

# Inter-firm Bundling and Vertical Product Differentiation\*

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February 4, 2011

## Abstract

This paper studies the competitive effects of bundled discounts when each component good is sold by a different single-product firm. In a setting with vertically differentiated goods and firms deciding simultaneously about their participation in a discounting scheme, it is shown that, in equilibrium, all firms offer bundled discounts and, relative to the no-bundling benchmark: (i) all headline prices rise; (ii) all bundle prices, net of the discount, rise; and (iii) all firms earn higher profits. Furthermore, the equilibrium corresponds to the worst scenario in terms of consumer and social welfare, when compared to bundled discounts only offered by a single pair of firms or to the no-bundling benchmark.

**Keywords:** Bundled Discounts, Bilateral Bundling, Vertical Differentiation.

**JEL Classification:** *D43; L13; L41.*

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\*An early version of the paper has circulated under the (long) title “Bundled Discounts by Independent Producers of Vertically Differentiated Goods.” (AdC WP n° 38).

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# 1 Introduction

Bundled discounts provide purchasers the opportunity to pay less for a bundle than the sum of the prices of the bundled products when purchased separately. These discounting schemes thus confront consumers with the choice between meeting all their requirements by buying a package at a discounted price and *à la carte* offerings.

Examples of companies offering bundled discounts include fast food restaurants, telephone companies, book stores, grocery stores and gasoline retailers, to name a few. Despite the fact that bundled discounts are a widespread business practice, the academic literature has devoted limited attention to this issue. So, little is known as to whether this type of discounting schemes should raise anticompetitive concerns.<sup>1</sup>

Bundled discounts affect the market outcome in at least two important ways. On the one hand, they can change the market structure by affecting firms' incentives to enter or exit the market. On the other hand, they introduce an additional instrument that enables some degree of price discrimination and, consequently, may have an important impact on both consumer surplus and social welfare. Accordingly, one can divide the literature on bundled discounts in two related strands. The first strand has investigated the case in which the bundled discount is offered by a multiproduct firm offering two or more goods or services as a package for a lower price than the aggregate price of its constituent parts. In particular, by examining a setting wherein a monopoly seller in one market faces competition in a second market, a set of recent models (e.g. Peitz (2008), Greenlee *et al.* (2008), Nalebuff (2005) and Nalebuff (2004)) has shown that the use of bundled discounts can lead to the exclusion of an existing (or potential) equally efficient rival that does not offer an equally diverse group of products. The second strand of the literature analyzes the implications of bundled discounts, *per se*, on consumer surplus and on social welfare and dates back to Carbajo *et al.* (1990) and Matutes and Regibeau (1992). Carbajo *et al.* (1990) show that imperfect competition creates a strategic incentive to bundle which is absent under the polar cases of competitive and monopolized tied markets. Their key argument is that the decision to bundle typically alters the behaviour of rivals in the non-competitive tied market. In their setting, bundling works as a product differentiation device resulting in less aggressive pricing by rivals (competition is relaxed) and lower consumer surplus. Matutes and Regibeau (1992)

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<sup>1</sup>As pointed out by Nalebuff (2005, p.364), “[t]he practice of bundle discounts is prevalent, but their effects on competition are not well understood.” Along these lines, Kobayashi (2005) highlights that his “review of the economic literature [on commodity bundling] generally confirms the US Solicitor General’s view in *3M v. LePage*’s regarding the underdeveloped state of the economics literature (...).”

analyze the behavior of duopolistic firms, both supplying the two necessary components to make up a system. More specifically, they study the incentives these firms have with respect to: (i) making the components compatible with those produced by the rival firm; and (ii) offering a discount to those consumers who decide to purchase the whole system from them. In their setting, the four possible combinations that make up the system are located at the corners of a unit square where consumers are uniformly distributed. The main conclusions are that, in most cases, firms will produce compatible goods and offer discounts to those consumers who purchase both components from them. However, there is a prisoner's dilemma in the sense that firms would have a higher payoff if they could commit not to offer discounts. In addition, when in equilibrium all firms offer discounts, welfare is lower than when there are no discounts.

This pathbreaking work by Matutes and Regibeau (1992) has been extended in several ways. In particular, Thanassoulis (2007) builds on Matutes and Regibeau (1992) by introducing consumers who only value one of the components and by making a distinction between firm-specific and product-specific preferences. As in Matutes and Regibeau (1992), there is a prisoners' dilemma: firms lose from mixed bundling. Armstrong and Vickers (2010), on the other hand, show that when Matutes and Regibeau's (1992) model is developed so that consumers need to pay an extra "shopping cost" when purchasing products from more than one firm and consumer preferences for product brands are correlated, then mixed bundling is more likely to lead to welfare gains.

All the above mentioned papers consider the case of bundling by multi-product firms. When this is the case, the discount is given to those consumers who purchase all relevant products from a single firm. However, there are several examples of discounts for bundles where each component good is sold by a different and independent firm. A typical example is the case of supermarket and retail gasoline chains that frequently offer bundled discounts to consumers who purchase from them. These discounts are usually a fixed amount off the headline (or stand-alone) prices that partner firms continue to set independently. This specific example has motivated the work by Gans and King (2006), the most similar previous work to ours.<sup>2</sup> Gans and King (2006) investigate the case where each of the two components is produced by two single product independent firms.<sup>3</sup> In their setting, unilateral bundling,

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<sup>2</sup>For a discussion of the Australian case, see Gans and King (2004).

<sup>3</sup>Other work that focuses on bundled discounts by independent producers is Maruyama and Minamikawa (2009), who study the incentives for vertical integration. Also, Armstrong (2010) discusses the impact on profits, consumer surplus and welfare of inter-firm discounts, focusing on independent, partially substitutable and complementary products.

i.e., bundling by a single pair of firms, increases the profits of the partner firms to the detriment of the remaining non-bundling firms. Nevertheless, if both pairs of independent firms offer bundled discounts, i.e., if there is bilateral bundling, then each firm's profits and output end up being *the same* as in the case where there are no bundled discounts. Moreover, bilateral bundling leads to a social-welfare reduction, as some consumers simply find themselves consuming a sub-optimal branding mix.<sup>4</sup>

The main differences between the results regarding models of bundling by independent and otherwise unrelated firms and those obtained in models of bundling by a multi-product firm are driven by the combination of the two following effects. First, firms' objective functions are different in the sense that the multiproduct firm takes into consideration how changing the price of a given product affects the demand for the other products in its portfolio. Second, when the discount is shared by independent firms, there is the need to contract upon its magnitude and also upon how the corresponding cost is divided between the partner firms. This negotiation must occur before the price setting stage and, hence, there are additional strategic considerations to be dealt with when setting the optimal discount.

One of the assumptions of Gans and King (2006), which is shared by Matutes and Regibeau (1992), Thanassoulis (2007), Peitz (2008) and Armstrong and Vickers (2010), is that products are horizontally differentiated.<sup>5</sup> However, there are several examples of bundled discounts in industries where, at least with respect to one of the products in the bundle, differentiation is clearly vertical. As mentioned, many supermarkets and discount stores offer a grocery-gasoline bundled discount whereby customers receive a discount on their grocery-gasoline purchases from the firms involved in the discounting scheme. It should be noted, however, that in this example, there is vertical differentiation, at least, on the groceries' side: most consumers consider that supermarkets chains and discount stores offer products and services of different quality. Another case in point refers to bundled discounts offered to consumers who look for airline tickets and car-rental. Low cost airlines and national carriers are also, in most cases, vertically differentiated, as well as local rent-a-car versus multinational rent-a-car agencies. To the best of our knowledge, however, vertical

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<sup>4</sup>In a recent empirical study using a detailed dataset regarding an Australian market - the Perth metropolitan area -, Wang (2009) finds that the Gans and King (2006) model captures the gasoline pricing behavior of those retail stations that are allied with, but are not operated by, supermarkets.

<sup>5</sup>Also, Caminal and Claici (2007) use a two-period horizontal differentiation model in which firms are able to discriminate between first time and repeat buyers. In the authors' opinion, "[l]oyalty programs can perhaps be interpreted as a form of price discrimination analogous to quantity and bundled discounts." (p.658)

differentiation has been neglected by the extant literature on bundled discounts.

In this paper we contribute to cover this gap in the literature by proposing a model of strategic interaction between four producers of two different and unrelated products to study the likely competitive effects of bundled discounts in the presence of vertical differentiation. In our setting, we follow Gans and King (2006) except for the type of product differentiation. In particular, we assume that consumers are arrayed on a unit square where each axis measures consumers' valuations for quality of the two products. Each good is produced by two firms, a high quality producer and a low quality producer. Pairs of firms may then agree to offer jointly a bundled discount across the two products and to share the costs of that discount. We address, apart from the no discounting benchmark case, three different scenarios: *(i)* unilateral bundled discount by the low quality firms; *(ii)* unilateral bundled discount by the high quality firms; and *(iii)* bilateral bundling.

We assume that only firms of the same quality level may offer bundled discounts. The motivation for this assumption is that a producer offering a high quality product is probably not interested in being allied with a low quality producer (of the other good) as this may seriously hurt the reputation of its firm. An advantage of this assumption is that it allows to keep the equilibrium analysis tractable.<sup>6</sup>

This theoretical framework enables us to raise a number of interesting questions: What are the welfare effects of bundled discounts in each of the three scenarios described above? Are bundled discounts consumer-surplus-enhancing? Should bundled discounts be free of antitrust concerns in this context? Which firms have the highest incentives to offer the discounts? Does the prisoners' dilemma identified in the literature carry over to the case of vertical differentiation? The answer to this and other related questions is, to our knowledge, not yet known and is the main focus of this paper.

We start by studying the competitive effects of the introduction of bundled discounts in each of the three scenarios identified above. Our main results are the following. First, and relative to the no-discounting benchmark case, the headline prices of the bundling firms rise whereas the headline prices of the firms not involved in discounting (if any) decrease. Second, whatever the scenario considered, bundled discounts always induce a decrease both in consumer surplus and in total welfare, with those reductions being more pronounced in the bilateral bundling case. This leads to the conclusion that in none of the three scenarios should bundled discounts be free of antitrust concerns. Third, in the case of bilateral

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<sup>6</sup>In particular, the determination of the optimal way to share the discount between a high quality and a low quality producer is an untractable problem.

bundling, both the price of the high quality bundle and the price of the low quality bundle, net of the corresponding discounts, rise. This last result is in sharp contrast with Gans and King (2006) since, in their model, under bilateral bundling the headline price for each separate good is increased by the exact value of the discount.

We then turn to the study of the firms' simultaneous decisions regarding their eventual participation in a bundled discount scheme. It turns out that offering a bundled discount is a dominant strategy both for the high quality firms and for the low quality firms. Hence, the Nash equilibrium of this game corresponds to the scenario of bilateral bundling, as in Gans and King (2006), where there are no consumers buying unpaired products: all consumers find it optimal to purchase a bundle and benefit from the corresponding discount. However, and in contrast with Matutes and Regibeau (1992), Gans and King (2006), Thanassoulis (2007) or Maruyama and Minamikawa (2009), in this bilateral bundling scenario all firms earn *higher profits* than in the *status quo* no-discounting situation and, therefore, firms do not find themselves in a prisoner's dilemma situation: allowing for vertical differentiation results in the elimination of the Bertrand bundling super-trap, as identified by Gans and King (2006).<sup>7</sup> In our setting, all firms have very strong incentives to participate in bilateral bundling. Nevertheless, this scenario is shown to be the one leading to the most adverse consequences both in terms of consumer welfare and in terms of social welfare. This then suggests that competition authorities should scrutinize in detail bundling discounting by independent producers of vertically differentiated goods.

