improving the efficiency of one player benefits consumers, as expected.

3.2 Merger Paradox

In this subsection we analyze how the two elements of the merger paradox – profitability and incentive to free ride – are affected by the presence of cost convexity.

3.2.1 Profitability

Profitability is defined as $\triangle\pi_M(a, c, l, n) = \pi_M - 2\pi_F^B > 0$. In the case of $c = 0$ we simply have

$$\Delta\pi_M = \frac{10l + 3l^2 + \left(2ln^2 - 4ln\right) + \left(l^2n^2 + n^2 - 6n\right) + 9}{(n+1)^2 (n-1)(l+1)^2 (l+2)^2},$$

which is positive, that is, the merger is always profitable, as previously shown by Huck et al (2001) and Feltovitch (2001). When $c > 0$, a cost reduction effect is added, so the merger is also expected to be profitable.[9]

Lemma 3: *A merger whereby two followers become a leader is always profitable.* ■

This result solves the first part of the merger paradox: any two followers always have an incentive to merge and become a leader. This contrasts with

[8] However, this is not monotonous in c, because $\frac{\partial l^{**}}{\partial c} < 0$ if and only if $c > 1$. Hence, for a large c, further increases in the degree of cost convexity make it more likely that the price will rise with the merger.

[9] This is in line with the case of Cournot competition with convex costs. As the degree of cost convexity increases it becomes more likely that a two-firm merger is profitable.

Heywood and McGinty (2007b), where profitability of mergers involving two leaders or two followers that stay a follower hinges on a sufficiently high c.

3.2.2 Free riding

In this subsection we analyze whether outsider followers profit more from the merger than the participating firms, i.e, whether $\pi_M - 2\pi_F^A < 0$.[10] If this is the case, although participant profits increase with the merger, firms would prefer to wait for their rivals to merge, thus benefitting from the higher prices without having to reduce production. This may stop profitable mergers from taking place.[11]

A priori, one would expect that the presence of merger created cost savings, i.e. a positive c, would help solving the free rider problem. As Heywood and McGinty (2008) put it, "In traditional models without merger-created cost efficiencies, the profit of the excluded firms increases. (...) However, with convex costs, the merged Stackelberg leader may actually increase output beyond that of its pre-merger constituent firms. When that happens, excluded rivals see their output and profits fall". Nevertheless, as we will show, a positive c, when compared to the case of linear costs, may actually create the free rider problem.

The next Lemma establishes under which conditions it is better to free ride than to participate in the merger. As we only allow for a merger between two followers, the incentives to free ride, or the insiders' dilemma, need only be considered for this type of firms.

Lemma 4: *Outside followers profit more from the merger than the participating firms if and only if* $c > n - 3$. ■

A direct implication of Lemma 4 is that under linear costs there is no free rider problem. However, the presence of convex costs may restore it.

As before, we can divide the effects of the merger into three steps. We want to compare how the different steps affect the joint profit of two followers that merged and the joint profits of any two followers that remained independent.

The first step corresponds to the suppression of a follower. This has a negative impact on the profit of the merging followers because one of them is eliminated and the gains of the other do not compensate for this. However, the suppression of a rival is always positive for any two followers that did not merge.

[10] We are assuming that the gain from merging is equally divided among participants. In that case, each participant would see its profit rise by $\pi_M/2 - \pi_F^B$, whereas an outsider follower would gain $\pi_F^A - \pi_F^B$. The former gains more than the latter if and only if $\pi_M/2 - \pi_F^B - (\pi_F^A - \pi_F^B) > 0 \Leftrightarrow \pi_M - 2\pi_F^A > 0$. An alternative approach would be to consider that one party acquired the other by paying it the pre-merger profit, i.e, one firm would capture all the additional profits. In that case, the gain to the acquiring firm would be $\pi_M - 2\pi_F^B$. This exceeds the one of an outsider follower if and only if $\pi_M - 2\pi_F^B - (\pi_F^A - \pi_F^B) > 0 \Leftrightarrow \pi_M - \pi_F^B - \pi_F^A > 0$. Given that there is no reason to assume that one firm would have a bargaining advantage *vis-à-vis* a symmetric competitor, we opted for the first approach.

[11] In the case of Cournot competition with convex costs there is always an incentive to free ride. This incentive decreases with the degree of cost convexity.

The second step turns a follower into a leader. This is profitable for the follower in question and has a negative impact on the joint profits of any two followers that remained independent if and only if $l < l^{**}(c)$. The reason is simple. Increasing by one the number of leaders always reduces each follower's profit. When the number of leaders is small, this increase has a more significant impact than when it is large. On the other hand, reducing by one the number of followers always increases each follower's profit. If l is small, the former effect dominates the latter and the follower's profits decrease. If l is large the opposite occurs. As $l^{**}(c) > l^{**}(0)$, the existence of convex costs makes it more likely that this effect is negative for the outsider followers.