The remainder of the paper is organized as follows. In Section 2, we lay down our general framework and specify the timing of the proposed game. In Section 3, we study the competitive effects of both unilateral bundling by low quality producers and by high quality producers as well as bilateral bundling, relative to the benchmark case where there is no bundling. Section 4 studies what we term as the discounting game, a simultaneous move game where each pair of firms decides upon its participation in a bundled discount scheme. Section 5 investigates the robustness of the main results obtained when the assumption that consumer valuations over the two products are uncorrelated is relaxed. In particular, this section considers the case of perfect positive correlation. Finally, Section 6 concludes the paper. All proofs are relegated to the Appendix.

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<sup>7</sup>The prisoner's dilemma is a recurrent result in the price discrimination literature. As Armstrong (2008) highlights, "(...) an oligopolistic firm is always better off if it can price discriminate compared to when it cannot, for *given* prices offered by its rivals. However, as in many instances of strategic interaction, once account is taken of what rivals too will do, firms in equilibrium can be worse off when price discrimination is permitted. Firms then find themselves in a classic prisoner's dilemma."

## 2 The model

### 2.1 Firms

We consider the case of two distinct products,  $X, Y$ , each sold by two firms, a high quality producer and a low quality producer. Denote by  $A_X, A_Y$  the two high quality producers and by  $B_X, B_Y$  the two low quality producers of products  $X$  and  $Y$ , respectively. There are no costs associated with the production of either product or quality level. We denote the price of the higher quality product by  $P_i$  and the price of the lower quality product by  $p_i$ , with  $i = X, Y$ .

Each pair of producers may agree to participate in a bundled discount scheme, where we assume that only firms of the same quality level may offer the bundled discounts together. Hence, we consider four different scenarios. Scenario 0 is the benchmark case in which there are no discounts. Scenarios 1 and 2 are unilateral bundling scenarios and refer, respectively, to the cases of a discount given by the low quality or by the high quality firms. Scenario 3 refers to the case of bilateral bundling: simultaneous bundled discounts offered by both the low and the high quality firms. Let  $\gamma_j$ , with  $j = A, B$  denote the discount offered by the producers of  $j_X$  and  $j_Y$ . When  $\gamma_j > 0$  we say that firms  $j_X$  and  $j_Y$  are partner firms in the discounting scheme. For instance, consumers that purchase the high quality bundle will pay  $P_X + P_Y - \gamma_A$ .<sup>8</sup>

In what follows,  $s_j$  represents an index of the quality of the product sold by firm  $j_i$ , with  $j = A, B$  and  $i = X, Y$ . Let the quality difference be denoted by  $s := s_A - s_B > 0$ . As Gans and King (2006), we are interested in the profitability of relatively small discounts; thus we assume that no discount can be larger than the average market price before the introduction of the discount. We denote by  $\alpha$  the percentage of the discount financed by the producer of  $X$ .

### 2.2 Consumers

The way we model consumers' preferences for quality follows Gabszewicz and Thisse (1979). Consumers purchase at most one unit of each good. Consumer's net utility when purchasing product  $i$  from producer  $A_i$  is given by  $V_i + \theta_i s_A - P_i$ , whereas consumer's net utility when purchasing product  $i$  from producer  $B_i$  is given by  $V_i + \theta_i s_B - p_i$ , with

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<sup>8</sup>As Armstrong (2010) points out, this additive bundled discount is "probably more easily implemented in practice relative to a system of choosing a rigid bundle price (...) and then negotiating how to share that revenue."

$i = X, Y$ . Throughout the paper we assume that  $V_i$  is sufficiently large so that the market is fully covered, i.e., every consumer purchases one unit of each good. Consumers do not get any extra benefit or any transaction costs reduction from purchasing goods of the same quality. Hence, in the absence of a discount, the demand for one product is independent of the demand for the other.

We assume that consumers' valuations for the quality of both products,  $(\theta_X, \theta_Y)$ , are uniformly distributed in  $[0, 1] \times [0, 1]$ .<sup>9</sup> Let

$$\begin{aligned}\theta_i^{*a} &: = \frac{P_i - p_i - \gamma_A}{s}, \\ \theta_i^{*b} &: = \frac{P_i - p_i + \gamma_B}{s},\end{aligned}$$

with  $i = X, Y$ .

The following lemma presents the relevant demand functions.

**Lemma 1:**

(i) Assume that  $(\theta_Y^{*a}, \theta_X^{*a}, \theta_Y^{*b}, \theta_X^{*b}) \in [0, 1]^4$ . Then, the demand functions for each possible pair of products are given by:

$$\begin{aligned}Q_{A_X, B_Y} &= \left(1 - \frac{P_X - p_X + \gamma_B}{s}\right) \left(\frac{P_Y - p_Y - \gamma_A}{s}\right), \\ Q_{B_X, A_Y} &= \left(1 - \frac{P_Y - p_Y + \gamma_B}{s}\right) \left(\frac{P_X - p_X - \gamma_A}{s}\right), \\ Q_{A_X, A_Y} &= \left(1 - \frac{P_Y - p_Y - \gamma_A}{s}\right) \left(1 - \frac{P_X - p_X - \gamma_A}{s}\right) - \frac{(\gamma_A + \gamma_B)^2}{2s^2}, \\ Q_{B_X, B_Y} &= \left(\frac{P_Y - p_Y + \gamma_B}{s}\right) \left(\frac{P_X - p_X + \gamma_B}{s}\right) - \frac{(\gamma_A + \gamma_B)^2}{2s^2}.\end{aligned}$$

(ii) Assume that  $\theta_Y^{*a} < 0$ ,  $\theta_X^{*a} < 0$  and  $\theta_Y^{*b} + \theta_X^{*a} \in [0, 1]$ . Then, the demand functions for each possible pair of products are given by:

$$\begin{aligned}Q_{A_X, A_Y} &= 1 - \left(\frac{P_X - p_X + P_Y - p_Y + \gamma_B - \gamma_A}{s}\right)^2 / 2, \\ Q_{B_X, B_Y} &= \left(\frac{P_X - p_X + P_Y - p_Y + \gamma_B - \gamma_A}{s}\right)^2 / 2, \\ Q_{A_X, B_Y} &= Q_{B_X, A_Y} = 0.\end{aligned}$$

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<sup>9</sup>In Section 5 it is shown that the main results derived under the assumption that consumer valuations over the two products are uncorrelated extend to the case of perfectly (positively) correlated valuations.



Now, making use of Lemma 1, the demand function for each individual product can be obtained from:

$$\begin{aligned} Q_{A_X} &= Q_{A_X, B_Y} + Q_{A_X, A_Y} \text{ and } Q_{B_X} = Q_{B_X, A_Y} + Q_{B_X, B_Y} \\ Q_{A_Y} &= Q_{B_X, A_Y} + Q_{A_X, A_Y} \text{ and } Q_{B_Y} = Q_{A_X, B_Y} + Q_{B_X, B_Y}. \end{aligned}$$

Clearly, all quantities are a function of prices and discounts. For the sake of brevity, however, we write  $Q_{A_X, B_Y}(P_X, p_X, P_Y, p_Y, \gamma_A, \gamma_B)$  as  $Q_{A_X, B_Y}$  and so forth. Also, when we say that a given consumer purchases, say,  $A_X, B_Y$  we mean that this consumer purchases the high quality version of good  $X$  and the low quality version of good  $Y$ .

### 2.3 Timing

The timing of the game played between firms is as follows:

1. The two pairs of firms of similar quality simultaneously agree to their bundled discount, if any.
2. Given the bundled discount(s), all four firms set their headline prices simultaneously.
3. Given prices and discounts, customers decide from which producers to make their purchases.

So, partner firms choose the discount and the percentage of the discount financed by each producer so as to maximize their joint profits. Afterwards, each single-product firm sets its headline price. This can be thought of a joint organization setting the discount on behalf of the two firms involved. Since the discount is set before the headline prices, we are implicitly assuming that it is easier for a firm to change its own price than to change the bundled discount. This is a natural assumption given that any firm is free to unilaterally change its headline price at a short notice, whereas changing the discount would involve a renegotiation with the partner firm. Moreover, in the gasoline-groceries example mentioned in the Introduction, it is often the case that firms advertise the discounting scheme but not the headline prices. This is a way of committing in advance to the agreed upon discount while still allowing for future changes in the headline prices.

## 3 Results

In this section, we investigate the consequences of bundled discounts relative to the situation where there is no bundling (and, thus, no discounts). More specifically, in what

follows, we discuss, apart from the no-discounting equilibrium, three different scenarios: two involving unilateral bundling and a final one involving bilateral bundling. We assume upfront that the discount is equally financed by the partner firms. In Appendix B we show that this allocation rule is optimal for both cases of unilateral bundling: regardless of the level of the discount, firms optimally agree to fund it equally, which is a natural consequence of the symmetry assumptions in the model. In what follows, we write the discount as  $\gamma_j = \beta_j s$ , with  $j = A, B$ .

### 3.1 Scenario 0: No discounting benchmark case

Consider first the benchmark case where  $\beta_A = \beta_B = 0$ . The corresponding demand functions are given by

$$Q_{A_i} = \frac{(s - P_i + p_i)}{s} \text{ and } Q_{B_i} = \frac{(P_i - p_i)}{s},$$

with  $i = X, Y$ . As expected, the demand for the high quality producer of good  $i$  depends only on the high and low quality prices of this specific good. In the absence of discounts, the decision to purchase a high or low quality version of one product is independent of the prices of the two different quality variants of the other product.

The equilibrium prices in this benchmark case are given by

$$P_X = P_Y = 2s/3 \text{ and } p_X = p_Y = s/3.$$

The corresponding quantities and profits are, respectively, equal to:

$$\begin{aligned} Q_{A_X, B_Y} &= Q_{B_X, A_Y} = 2/9; Q_{A_X, A_Y} = 4/9 \text{ and } Q_{B_X, B_Y} = 1/9 \\ \Pi_{A_X} &= \Pi_{A_Y} = 4s/9 \text{ and } \Pi_{B_X} = \Pi_{B_Y} = s/9 \end{aligned}$$

with  $\theta_X^{*b} = \theta_Y^{*b} = \theta_X^{*a} = \theta_Y^{*a} = 1/3$ . Consumers' purchasing choices in this no-discounting equilibrium are illustrated in Figure 1.

Note that the average price for product  $i$ ,  $i = X, Y$ , is  $\bar{p}_i = 5s/9$ . This is our assumed upper bound on the set of admissible discounts in the analysis of the three following scenarios.

### 3.2 Scenario 1: Unilateral bundling by the low quality firms

In this section, we consider the effects of bundled discounts by the pair of low quality producers, assuming that the pair of high quality producers does not offer a discount. Hence,  $\beta_B > 0$  while  $\beta_A = 0$ . The resulting division of consumers is as in Figure 2.

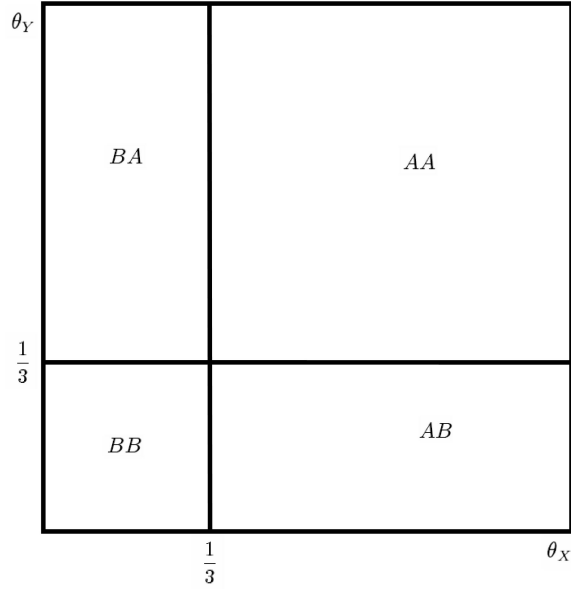


Figure 1: Consumers' choices with no discount.

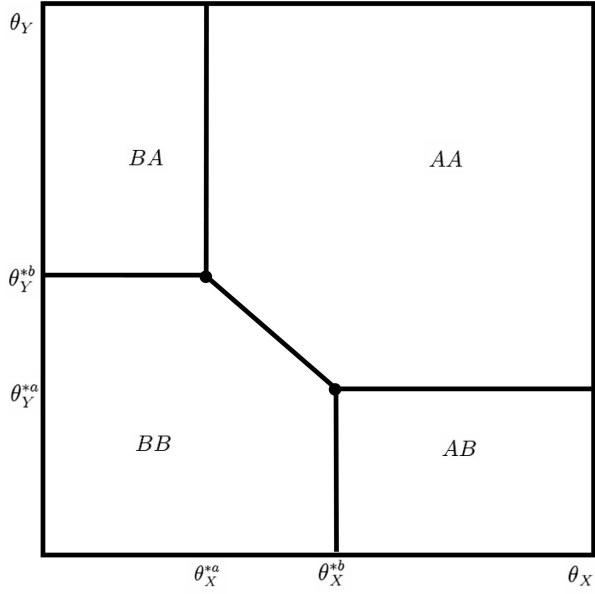


Figure 2: Consumers' choices with an unilateral bundled discount.

Equilibrium prices result from the individual maximization of the following objective functions:

$$\begin{aligned}
\Pi_{A_X} &= P_X (Q_{A_X, B_Y} + Q_{A_X, A_Y}) \\
\Pi_{A_Y} &= P_Y (Q_{B_X, A_Y} + Q_{A_X, A_Y}) \\
\Pi_{B_X} &= p_X (Q_{B_X, A_Y} + Q_{B_X, B_Y}) - \frac{\beta_B s}{2} Q_{B_X, B_Y} \\
\Pi_{B_Y} &= p_Y (Q_{A_X, B_Y} + Q_{B_X, B_Y}) - \frac{\beta_B s}{2} Q_{B_X, B_Y}
\end{aligned}$$

yielding<sup>10</sup>

$$P_X = P_Y = \frac{(6\beta_B + \beta_B^3 + 8)s}{2(5\beta_B + 6)} \text{ and } p_X = p_Y = \frac{(6\beta_B + 6\beta_B^2 + \beta_B^3 + 4)s}{2(5\beta_B + 6)}.$$

Analyzing the equilibrium prices, we conclude that, for all admissible discounts:

- (i) the low quality headline price increases with  $\beta_B$ . This is as expected. All else constant, a higher discount offered by the low quality producers increases the demand for the bundle, which means that demand for each low quality component increases, thus leading to higher headline prices for these component goods. Also, the introduction of the discount can be interpreted as a unit cost, partially incurred by each partner firm, for the units entitled to the discount.
- (ii) the low quality bundle price, net of the discount, is decreasing in  $\beta_B$ . Despite the fact that both headline low quality prices increase with the discount, their sum increases at a lower rate than the discount itself and, as a result, the “net” bundle price decreases with the discount.
- (iii) the high quality headline price is a U-shaped function of  $\beta_B$ . This results from the interaction of two effects with opposite signs. On the one hand, the price of the low quality bundle is decreasing in  $\beta_B$ , which will make some consumers switch from  $A_X, B_Y$  or  $B_X, A_Y$  to  $B_X, B_Y$ , thereby reducing the demand for the high quality component. However, the increase in the headline price of the low quality component, will also make some consumers switch from  $A_X, B_Y$  or  $B_X, A_Y$  to  $A_X, A_Y$ , thus increasing the demand for the high quality component. Each effect may then dominate the other, but it turns out that, for any admissible discount, the high quality headline price is always lower when the low quality firms bundle than in the no discount benchmark.

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<sup>10</sup>In order to compare Figures 1 and 2, note that, in equilibrium,  $\theta_X^{*a} = P_X/s - p_X/s \leq \frac{1}{3}$  and that  $\theta_X^{*b} = P_X/s - p_X/s + \beta_B \geq \frac{1}{3}$ . Furthermore,  $\frac{1}{3} - \theta_X^{*a} < \theta_X^{*b} - \frac{1}{3}$ . When  $\beta_B$  is equal to 0, we obtain the standard benchmark solution wherein  $P_X = P_Y = 2s/3$  and  $p_X = p_Y = s/3$ .

(iv) the sum of the headlines prices of two different quality products increases with the discount. When both of these headline prices increase with the discount, this effect is obvious. When, however, the high quality product headline price decreases in  $\beta_B$  whereas the low quality product headline price increases in  $\beta_B$ , it turns out that the latter effect is stronger.

Two remarks are worth making regarding the equilibrium quantities, which follow directly from the price effects described above. First, both  $Q_{A_X, A_Y}$  and  $Q_{B_X, B_Y}$  increase with the level of the discount given. Second, the share of consumers opting for purchasing products of different qualities decreases with the discount level.

The optimal discount is obtained by maximizing the low quality firms' joint profit evaluated at the equilibrium prices,

$$\Pi_B(\beta_B) = \frac{32\beta_B - 28\beta_B^3 + 10\beta_B^4 + 11\beta_B^5 - \beta_B^6 + 16}{2(5\beta_B + 6)^2}s,$$

with respect to  $\beta_B$  and subject to  $\beta_B \in [0, 5/9]$ . This yields  $\beta_B^* = 0.14058$  (obtained numerically).<sup>11</sup>

This optimal discount results from the trade-off between several effects, which we discuss in turn. The discount impacts low-quality firms' aggregate profit directly but also strategically, via the equilibrium prices. The *direct effect* is given by:

$$\frac{d\Pi_B}{d\beta_B} = \frac{\partial\Pi_B}{\partial Q_{B_X, A_Y}} \frac{\partial Q_{B_X, A_Y}}{\partial\beta_B} + \frac{\partial\Pi_B}{\partial Q_{A_X, B_Y}} \frac{\partial Q_{A_X, B_Y}}{\partial\beta_B} + \frac{\partial\Pi_B}{\partial Q_{B_X, B_Y}} \frac{\partial Q_{B_X, B_Y}}{\partial\beta_B} + \frac{\partial\Pi_B}{\partial\beta_B}.$$

The direct effects on demand are illustrated in Figure 3, where the grey area represents the increase in the market share of the low quality bundle.

This effect works as follows. When the discount increases, some consumers purchasing  $B_X, A_Y$  and  $A_X, B_Y$ , as well as  $A_X, A_Y$ , will switch to the bundle  $B_X, B_Y$ . Hence, the demand for the low quality firms will increase. However, all those consumers that were previously purchasing  $B_X, B_Y$  will now benefit from the increased discount.

Evaluated at the no-discount equilibrium, the net direct effect is positive and boils down to:

$$\frac{s}{3} \left( \frac{-1}{3} \right) + \frac{s}{3} \left( \frac{-1}{3} \right) + \frac{2s}{3} \left( \frac{2}{3} \right) - \frac{1}{9}s = \frac{1}{9}s.$$

If the discount and prices were set simultaneously, as in most of the previous literature on bundling by a multi-product firm, the *direct effect* would characterize completely the

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<sup>11</sup>For an interior solution, all  $\theta_X^{*b}, \theta_Y^{*b}, \theta_X^*, \theta_Y^*$  must belong to the  $[0, 1]$  interval. This is true if and only if  $\beta_B < \sqrt{6}/3$  which holds in the feasible range of discounts.

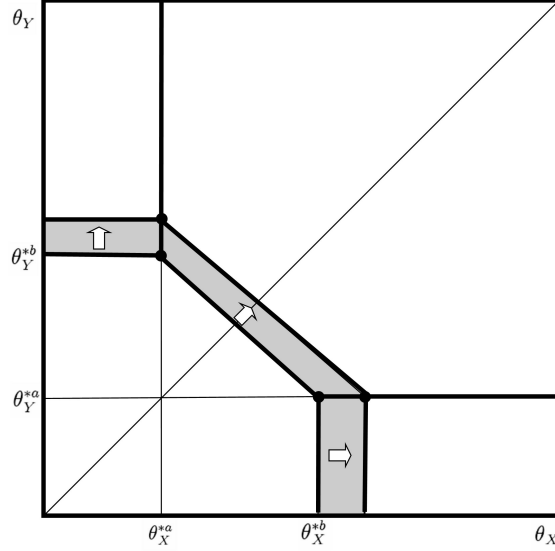


Figure 3: Direct effect on demand of an increase in bundle discount by the low quality producers.

incentives to offer discounts. However, in the case of single-product independent and otherwise unrelated partner firms, there is a pre-commitment to a discount and prices are set later. Hence, when setting the discount, firms must also consider the impact this discount will have both on the price set by the non-partner firms as well as on the price charged by the partner firms.

Let us first discuss the effect on the partner firms' profits that is due to the change in the high quality prices in response to an increase in the discount. As this results from changes in the prices of the non-partner firms, we call this the *external strategic effect*:

$$\begin{aligned} & \sum_{i=X,Y} \frac{\partial (\Pi_{B_X} + \Pi_{B_Y})}{\partial P_i} \frac{\partial P_i}{\partial \beta_B} \\ &= \sum_{i=X,Y} \left( p_X \frac{\partial (Q_{B_X, A_Y} + Q_{B_X, B_Y})}{\partial P_i} + p_Y \frac{\partial (Q_{A_X, B_Y} + Q_{B_X, B_Y})}{\partial P_i} - \beta_B s \frac{\partial Q_{B_X, B_Y}}{\partial P_i} \right) \frac{\partial P_i}{\partial \beta_B}. \end{aligned}$$

Evaluated at the no-discount equilibrium, this external strategic effect boils down to:

$$\left( s^{-2}(s) \left( \frac{s}{3} + \frac{s}{3} \right) - s^{-2} \left( \frac{2s}{3} \right) * 0 \right) \frac{-2s}{(6)^2} = -\frac{1}{27}$$

An increase in the discount will induce a decrease in the headline prices for the high quality products. As a result, some consumers will switch from  $B_X, B_Y$ ;  $B_X, A_Y$  and  $A_X, B_Y$  to  $A_X, A_Y$ , while others will switch from  $B_X, B_Y$  to  $B_X, A_Y$  and  $A_X, B_Y$ . This will result in a decrease in the quantity demanded from the low quality firms but also in a reduction in the

number of customers entitled to the discount. Despite this trade-off, the effect is negative and mitigates the previous (direct) one.

Finally, we present the (cross) effect in the partner firms' profits that is due to the change in the low quality products' prices in response to an increase in the discount. We call this the *internal strategic effect*.<sup>12</sup>

$$\begin{aligned} & \frac{\partial \Pi_{B_Y}}{\partial p_X} \frac{\partial p_X}{\partial \beta_B} + \frac{\partial \Pi_{B_X}}{\partial p_Y} \frac{\partial p_Y}{\partial \beta_B} \\ = & \left( p_Y \left( \frac{\partial Q_{A_X, B_Y}}{\partial p_X} + \frac{\partial Q_{B_X, B_Y}}{\partial p_X} \right) - \frac{\beta_B s}{2} \frac{\partial Q_{B_X, B_Y}}{\partial p_X} \right) \frac{\partial p_X}{\partial \beta_B} + \\ & + \left( p_X \left( \frac{\partial Q_{B_X, A_Y}}{\partial p_Y} + \frac{\partial Q_{B_X, B_Y}}{\partial p_Y} \right) - \frac{\beta_B s}{2} \frac{\partial Q_{B_X, B_Y}}{\partial p_Y} \right) \frac{\partial p_Y}{\partial \beta_B}. \end{aligned}$$

Evaluated at the no-discount equilibrium, this internal strategic effect boils down to

$$\left( \frac{s}{3} \left( \frac{s}{s^2} - \frac{s}{s^2} \right) + \frac{s}{3} \left( \frac{s}{s^2} - \frac{s}{s^2} \right) \right) \frac{8s}{36} = 0$$

An increase in the discount will lead to an increase in the headline prices for the low quality products. The low quality producer of product  $Y$ , firm  $B_Y$ , will be affected by the increase in the headline price of the low quality version of product  $X$  in the following way. Some consumers previously purchasing  $B_X, B_Y$  will switch to  $A_X, B_Y$ , leaving the total demand of the low quality producer of  $Y$  unchanged.<sup>13</sup> There is also a positive effect which is linked to the reduction of customers entitled to the discount. However, at the no-discount equilibrium, this last effect does not change the profit of the low quality producer of  $Y$ . Likewise for the low quality producer of  $X$ .