Finally, the third step corresponds to the change in the merged firm cost function, which is profitable for the insiders but unprofitable for any pair of outsider followers. Table 4 summarizes the sign of these effects, where (++) means more positive than (+).

	1st eff.	2nd eff.		3rd eff.	Total	
		$l < l^{**}(c)$	$l > l^{**}(c)$	$c > 0$	$c < n-3$	$c > n-3$
$\Delta\pi_M$	(−)	(+)	(+)	(+)	(++)	(+)
$2\Delta\pi_F^A$	(+)	(−)	(+)	(−)	(+)	(++)

Table 4: Decomposition of the incentive to free ride.

Hence, if a positive c leads to a third effect that reduces the incentive to free ride and makes it more likely that the second effect also reduces the incentive to free ride, then it must be the case that cost convexity amplifies the first effect, so that sufficiently high values of c restore the insiders dilemma.

Finally, it can be shown that outsider followers always lose with the merger if and only if the merger results in a price decrease. In these circumstances, there is no incentive to free ride, so the merger is expected to occur and consumers will benefit from it. However, it is also possible to have a case where outsider followers benefit from the merger by less than the merging parties. In this situation, there is also no incentive to free ride, the merger still is expected to take place, but, contrary to the previous case, consumers will be worse off.

4 Conclusion

This paper analyzes mergers in Stackelberg markets, with quadratic production cost. We consider the case of a merger between two followers that gives rise to a new leader. Contrary to what might be expected, it is possible, with cost convexity, that (i) a merger may result in an increase in price and (ii) there is an incentive to free ride, when without cost convexity exactly the opposite would occur. This contrasts with the literature where cost convexity has been presented as a solution to the merger paradox and where merger induced cost savings are claimed to make it more likely that prices will decrease.

Appendix

Proof of Lemma 1: It can be shown that $\Delta P > 0$ if and only if $-c^5 + c^4(-l-n-5) + c^3(n^2 - 6n - ln - 3l - 8) + c^2(n^3 - 6n - 2ln - n^2 - 5l - 8) + c(2n^2 - 4n - ln - 5l - 7) + n - 2l - 3 > 0$. This condition is linear in l and can be written as $l > \frac{(n-c-3)(2c+cn+c^2+1)(cn+c^2+1)}{(c+1)^2(c+cn+c^2+2)}$. ■

Proof of Lemma 2: The sign of $\Delta_1 P(c) - \Delta_1 P(0)$ is equal to the sign of $Al^2 + Bl + C$, where

$$\begin{aligned} A &= (c+1)^2(n^2 - n - c - 1) \\ B &= (c+1)\begin{pmatrix} c^3(n^2 + n - 2) + c^2(2n^3 + 5n^2 - 3n - 5) \\ +c(n^4 + 4n^3 + 2n^2 - 5n - 5) + 2n^2 - 2n - 2 \end{pmatrix} \\ C &= (2c + cn + c^2 + 1)(c + cn + c^2 + 1)(n^2 - n - c - 1) \end{aligned}$$

For $c \leq n^2 - n - 1$ we have $A \geq 0$, $B > 0$ and $C \geq 0$. Hence, $\Delta_1 P(c) > \Delta_1 P(0)$.

For $c > n^2 - n - 1$ an example shows that we may have $\Delta_1 P(c) > \Delta_1 P(0)$ or $\Delta_1 P(c) < \Delta_1 P(0)$. Consider the case of $n = 10$ and $c = 200$. It is easy to check that $l^*(c) < 1$. The relevant interval then becomes $[1, l^*(0)]$. As $A \times 1^2 + B \times 1 + C < 0$ and $Al^*(0)^2 + Bl^*(0) + C > 0$ we conclude that $\Delta_1 P(c) > \Delta_1 P(0)$ or $\Delta_1 P(c) < \Delta_1 P(0)$ are both possible for $l \in [1, l^*(0)]$ ■

Proof of Lemma 3: Proposition 1 in Heywood and McGinty (2008) establishes that $\pi_M - \pi_F^B - \pi_L^B > 0$. As $\pi_F^B < \pi_L^B$, this implies that $\pi_M - 2\pi_F^B > 0$. ■

Proof of Lemma 4: One can show that

$$\pi_M - 2\pi_F^A = \frac{(cn + c^2 + 1)^2 (5c + cn + 3c^2 + 4)(n - c - 3)a^2}{(c + n - 1)^2 \Psi(c, n, l)^2}$$

from where the result is immediate. ■

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