As note above, the two latter effects only exist when the discount is set in advance with respect to prices. For small discounts, the sum of the two effects is negative and, hence, the optimal discount is lower than in the case where price and discount decisions are simultaneous.<sup>14</sup>

The following proposition summarizes our results regarding unilateral bundling by the low quality firms:

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<sup>12</sup>This effect would not exist in the case of bundling by a multiproduct firm. In that case, both prices and the discount are set to maximize the same objective, the profit of the low quality firm.

<sup>13</sup>Also, some consumers of  $B_X, A_Y$  will switch to  $A_X, A_Y$  but this does not affect the total demand for  $B_Y$ .

<sup>14</sup>With simultaneous price and discount decisions one would have five independent entities making decisions: four firms setting prices and an "entity" that maximizes the low quality firms' joint profit with respect to the discount. In this case, the profits of the four firms would be lower than when the discount is set in advance. In our opinion this scenario is less plausible than the one analyzed in the text. We discuss it, however, in order to help isolate the impact of pre-commitment to a discount, which is one of the consequences of introducing discounts by a pair of single product independent firms.

**Proposition 1:** *If only the low quality firms offer the bundled discount, then, in equilibrium, and relative to the situation without bundling:*

- (i) *the headline prices for the low quality bundling firms will rise.*
- (ii) *the headline prices for the high quality firms will fall.*
- (iii) *the price of the bundle, net of the discount, will fall.*
- (iv) *both consumer surplus and welfare will fall.*
- (v) *the profit of the bundling firms will rise and the profit of the non-bundling firms will fall.* ■

It is straightforward to conclude that with unilateral bundling by the low quality producers, the average equilibrium price decreases. It should be remarked, however, that, in our setting, price is only a good indicator to evaluate the discount effects on consumer welfare if one restricts attention to those consumers who keep the same purchase option after the discount introduction (for the other consumers, quality will change as well). The following table illustrates the prices paid before and after the introduction of the discount for those consumers whose purchase options were not affected by the discount.<sup>15</sup>

Total price paid for product $X$ and $Y$				
	Buy $B_X, B_Y$	Buy $A_X, A_Y$	Buy $B_X, A_Y$ or $A_X, B_Y$	Average Price
Without discount	0.6666	1.3333	1.0000	1.1111
With discount	0.6001	1.3198	1.0302	1.0982
% increase in price	−9.98%	−1.01%	+3.02%	−1.16%
% consumers	11.11%	44.30%	33%	—

Consumers who either purchase the low quality bundle or the two high quality products will pay a lower total price while the remainder will pay a higher total price. Additionally, some consumers will make different options after the introduction of the bundled discount and, thus, will face a price/quality trade-off. Consumer surplus takes all the induced changes in prices and quality into account as well as consumer heterogeneity. As the previous proposition shows, it turns out that, overall, consumers will be worse off after the discount introduction.

<sup>15</sup>The equilibrium configuration of demand can be observed in Figure 2, where  $\theta_X^{*a} = \theta_Y^{*a} := 0.28953$  and  $\theta_X^{*b} = \theta_Y^{*b} := 0.43011$ .



Offering a bundled discount to those consumers opting for the acquisition of the low quality bundle will increase the individual profit of the firms offering the discount and, at the same time, will decrease the profit of the high quality producers (not involved in a similar discounting scheme). In addition, aggregate industry profit will fall and, given that consumer surplus will also fall, total welfare will be reduced.

### 3.3 Scenario 2: Unilateral bundling by the high quality firms

We now analyze the scenario in which  $\beta_A > 0$  while  $\beta_B = 0$ . In this scenario, equilibrium prices result from the maximization of following objective functions,

$$\begin{aligned}\Pi_{A_X} &= P_X (Q_{A_X, B_Y} + Q_{A_X, A_Y}) - \frac{\beta_A s}{2} Q_{A_X, A_Y} \\ \Pi_{A_Y} &= P_Y (Q_{B_X, A_Y} + Q_{A_X, A_Y}) - \frac{\beta_A s}{2} Q_{A_X, A_Y} \\ \Pi_{B_X} &= p_X (Q_{B_X, A_Y} + Q_{B_X, B_Y}) \\ \Pi_{B_Y} &= p_Y (Q_{A_X, B_Y} + Q_{B_X, B_Y})\end{aligned}$$

yielding,<sup>16</sup>

$$P_X = P_Y = \frac{(\beta_A + 2)^3 s}{2(5\beta_A + 6)} \text{ and } p_X = p_Y = \frac{(2\beta_A + \beta_A^3 + 4)s}{2(5\beta_A + 6)}.$$

Analyzing the equilibrium prices, we conclude that, for all admissible discounts:

- (i) the high quality headline price increases with  $\beta_A$ . This occurs for the same reason that the low quality headline price increased with  $\beta_B$  in the previous subsection.
- (ii) the high quality bundle price, net of the corresponding discount, is U-shaped (convex) in  $\beta_A$ , which contrasts with the case of unilateral bundling by the low quality firms. Comparison of Figures 3 and 4 illustrates that the effect of an increase in the discount on a given firm's demand is larger in the case of a discount by the high quality producers. Hence, the headline price of a high quality product increases more and may more than compensate for the increase in the discount.
- (iii) the low quality headline price decreases in  $\beta_A$ . This may be counterintuitive, especially when the high quality headline price and the high quality bundle's "net" price are both increasing with the discount level. The intuition is, however, simple. When both of these prices are increasing, the headline high quality price increases at a higher rate

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<sup>16</sup> As in scenario 1, we have that, in equilibrium,  $\theta_X^{*b} = P_X/s - p_X/s \geq \frac{1}{3}$  and that  $\theta_X^{*a} = P_X/s - p_X/s - \beta_A \leq \frac{1}{3}$ . Furthermore,  $\theta_X^{*b} - \frac{1}{3} > \frac{1}{3} - \theta_X^{*a}$ . Hence, Figure 2 also applies qualitatively to this case.

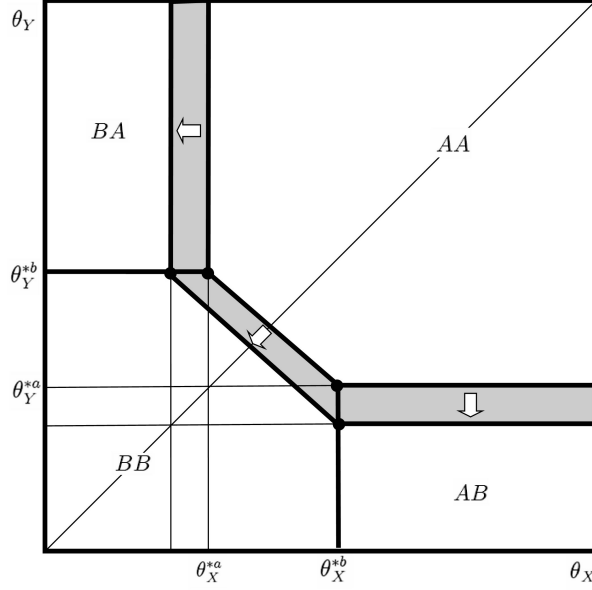


Figure 4: The direct effect on demand of increasing the bundled discount by the high quality producers.

than the bundle “net” price. Hence, some consumers that were previously purchasing a mixed quality pair (i.e.,  $A_X, B_Y$  or  $B_X, A_Y$ ) will switch to the high quality bundle while others will switch to the low quality pair. The net effect on the demand for the low quality firms may thus be negative when the consumers switching from  $A_X, B_Y$  or  $B_X, A_Y$  to  $A_X, A_Y$  outnumber those switching to  $B_X, B_Y$ . Direct inspection of Figure 1 suggests that, indeed, for most consumers initially buying  $A_X, B_Y$  or  $B_X, A_Y$ , the consumers purchasing the  $A_X, A_Y$  bundle are “closer” in terms of preferences for quality than those purchasing the  $B_X, B_Y$  pair. The same reasoning applies to the case when the price of the high quality bundle is decreasing, wherein the switching from  $A_X, B_Y$  or  $B_X, A_Y$  to  $A_X, A_Y$  is reinforced.

(iv) The sum of the headlines prices of two goods of different qualities increases with  $\beta_A$ .

With respect to quantities, the impact of the discount on equilibrium quantities is analogous to the one regarding the case of bundling by the low quality firms.

Now, the optimal discount is obtained by maximizing joint profit,

$$\Pi_A(\beta_A) = \frac{(16\beta_A^2 - 64\beta_A + 4\beta_A^3 - 12\beta_A^4 + \beta_A^5 - 64)(\beta_A + 1)}{-2(5\beta_A + 6)^2},$$

with respect to  $\beta_A$ , subject to  $\beta_A \in [0, 5/9]$ , which yields the corner solution  $\beta_A^* = 5/9$ .<sup>17</sup>

<sup>17</sup>For all  $\beta_A \in [0, 5/9]$ ,  $\theta_X^{*b}, \theta_Y^{*b}, \theta_X^*, \theta_Y^*$  belong to the  $[0, 1]$  interval

The trade-off faced by the partner high-quality producers when deciding on the optimal level for the discount is qualitatively similar to the one discussed in the previous section for the low quality partner firms. There are, however, the following differences when the effects are evaluated at the no-discount equilibrium:

- The initial number of consumers entitled to the discount is four times larger, which results from the fact that the high quality firms have a larger market share;
- Each consumer that switches to the high quality components is charged a price which is twice as large as the price paid by consumers who switch to low quality components. The unit cost, however, is the same for both types of quality, and equal to 0.
- The effects of the discount on the corresponding quantities are larger (again, twice as large) - see Figures 3 and 4.
- The effect of the discount on the equilibrium headline prices of the bundling partners (positive effect) and of the non bundling partners (negative effect) is larger in the case of bundling by the high quality firms (again, twice as large).

As a result, the overall effect (and also the direct and the strategic effects when taken separately) turns out to be four times larger in the scenario under analysis than in the case of bundling by the low quality firms. This helps explaining why the magnitude of the equilibrium discount is larger in the case of unilateral bundling by the high quality firms.<sup>18</sup>

The results are summarized in the following proposition.

**Proposition 2:** *If only the high quality firms offer a bundled discount, then, in equilibrium and relative to the situation without bundling:*

- (i) *the headline prices for the high quality bundling firms will rise.*
- (ii) *the headline prices for the low quality firms will fall.*
- (iii) *the price of the bundle, net of the discount, will rise.*
- (iv) *both consumer surplus and welfare will fall.*
- (v) *the profit of the bundling firms will rise and the profit of the non-bundling firms will fall.* ■

Contrary to what happened for the case discussed in the previous section, in the current scenario, the introduction of a bundled discount (by the high quality producers) leads to

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<sup>18</sup>The equilibrium configuration of demand can be observed in Figure 2, where  $\theta_X^{*a} = \theta_Y^{*a} := 67/711$  and  $\theta_X^{*b} = \theta_Y^{*b} := 154/237$ .

an increase in the average price. The prices paid before and after the introduction of the discount by those consumers who make the same purchasing options after the discount becomes available are summarized in the following table.

Total price paid by product $X$ and $Y$				
	Buy $B_X, B_Y$	Buy $A_X, A_Y$	Buy $B_X, A_Y$ or $A_X, B_Y$	Average Price
Without discount	0.6666	1.3333	1.0000	1.1111
With discount	0.6018	1.3458	1.2516	1.1403
% increase in price	-9.72%	+0.94%	+25.16%	+2.63%
% consumers	11.11%	44.15%	6.60%	—

Hence, within the group of consumers keeping their purchase decision, the only consumers who end up benefiting from the discount introduction are those who *do not* purchase any product from the bundling firms. In addition, and in line with the scenario studied in the previous section, overall, consumer surplus will be negatively affected by the introduction of a bundled discount by the high quality producers.

The bundling firms will benefit from an increase in profits whereby their competitors will see profits declining. In addition, and despite the fact that aggregate profits increase, total welfare decreases upon the introduction of the discount.

### 3.4 Scenario 3: Bilateral bundling

We now analyze the case in which  $\beta_A$  and  $\beta_B$  are both positive. In this case, firms maximize the following objective functions:

$$\begin{aligned}
\Pi_{A_X} &= P_X (Q_{A_X, B_Y} + Q_{A_X, A_Y}) - \frac{\beta_A s}{2} Q_{A_X, A_Y} \\
\Pi_{A_Y} &= P_Y (Q_{B_X, A_Y} + Q_{A_X, A_Y}) - \frac{\beta_A s}{2} Q_{A_X, A_Y} \\
\Pi_{B_X} &= p_X (Q_{B_X, A_Y} + Q_{B_X, B_Y}) - \frac{\beta_B s}{2} Q_{B_X, B_Y} \\
\Pi_{B_Y} &= p_Y (Q_{A_X, B_Y} + Q_{B_X, B_Y}) - \frac{\beta_B s}{2} Q_{B_X, B_Y}
\end{aligned}$$

yielding, at a symmetric equilibrium:

$$P_X = P_Y = \left( \frac{1}{2}\beta_A + \frac{1}{3}\sqrt{6} \right) s \text{ and } p_X = p_Y = \left( \frac{1}{2}\beta_B + \frac{1}{6}\sqrt{6} \right) s$$

Analyzing these equilibrium prices, we conclude that, for all admissible discounts:

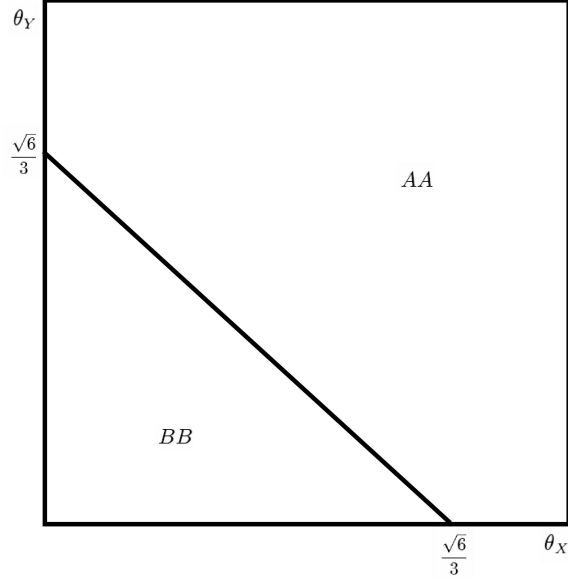


Figure 5: Consumers' choices with bilateral bundled discounts.

- (i) the high quality and the low quality headline prices increase with the respective discount.
- (ii) the high and low quality bundle prices are *independent* of the discount, with  $p_X + p_Y - \beta_B s = \frac{\sqrt{6}}{3}s$  and  $P_X + P_Y - \beta_A s = \frac{2\sqrt{6}}{3}s$ .
- (iii) the sum of the headlines prices of the two different quality products increases with the discounts.
- (iv) consumers purchase either the high quality pair of products or the low quality pair of products.

The following proposition presents the equilibrium under bilateral bundling.

**Proposition 3:** *If both the low and high quality firms can offer the bundled discount, then, in equilibrium and relative to the situation without bundling:*

- (i) *the headline prices for the high quality bundling firms will rise.*
- (ii) *the headline prices for the low quality firms will rise.*
- (iii) *the price of the high quality bundle, net of the discount, will rise .*
- (iv) *the price of the low quality bundle, net of the discount, will rise.*
- (v) *both consumer surplus and welfare will fall.*
- (vi) *the individual profit of all firms will rise.* ■

Figure 5 illustrates the outcome under bilateral bundling. Interestingly, the competition

for customers drives the discount levels up to a point in which every consumer decides to buy one of the available bundles. After the introduction of the two discounts, the prices of the separate products of different quality become unattractive and, as a result, no consumer decides, in equilibrium, to mix-and-match (i.e. to purchase products of different quality).

Note that the individual equilibrium profits are independent of the discount level. This is because the discount cancels out as equilibrium prices equal the discount plus a constant. This results from the fact that each pair of partner firms has three instruments, the two headline prices and the discount, to determine how much each partner firm will receive from selling the bundle. The headline prices serve no additional purpose because there are no consumers that exclusively pay these prices.

With bilateral bundled discounts, average price increases. The following table summarizes the prices paid by those consumers who keep the same purchasing decisions before and after the introduction of both discounts.

Total price paid by product $X$ and $Y$				
	Buy $B_X, B_Y$	Buy $A_X, A_Y$	Buy $B_X, A_Y$ or $A_X, B_Y$	Average Price
Without discount	0.6666	1.3333	1.0000	1.1111
With discount	0.8165	1.6330	—	1.3608
% increase in price	+22.49%	+22.48%	—	+22.47%
% consumers	11.11%	43.32%	—	—

Note that consumers who formerly purchased goods of the same quality will be worse off with the introduction of the two bundled discounts. In addition, in this bilateral bundling scenario, both pairs of firms will be better off. However, total welfare will decrease.

## 4 The Discounting Game

In this section, we discuss the simultaneous decisions by the high quality and low quality firms on whether to participate in a bundled discount scheme.

**Proposition 4:** *In equilibrium, both the high and low quality pairs of firms will offer bundled discounts.* ■

The following table, which results from Propositions 1 to 3, presents the absolute and relative increases in profit in each of the three scenarios considered. In case of unilateral

bundling, the high quality pair has a higher incentive, both in absolute or in relative terms, to offer the bundled discount. Curiously, however, in case of bilateral bundling, the relative increase in profit is almost the same for the high and low quality firms.

	Low quality firms		High quality firms	
	$\Delta\Pi_{B_i}$	$\Delta\Pi_{B_i}/\Pi_{B_i}$	$\Delta\Pi_{A_i}$	$\Delta\Pi_{A_i}/\Pi_{A_i}$
Scenario 1	+2.54	+2.29%	−9.00	−2.03%
Scenario 2	−20.10	−18.55%	+35.16	+7.91%
Scenario 3	+24.97	+22.47%	+99.89	+22.48%

Clearly, offering a bundled discount is a dominant strategy both for the high quality firms and for the low quality firms. Hence, the Nash equilibrium of this game corresponds to the scenario of bilateral bundling, as in Gans and King (2006), where there are no consumers buying unpaired products: all consumers find it optimal to purchase a bundle and benefit from the corresponding discount. However, and in contrast with Matutes and Regibeau (1992), Gans and King (2006), Thanassoulis (2007) or Maruyama and Minamikawa (2009), in this bilateral bundling scenario all firms earn *higher profits* than in the *status quo* no-discounting situation. So, in our setting, firms do not find themselves in a prisoners' dilemma: all firms have very strong incentives to participate in bilateral bundling.

Nevertheless, and as shown above, this scenario is the one leading to the most adverse consequences both in terms of consumer welfare and in terms of social welfare. This then suggests that competition authorities should be more vigilant with regards to bundling discounting by independent producers of vertically differentiated goods.

We conclude this section by discussing the major difference between our results and those of Gans and King (2006). In their model, any pair of firms also has an incentive to offer bundled discounts, but in the bilateral bundling scenario, the individual equilibrium profits remain the same when compared to the benchmark case where prices are set independently.

The reason why in their setting firms end up having the same profit is that the equilibrium prices, net of the discount, are exactly the same as in the no-discounting benchmark. Although the distribution of consumers is different, their model's symmetry assumptions imply that firms have exactly the same demand both before and after the discounts are introduced. With vertical differentiation, however, there is no such symmetry.

To understand the differences, consider those consumers who, in the absence of bundled discounts, choose the low quality version of product  $X$  and the high quality version of product  $Y$ .<sup>19</sup> Assume now that the headline prices increase significantly, inducing consumers

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<sup>19</sup>These consumers are those with  $(\theta_X, \theta_Y) \in [0, \frac{1}{3}] \times [\frac{1}{3}, 1]$ .

not to buy unbundled products but that discounts are such that the prices net of the discount remain unchanged. When deciding between the high quality and the low quality bundles, the above mentioned consumers will select the high quality bundle if and only if  $\theta_X + \theta_Y > \frac{2}{3}$ .<sup>20</sup> Given our assumption of a uniform distribution, the number of consumers that will change to the high quality bundle is three times larger than the number of consumers that will change to the low quality bundle. This means that, at the original (net) prices and after this redistribution of consumers, the number of inframarginal consumers is very large for the high quality firms. Hence, the high quality firms have a strong incentive to hike their prices net of the discount above the initial equilibrium level and, in equilibrium, are followed by the low quality firms.

## 5 Extension: positive correlation

So far, we have assumed that consumers' valuation for quality is uncorrelated across products. In this section, we will consider bilateral bundling in the extreme case of perfectly positively correlated valuations. As the following analysis demonstrates, the main qualitative results obtained in our baseline model extend to this case.

Assume for all consumers that  $\theta_X = \theta_Y = \theta$ , with  $\theta$  uniformly distributed in the interval  $[0, 1]$ . Assume also that  $\theta_Y^{*a} < \theta_X^{*b}$  and  $\theta_Y^{*b} > \theta_X^{*a}$ .<sup>21</sup> Then, no consumer will purchase products of different quality.<sup>22</sup> Hence,

$$\begin{aligned} Q_{A_X, B_Y} &= Q_{B_X, A_Y} = 0 \\ Q_{A_X, A_Y} &= \left( 1 - \frac{P_X + P_Y - p_X - p_Y - \gamma_A + \gamma_B}{2s} \right) \\ Q_{B_X, B_Y} &= \frac{P_X + P_Y - p_X - p_Y - \gamma_A + \gamma_B}{2s} \end{aligned}$$

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<sup>20</sup>Moving from  $B_X A_Y$  to  $A_X A_Y$  represents an additional quality valuation of  $\theta_X s$  and an increase in price of  $\frac{1}{3}s$ . Moving from  $B_X A_Y$  to  $B_X B_Y$  represents a decrease in quality valued at  $-\theta_Y s$  but also a decrease in price of  $\frac{1}{3}s$ . Hence, consumers such that  $\theta_X s - \frac{1}{3}s > -\theta_Y s + \frac{1}{3}s \Leftrightarrow \theta_X + \theta_Y > \frac{2}{3}$  will change to the high quality bundle and the remainder will change to the low quality bundle.

<sup>21</sup>Note that these conditions cannot hold simultaneously if  $\gamma_A = \gamma_B = 0$ . Therefore, the no discount case cannot be obtained as a special case of what follows. Appendix C shows that it is impossible to have  $\theta_Y^{*a} > \theta_X^{*b}$  or  $\theta_Y^{*b} < \theta_X^{*a}$ .

<sup>22</sup>This follows from the fact that there is no value for  $\theta$  such that:

$\theta(s_A + s_B) - P_X - p_Y > \theta(s_B + s_B) - p_X - p_Y + \gamma_B$  and  $\theta(s_A + s_B) - P_X - p_Y > \theta(s_A + s_A) - P_X - p_Y + \gamma_A$ . This can be simplified to  $\theta > \theta_X^{*b} := \frac{P_X - p_X + \gamma_B}{s}$  and  $\theta < \theta_Y^{*a} := \frac{P_Y - p_Y - \gamma_A}{s}$ , which is impossible since  $\theta_Y^{*a} < \theta_X^{*b}$  (and likewise for the case of high quality Y and low quality X).



Equilibrium prices that result from the maximization of the firms' objective functions are

$$\begin{aligned} P_X(\beta_A, \alpha_A) &= P_Y(\beta_A, 1 - \alpha_A) = \frac{6}{5}s + \alpha_A\beta_A s \\ p_X(\beta_B, \alpha_B) &= p_Y(\beta_B, 1 - \alpha_B) = \frac{4}{5}s + \alpha_B\beta_B s \end{aligned}$$

and the corresponding quantities are  $Q_{A_X, A_Y} = 3/5$  and  $Q_{B_X, B_Y} = 2/5$ .

The equilibrium profits are:

$$\Pi_{A_X} = \Pi_{A_Y} = \frac{18}{25}s \text{ and } \Pi_{B_X} = \Pi_{B_Y} = \frac{8}{25}s.$$

It should be noted that the conditions on the  $\theta$ 's are verified ex-post, for any pair of positive discounts.<sup>23</sup> The following results still hold, when compared to the case of no discount:

- (i) the price of the high quality bundle, net of the discount, will rise.
- (ii) the price of the low quality bundle, net of the discount, will rise.
- (iii) consumer surplus and welfare will fall.<sup>24</sup>
- (iv) there is no prisoner's dilemma.

## 6 Conclusion

Bundled discounts provide purchasers the opportunity to pay less for a bundle of products than if they purchased each item in the package separately at the corresponding headline price. Despite the fact that this business practice is ubiquitous in today's society, economic theory has devoted very scarce attention to this issue until recently.

The present paper studies the consequences of bundled discounts in an oligopoly setting where pairs of firms sell vertically differentiated and otherwise unrelated products. More specifically, we investigate the effects induced by the introduction of bundled discounts under three different scenarios: (i) unilateral bundled discount by the low quality firms; (ii) unilateral bundled discount by the high quality firms; and (iii) bilateral bundling.

Some interesting results are obtained regarding the competitive effects of bundled discounts, allowing to shed some light on the understanding of the potential antitrust risks associated with this particular type of discount arrangements. First, whenever bundled

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<sup>23</sup>The conditions, in equilibrium, are:  $\theta_Y^{*b} > \theta_X^{*a} \Leftrightarrow \beta_A(1 - \alpha_A) + \alpha_B\beta_B > 0$  and  $\theta_Y^{*a} < \theta_X^{*b} \Leftrightarrow \beta_B(\alpha_B - 1) - \alpha_A\beta_A < 0$ .

<sup>24</sup>The expressions regarding consumer surplus and welfare are presented in Appendix C.

discounts are offered by (one or two pairs of) firms, then, relative to the no-discounting benchmark case, the headline prices of the bundling firms always increase whereas the headline prices of the firms not involved in discounting (if any) decrease. Second, in none of the three studied scenarios should bundled discounts be free of antitrust concerns: bundled discounts always induce a decrease both in consumer surplus and in total welfare. Third, when firms make simultaneous decisions regarding their eventual participation in a bundled discount scheme, it turns out that offering a bundled discount is a dominant strategy both for the high quality firms and for the low quality firms. In addition, in this bilateral bundling (Nash) equilibrium, and relative to the no-discounting benchmark case: (i) both the price of the high quality bundle and that of the low quality bundle, net of the corresponding discount, rise; and (ii) all firms earn higher profits.

These results differ from the previous literature on bundling discounts by independent firms in two important ways: the effect on (consumers and total) welfare and the inexistence of a prisoner's dilemma.

As Armstrong (2010) highlights, “when firms offer independent products (i.e. the purchase of one product has no impact on a consumer's willingness to pay for a second product), Pareto improvements are possible if firms coordinate their pricing policies. This is achieved by means of bundling so that a consumer is offered a discount if she buys both products. Such bundling discounts, if they can be implemented by separate firms, can improve both profits and consumer surplus.” Our contribution to this discussion is the identification of a context – bundling by vertically differentiated competitors – in which, while profitable, bundling discounts by independent firms harm consumers. Additionally, the typical prisoner's dilemma identified in the literature is absent in this case, meaning that the practice of mixed bundling is profitable for all firms. The equilibrium with bilateral bundled discounts corresponds to the worst scenario in terms of both consumer surplus and social welfare. This then suggests that competition authorities should scrutinize in detail the use of bundled discounts in industries where suppliers offer vertically differentiated products.

## Appendix A - Proofs

**Proof of Lemma 1:** Assume initially that there are no discounts. Consumers purchase  $i = X, Y$  from  $A_i$  if and only if

$$V + \theta_i s_A - P_i > V + \theta_i s_B - p_i \Leftrightarrow \theta_i > \theta_i^* := \frac{P_i - p_i}{s}$$

Assume now that the high quality firms introduce a discount  $\gamma_A$ . Then:

If  $\theta_X > \theta_X^* = \frac{P_X - p_X}{s}$  and  $\theta_Y > \theta_Y^* = \frac{P_Y - p_Y}{s}$  will still purchase from  $A_X, A_Y$ .

If  $\theta_X > \theta_X^* = \frac{P_X - p_X}{s}$  and  $\theta_Y < \theta_Y^* = \frac{P_Y - p_Y}{s}$  will purchase from  $A_X, A_Y$  if

$$\theta_X s_A - P_X + \theta_Y s_A - P_Y + \gamma_A > \theta_X s_A - P_X + \theta_Y s_B - p_Y \Leftrightarrow \theta_Y > \theta_Y^{*a} := \frac{P_Y - p_Y - \gamma_A}{s}.$$

If  $\theta_X < \theta_X^* = \frac{P_X - p_X}{s}$  and  $\theta_Y < \theta_Y^* = \frac{P_Y - p_Y}{s}$  will purchase from  $A_X, A_Y$  if

$$\theta_X s_A - P_X + \theta_Y s_A - P_Y + \gamma_A > \theta_X s_B - p_X + \theta_Y s_B - p_Y \Leftrightarrow \theta_X > \frac{P_X - p_X + P_Y - p_Y - \gamma_A}{s} - \theta_Y.$$

If  $\theta_X < \theta_X^* = \frac{P_X - p_X}{s}$  and  $\theta_Y > \theta_Y^* = \frac{P_Y - p_Y}{s}$  will purchase from  $A_X, A_Y$  if

$$\theta_X s_A - P_X + \theta_Y s_A - P_Y + \gamma_A > \theta_X s_B - p_X + \theta_Y s_A - P_Y \Leftrightarrow \theta_X > \theta_X^{*a} = \frac{P_X - p_X - \gamma_A}{s}.$$

Assume now that the low quality firms also introduce a discount,  $\gamma_B$ . Then:

If  $\theta_X < \theta_X^{*a}$  and  $\theta_Y > \theta_Y^*$  will purchase from  $B_X, B_Y$  if

$$\theta_X s_B - p_X + \theta_Y s_B - p_Y + \gamma_B > \theta_X s_B - p_X + \theta_Y s_A - P_Y \Leftrightarrow \theta_Y < \theta_Y^{*b} := \frac{P_Y - p_Y + \gamma_B}{s}.$$

If  $\theta_X > \theta_X^*$  and  $\theta_Y < \theta_Y^{*a}$  will purchase from  $B_X, B_Y$  if

$$\theta_X s_B - p_X + \theta_Y s_B - p_Y + \gamma_B > \theta_X s_A - P_X + \theta_Y s_B - p_Y \Leftrightarrow \theta_X < \theta_X^{*b} := \frac{P_X - p_X + \gamma_B}{s}.$$

If  $\theta_X > \theta_X^{*a}$  and  $\theta_Y > \theta_Y^{*a}$  and  $\theta_X > \frac{P_X - p_X + P_Y - p_Y - \gamma_A}{s} - \theta_Y$  will purchase from  $B_X, B_Y$  if

$$\theta_X s_B - p_X + \theta_Y s_B - p_Y + \gamma_B > \theta_X s_A - P_X + \theta_Y s_A - P_Y + \gamma_A.$$

which is equivalent to

$$\theta_X < \frac{P_X - p_X + P_Y - p_Y + \gamma_B - \gamma_A}{s} - \theta_Y = \theta_X^{*a} + \theta_Y^{*b} - \theta_Y.$$

The demand functions result from calculating the relevant areas in the  $(\theta_X, \theta_Y)$ -space with  $(\theta_X, \theta_Y) \in [0, 1]^2$ . ■

**Proof of Proposition 1:** Parts (i) to (iii) follow directly from the comparison between the equilibrium prices at the optimal discount level and their benchmark counterparts. With  $\beta_B^* = 0.14058$  we have:

$$P_X = P_Y = 0.65988s \text{ and } p_X = p_Y = 0.37035s,$$

$$Q_{A_X, B_Y} = Q_{B_X, A_Y} = 0.165; Q_{A_X, A_Y} = 0.49488 \text{ and } Q_{B_X, B_Y} = 0.17512.$$

As for (iv), aggregate consumer surplus,  $CS_1$ , is given by the following expression

$$\begin{aligned} & \int_{\theta_Y^{*b}}^1 \int_0^{\theta_X^{*a}} (\theta_X s_B + \theta_Y s_A - p_X - P_Y) d\theta_X d\theta_Y + \int_0^{\theta_Y^{*a}} \int_{\theta_X^{*b}}^1 (\theta_X s_A + \theta_Y s_B - P_X - p_Y) d\theta_X d\theta_Y + \\ & \int_0^{\theta_Y^{*a}} \int_0^{\theta_X^{*b}} (s_B (\theta_X + \theta_Y) - p_X - p_Y) d\theta_X d\theta_Y + \int_{\theta_Y^{*a}}^{\theta_Y^{*b}} \int_0^{\theta_X^{*b} + \theta_Y^{*a} - \theta_Y} (s_B (\theta_X + \theta_Y) - p_X - p_Y) d\theta_X d\theta_Y + \\ & \int_{\theta_Y^{*b}}^1 \int_{\theta_X^{*a}}^1 (s_A (\theta_X + \theta_Y) - P_X - P_Y) d\theta_X d\theta_Y + \int_{\theta_Y^{*a}}^{\theta_Y^{*b}} \int_{\theta_X^{*b} + \theta_Y^{*a} - \theta_Y}^1 (s_A (\theta_X + \theta_Y) - P_X - P_Y) d\theta_X d\theta_Y, \end{aligned}$$

from where we obtain<sup>25</sup>

$$CS_1 = \frac{6.7578 - 1.3193 \frac{s_A}{s_B}}{5.4385}.$$

So,

$$CS_1 - CS_0 = \frac{6.7578 - 1.3193 \frac{s_A}{s_B}}{5.4385} - \left(11 - 2 \frac{s_A}{s_B}\right) / 9 = \left(-\frac{9967}{489465}\right) \left(\frac{s_A}{s_B} - 1\right) < 0.$$

Aggregate welfare is given by

$$W_1 = \frac{8.5678 \frac{s_A}{s_B} + 1.1658}{9.7335}.$$

This is obtained merely by removing the prices in the expression for consumer surplus. So,

(iv) follows from:

$$W_1 - W_0 = \frac{8.5678 \frac{s_A}{s_B} + 1.1658}{9.7335} - (8 \frac{s_A}{s_B} + 1) / 9 = \left(-\frac{842}{97335}\right) \left(\frac{s_A}{s_B} - 1\right) < 0.$$

Finally, simple algebra shows that equilibrium profits are given by:

$$\Pi_{A_X} = \Pi_{A_Y} = 0.43544s \text{ and } \Pi_{B_X} = \Pi_{B_Y} = 0.11365s.$$

This completes the proof of (v). ■

**Proof of Proposition 2:** Parts (i) to (iii) follow directly from the comparison between the equilibrium prices at the optimal discount level and their benchmark counterparts. At the optimal discount level, i.e. when  $\beta_A^* = \frac{5}{9}$ , we have that:

$$\begin{aligned} P_X &= P_Y = \frac{12167}{12798}s \text{ and } p_X = p_Y = \frac{3851}{12798}s, \\ Q_{A_X, B_Y} &= Q_{B_X, A_Y} = \frac{67 \times 83}{3^3 79^2}; Q_{A_X, A_Y} = \frac{673447}{3^4 79^2} \text{ and } Q_{B_X, B_Y} = \frac{439 \times 617}{3^4 79^2}. \end{aligned}$$

As for (iv), aggregate consumer surplus,  $CS_2$ , is given by the same expression as in Proposition 1 with the obvious substitution of the corresponding equilibrium prices and  $\theta_Y^{*a}, \theta_X^{*a}, \theta_Y^{*b}, \theta_X^{*b}$ 's:

$$CS_2 = \frac{22444673 - 8795606 \frac{s_A}{s_B}}{13649067}.$$

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<sup>25</sup>Note that we measure consumer surplus and welfare in units of  $s_B$ .

So,

$$CS_2 - CS_0 = \frac{22\,444\,673 - 8795\,606 \frac{s_A}{s_B}}{13\,649\,067} - \left(11 - 2 \frac{s_A}{s_B}\right)/9 = \left(-\frac{5762\,480}{13\,649\,067}\right) \left(\frac{s_A}{s_B} - 1\right) < 0.$$

Aggregate welfare is given by

$$W_2 = \frac{11\,819\,119 \frac{s_A}{s_B} + 1829\,948}{13\,649\,067}$$

So,

$$W_2 - W_0 = \frac{11\,819\,119 \frac{s_A}{s_B} + 1829\,948}{13\,649\,067} - (8 \frac{s_A}{s_B} + 1)/9 = \left(-\frac{313\,385}{13\,649\,067}\right) \left(\frac{s_A}{s_B} - 1\right) < 0$$

Finally, some algebra shows that the equilibrium profits are given by:

$$\begin{aligned} \Pi_{A_X} &= \Pi_{A_Y} = \frac{39\,276\,517}{81\,894\,402} s = 0.479\,60s, \\ \Pi_{B_X} &= \Pi_{B_Y} = \frac{14\,830\,201}{163\,788\,804} s = 9.054\,5 \times 10^{-2} s. \end{aligned}$$

This completes the proof of (v). ■

**Proof of Proposition 3:** We start by discussing the high quality firms best response function in terms of discounts. Let  $\tilde{\beta}_A(\beta_B) := \frac{1}{4} \left( \sqrt{10\beta_B + \beta_B^2 + 17} - 5\beta_B - 1 \right)$ . Assume initially that  $\beta_A < \min \left\{ \tilde{\beta}_A(\beta_B), \frac{5}{9} \right\}$  for all  $\beta_B$ . Given these discounts, we start by showing that equilibrium prices are such that there are *four distinct groups of consumers* (two groups buying only products of the same quality,  $A_X, A_Y$  or  $B_X, B_Y$ , and two other groups buying products of different quality,  $A_X, B_Y$  or  $B_X, A_Y$ ). The corresponding demand functions are as given by case (i) in Lemma 1. The prices that verify the first-order conditions are then:

$$\begin{aligned} P_X &= P_Y = \frac{(12\beta_A + 6\beta_B + 5\beta_A\beta_B + 6\beta_A^2 + \beta_A^3 + \beta_B^3 + 4\beta_A\beta_B^2 + 4\beta_A^2\beta_B + 8)}{2(5\beta_A + 5\beta_B + 6)} s \\ p_X &= p_Y = \frac{(2\beta_A + 6\beta_B + 5\beta_A\beta_B + \beta_A^3 + 6\beta_B^2 + \beta_B^3 + 4\beta_A\beta_B^2 + 4\beta_A^2\beta_B + 4)}{2(5\beta_A + 5\beta_B + 6)} s \end{aligned}$$

Aggregate profits are easily obtained by substituting the equilibrium prices and quantities in  $\Pi_A^4 = \Pi_{A_X}^4 + \Pi_{A_Y}^4$  and  $\Pi_B^4 = \Pi_{B_X}^4 + \Pi_{B_Y}^4$ , where the superscript 4 denotes the existence of four different groups of consumers.

At these prices, it should also be noted that

$$\theta_X^{*b} := \theta_Y^{*b} := \frac{5\beta_A + 6\beta_B + 5\beta_A\beta_B + 3\beta_A^2 + 2\beta_B^2 + 2}{5\beta_A + 5\beta_B + 6}$$

is always in  $[0, 1]$  and that

$$\theta_X^{*a} = \theta_Y^{*a} := \frac{2 - 5\beta_A\beta_B - 2\beta_A^2 - 3\beta_B^2 - \beta_A}{5\beta_A + 5\beta_B + 6}$$

is always lower than 1. Furthermore, when  $\beta_A < \tilde{\beta}_A(\beta_B)$  we have that both  $\theta_X^{*a}$  and  $\theta_Y^{*a}$  are positive. As  $\frac{\partial^2 \Pi_A}{\partial \beta_A \partial \beta_B} > 0$ , and using the result in Proposition 2, the high quality firms will choose the highest admissible discount,  $\beta_A^* = \min \left\{ \tilde{\beta}_A(\beta_B), \frac{5}{9} \right\}$ , for any  $\beta_B \geq 0$ .

Assume now that  $\beta_A \in \left( \tilde{\beta}_A(\beta_B), \frac{5}{9} \right)$ . This is only possible for  $\beta_B > 0.23708$  and, when this is the case, there will be no equilibrium with *four distinct groups of consumers*. In fact, with large discounts by both firms, there will be only *two groups of consumers*: consumers purchasing the high quality pair of products,  $A_X, A_Y$ , and consumers purchasing the low quality pair of products,  $B_X, B_Y$ . The corresponding demand functions are as given by case (ii) in Lemma 1. At a symmetric equilibrium, we obtain:

$$\begin{aligned} P_X &= P_Y = \left( \frac{1}{2}\beta_A + \frac{1}{3}\sqrt{6} \right) s \text{ and } p_X = p_Y = \left( \frac{1}{2}\beta_B + \frac{1}{6}\sqrt{6} \right) s \\ Q_{A_X, A_Y} &= \frac{2}{3}; Q_{B_X, B_Y} = \frac{1}{3} \text{ and } Q_{A_X, B_Y} = Q_{B_X, A_Y} = 0, \end{aligned}$$

which verify the second order conditions.<sup>26</sup>

Simple algebra now shows that equilibrium profits with two groups of consumers are:

$$\Pi_{A_X}^2 = \Pi_{A_Y}^2 = \frac{2\sqrt{6}}{9}s = 0.54433s \text{ and } \Pi_{B_X}^2 = \Pi_{B_Y}^2 = \frac{\sqrt{6}}{18}s = 0.13608s,$$

which are independent of both discount levels.

Some algebra shows that  $\Pi_{A_X}^2 > \Pi_{A_X}^4|_{\beta_A=\frac{5}{9}} \geq \Pi_{A_X}^4|_{\beta_A=\tilde{\beta}_A(\beta_B)}$  for all  $\beta_B > 0.23708$ . This means that if  $\beta_B > 0.23708$  the high quality firms will prefer to set any  $\beta_A^* \in \left( \tilde{\beta}_A(\beta_B), \frac{5}{9} \right]$ . In this case, we assume that the firms will opt for the highest discount in the interval. If instead  $\beta_B < 0.23708$  the high quality firms will prefer to set  $\beta_A^* = 5/9$ . So, for any  $\beta_B$  the high quality firm's best response is always  $\beta_A^* = 5/9$ .

We now turn to the low quality firms' best response to  $\beta_A^* = 5/9$ .

If  $\beta_B \in [0, 0.23708]$ , the low quality firms aggregate profit

$$\Pi_B^4|_{\beta_A=\frac{5}{9}} = \frac{9\beta_B(9\beta_B(81\beta_B(\beta_B(9\beta_B(9\beta_B-74)-2840)-1007)+25730)-2029393)-14830201}{-13122(45\beta_B+79)^2}s$$

is maximized at  $\beta_B = 0.12892$ , with  $\Pi_B^4|_{\beta_A=\frac{5}{9}, \beta_B=0.12892} = 0.18195$ . However, if  $\beta_B \in (0.23708, \frac{5}{9}]$ , the low quality firms will obtain a higher profit,  $\Pi_{B_X}^2 > 0.18195/2$ , regardless of the level of discount set by both pairs of firms. Assuming that the firms will opt for the highest discount, the equilibrium is  $\beta_A^* = \beta_B^* = 5/9$ . Although we obtain a corner solution recall that for any  $\beta_A \in \left( \tilde{\beta}_A(\beta_B), \frac{5}{9} \right]$ , the equilibrium is independent of the specific discount level.

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<sup>26</sup>Note that  $\theta_Y^{*b} + \theta_X^{*a} = \frac{\sqrt{6}}{3} \in [0, 1]$ .

As for (v), aggregate consumer surplus,  $CS_3$ , is given by

$$\begin{aligned}
CS_3 &= \int_0^{\frac{\sqrt{6}}{3}} \left( \int_0^{\frac{\sqrt{6}}{3}-\theta_Y} (s_B(\theta_X + \theta_Y) - p_X - p_Y + \gamma_B) d\theta_X \right) d\theta_Y + \\
&+ \int_{\frac{\sqrt{6}}{3}}^1 \left( \int_0^1 (s_A(\theta_X + \theta_Y) - P_X - P_Y + \gamma_A) d\theta_X \right) d\theta_Y + \\
&+ \int_0^{\frac{\sqrt{6}}{3}} \left( \int_{\frac{\sqrt{6}}{3}-\theta_Y}^1 (s_A(\theta_X + \theta_Y) - P_X - P_Y + \gamma_A) d\theta_X \right) d\theta_Y \\
&= \frac{1}{27} \left( 17\sqrt{6} + \frac{s_A}{s_B} (27 - 17\sqrt{6}) \right).
\end{aligned}$$

So,

$$CS_3 - CS_0 = \frac{1}{27} \left( 17\sqrt{6} + \frac{s_A}{s_B} (27 - 17\sqrt{6}) \right) - \left( 11 - 2\frac{s_A}{s_B} \right) / 9 = \frac{17\sqrt{6} - 33}{27} \left( 1 - \frac{s_A}{s_B} \right) < 0.$$

Finally, aggregate welfare is given by

$$W_3 = \frac{1}{27} \left( 2\sqrt{6} + \frac{s_A}{s_B} (27 - 2\sqrt{6}) \right).$$

So,

$$W_3 - W_0 = \frac{1}{27} \left( 2\sqrt{6} + \frac{s_A}{s_B} (27 - 2\sqrt{6}) \right) - (8\frac{s_A}{s_B} + 1)/9 = \frac{3 - 2\sqrt{6}}{27} \left( \frac{s_A}{s_B} - 1 \right) < 0$$

This completes the proof. ■

**Proof of Proposition 4:** The payoff matrix below, where all payoffs were multiplied by  $1000/s$ , follows directly from Propositions 1 to 3.

		Low quality firms	
		No discount	Discount
High quality firms	No discount	444.44; 111.11	435.44; 113.65
	Discount	479.60; 90.50	544.33; 136.08

It is straightforward to check that the Nash equilibrium corresponds to the scenario where the discount is introduced by both pairs of firms. ■

## Appendix B - The optimal discount funding rule

In this appendix we show that, in the unilateral scenarios, the optimal discount funding rule is when firms agree to fund the discount equally.

## Scenario 1: Unilateral bundling by the low quality firms

Equilibrium prices result from the individual maximization of the following objective functions:

$$\begin{aligned}\Pi_{A_X} &= P_X (Q_{A_X, B_Y} + Q_{A_X, A_Y}) \\ \Pi_{A_Y} &= P_Y (Q_{B_X, A_Y} + Q_{A_X, A_Y}) \\ \Pi_{B_X} &= p_X (Q_{B_X, A_Y} + Q_{B_X, B_Y}) - \gamma_B \alpha Q_{B_X, B_Y} \\ \Pi_{B_Y} &= p_Y (Q_{A_X, B_Y} + Q_{B_X, B_Y}) - \gamma_B (1 - \alpha) Q_{B_X, B_Y}\end{aligned}$$

yielding

$$\begin{aligned}P_X(\alpha, \gamma_B) &= \frac{12s^4 - \gamma_B^4 \alpha (3 - \alpha) + 2\gamma_B^3 s (1 - \alpha) (2 - \alpha) + \gamma_B^2 s^2 (2\alpha + 2\alpha^2 - 9) - 2\gamma_B s^3 (1 - \alpha)}{2s (9s^2 - 6\gamma_B^2 - \alpha\gamma_B^2 + \alpha^2\gamma_B^2)}, \\ p_X(\alpha, \gamma_B) &= \frac{6s^4 - \gamma_B^4 \alpha (3 - \alpha) - 2\gamma_B^3 s (2\alpha + 1) (2 - \alpha) + \gamma_B^2 s^2 (8\alpha + 2\alpha^2 - 3) + 2\gamma_B s^3 (2\alpha + 1)}{2s (9s^2 - 6\gamma_B^2 - \alpha\gamma_B^2 + \alpha^2\gamma_B^2)}.\end{aligned}$$

The remaining equilibrium prices can be obtained by making use of the fact that  $P_Y(\alpha, \gamma_B) = P_X(1 - \alpha, \gamma_B)$  and  $p_Y(\alpha, \gamma_B) = p_X(1 - \alpha, \gamma_B)$ .

By definition, the joint profit of the low quality producers is given by  $\Pi_B(\alpha) := \Pi_{B_X} + \Pi_{B_Y}$ . Now, maximizing  $\Pi_B(\alpha)$ , evaluated at the equilibrium prices specified above, with respect to  $\alpha$ , gives the relevant solution at  $\alpha = \frac{1}{2}$ . This is the unique solution to  $\partial\Pi_B(\alpha)/\partial\alpha = 0$  for any  $\gamma_B \in [0, 5s/9]$ . Second-order conditions are always verified in this range.

## Scenario 2: Unilateral bundling by the high quality firms

We now analyze the scenario in which  $\gamma_A > 0$  while  $\gamma_B = 0$ . In this scenario, equilibrium prices result from the maximization of following objective functions,

$$\begin{aligned}\Pi_{A_X} &= P_X (Q_{A_X, B_Y} + Q_{A_X, A_Y}) - \gamma_A \alpha Q_{A_X, A_Y} \\ \Pi_{A_Y} &= P_Y (Q_{B_X, A_Y} + Q_{A_X, A_Y}) - \gamma_A (1 - \alpha) Q_{A_X, A_Y} \\ \Pi_{B_X} &= p_X (Q_{B_X, A_Y} + Q_{B_X, B_Y}) \\ \Pi_{B_Y} &= p_Y (Q_{A_X, B_Y} + Q_{B_X, B_Y})\end{aligned}$$

yielding

$$\begin{aligned}P_X(\alpha, \gamma_A) &= \frac{(6s^3 - 4s\gamma_A^2 - s^2\gamma_A + \gamma_A\alpha(4s^2 - 3\gamma_A^2) + \gamma_A^2\alpha^2(2s + \gamma_A))(2s + \gamma_A)}{2(9s^2 - 6\gamma_A^2 - \alpha\gamma_A^2 + \alpha^2\gamma_A^2)s}, \\ p_X(\alpha, \gamma_A) &= \frac{s(6s^3 + 4\gamma_A^3 - 3s\gamma_A^2 - 4s^2\gamma_A) + \gamma_A\alpha(4s^3 - 3\gamma_A^3 - 6s\gamma_A^2) + \gamma_A^2\alpha^2(2s\gamma_A + 2s^2 + \gamma_A^2)}{2(9s^2 - 6\gamma_A^2 - \alpha\gamma_A^2 + \alpha^2\gamma_A^2)s},\end{aligned}$$



where, as before, the remaining two equilibrium prices can be easily obtained from  $P_Y(\alpha, \gamma_A) = P_X(1 - \alpha, \gamma_A)$  and  $p_Y(\alpha, \gamma_A) = p_X(1 - \alpha, \gamma_A)$ .

By definition, the joint profits of the high quality firms are  $\Pi_A(\alpha) := \Pi_{A_X} + \Pi_{A_Y}$ . By maximizing  $\Pi_A(\alpha)$ , evaluated at the equilibrium prices above, with respect to  $\alpha$ , one obtains the relevant solution at  $\alpha = \frac{1}{2}$ . This is the unique solution to  $\partial \Pi_A(\alpha) / \partial \alpha = 0$  for any  $\gamma_A \in [0, \frac{5}{9}s]$ . Second-order conditions are always verified in this range.

## Appendix C

In this appendix we address the cases left out of the analysis in Section 5. We also present the expression for the equilibrium consumer surplus and welfare for the case in which valuations are positively correlated.

Assume first that  $\theta_Y^{*a} > \theta_X^{*b} \Leftrightarrow \frac{P_Y - p_Y - \gamma_A}{s} > \frac{P_X - p_X + \gamma_B}{s}$ . Then

$$\begin{aligned} Q_{A_X, B_Y} &= \frac{P_Y - p_Y - \gamma_A}{s} - \frac{P_X - p_X + \gamma_B}{s}, \quad Q_{B_X, A_Y} = 0 \\ Q_{A_X, A_Y} &= \left(1 - \frac{P_Y - p_Y - \gamma_A}{s}\right), \quad Q_{B_X, B_Y} = \frac{P_X - p_X + \gamma_B}{s} \end{aligned}$$

Equilibrium prices result from the maximization of the following objective functions:

$$\begin{aligned} \Pi_{A_X} &= P_X (Q_{A_X, B_Y} + Q_{A_X, A_Y}) - \gamma_A \alpha_A Q_{A_X, A_Y} \\ \Pi_{A_Y} &= P_Y (Q_{A_X, A_Y}) - \gamma_A (1 - \alpha_A) Q_{A_X, A_Y} \\ \Pi_{B_X} &= p_X (Q_{B_X, B_Y}) - \gamma_B \alpha_B Q_{B_X, B_Y} \\ \Pi_{B_Y} &= p_Y (Q_{A_X, B_Y} + Q_{B_X, B_Y}) - \gamma_B (1 - \alpha_B) Q_{B_X, B_Y} \end{aligned}$$

yielding

$$\begin{aligned} P_X &= \left(\frac{2}{3}s - \frac{1}{3}\gamma_B + \frac{1}{3}\alpha_B\gamma_B\right) \text{ and } P_Y = \left(\frac{2}{3}s + \gamma_A - \frac{2}{3}\alpha_A\gamma_A\right) \\ p_X &= \left(\frac{1}{3}s + \frac{1}{3}\gamma_B + \frac{2}{3}\alpha_B\gamma_B\right) \text{ and } p_Y = \left(\frac{1}{3}s - \frac{1}{3}\alpha_A\gamma_A\right) \end{aligned}$$

Note that it is impossible to have  $\theta_Y^{*a} > \theta_X^{*b} \Leftrightarrow \frac{P_Y - p_Y - \gamma_A}{s} > \frac{P_X - p_X + \gamma_B}{s} \Leftrightarrow -\gamma_B(1 - \alpha_B) - \alpha_A\gamma_A > 0$ .

Assume now that  $\theta_Y^{*b} < \theta_X^{*a} \Leftrightarrow \frac{P_Y - p_Y + \gamma_B}{s} < \frac{P_X - p_X - \gamma_A}{s}$ . Then,

$$\begin{aligned} Q_{A_X, B_Y} &= 0 \\ Q_{B_X, A_Y} &= \frac{P_X - p_X - \gamma_A}{s} - \frac{P_Y - p_Y + \gamma_B}{s} \\ Q_{A_X, A_Y} &= 1 - \frac{P_X - p_X - \gamma_A}{s} \\ Q_{B_X, B_Y} &= \frac{P_Y - p_Y + \gamma_B}{s} \end{aligned}$$

Equilibrium prices result from the maximization of the following objective functions:

$$\begin{aligned} \Pi_{A_X} &= P_X (Q_{A_X, A_Y}) - \gamma_A \alpha_A Q_{A_X, A_Y} \\ \Pi_{A_Y} &= P_Y (Q_{A_X, A_Y} + Q_{B_X, A_Y}) - \gamma_A (1 - \alpha_A) Q_{A_X, A_Y} \\ \Pi_{B_X} &= p_X (Q_{B_X, B_Y} + Q_{B_X, A_Y}) - \gamma_B \alpha_B Q_{B_X, B_Y} \\ \Pi_{B_Y} &= p_Y (Q_{B_X, B_Y}) - \gamma_B (1 - \alpha_B) Q_{B_X, B_Y} \end{aligned}$$

yielding

$$\begin{aligned} P_X &= \left( \frac{2}{3}s + \frac{1}{3}\gamma_A + \frac{2}{3}\alpha_A\gamma_A \right) \text{ and } P_Y = \left( \frac{2}{3}s - \frac{1}{3}\alpha_B\gamma_B \right) \\ p_X &= \left( \frac{1}{3}s - \frac{1}{3}\gamma_A + \frac{1}{3}\alpha_A\gamma_A \right) \text{ and } p_Y = \left( \frac{1}{3}s + \gamma_B - \frac{2}{3}\alpha_B\gamma_B \right) \end{aligned}$$

Note that it is impossible to have  $\theta_Y^{*b} < \theta_X^{*a} \Leftrightarrow \frac{P_Y - p_Y + \gamma_B}{s} < \frac{P_X - p_X - \gamma_A}{s} \Leftrightarrow (1 - \alpha_A) \gamma_A + \alpha_B \gamma_B < 0$ .

The equilibrium consumer surplus and welfare expressions are

$$\begin{aligned} CS &= \int_0^{\frac{2}{5}} \left( \theta 2s_B - \frac{8}{5}s \right) d\theta + \int_{\frac{2}{5}}^1 \left( \theta 2s_A - \frac{12}{5}s \right) d\theta = \frac{56 - 31 \frac{s_A}{s_B}}{25}, \\ W &= \int_0^{\frac{2}{5}} (\theta 2s_B) d\theta + \int_{\frac{2}{5}}^1 (\theta 2s_A) d\theta = \frac{21 \frac{s_A}{s_B} + 4}{25}. \end{aligned}$$

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